



## Analysis of the activated sludge model (number 1)

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### ABSTRACT

We analyse a model for the activated sludge process occurring in a biological reactor without recycle. The biochemical processes occurring within the reactor are represented by the activated sludge model number 1 (ASM1). In the past the ASM1 model has been investigated via direct integration of the governing equations. This approach is time-consuming as parameter regions of interest (in terms of the effluent quality leaving the plant) can only be determined through laborious and repetitive calculations. In this work we use continuation methods to determine the steady-state behaviour of the system. In particular, we determine bifurcation values of the residence time, corresponding to branch points, that are crucial in determining the performance of the plant.

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### 1. Introduction

The activated sludge process is widely used in wastewater treatment plants to reduce effluent levels in contaminated wastewaters originating from both the municipal and industrial sectors. The process generally occurs in two units: an aerated biological reactor, in which bacteria are used to degrade pollutants, and a settling unit (or clarifier), in which the activated sludge settles to the bottom of the unit. Activated sludge, along with mixed liquor, is recycled from the bottom of the clarifier into the biological reactor.

In this work we investigate the processing of contaminated wastewaters in a biological reactor without recycle. A clear understanding of process behaviour in the absence of recycle is required to appreciate the effect that recycle has on operating characteristics of the process. The results presented in this work therefore provide a benchmark for such an investigation. The biochemical processes occurring within the reactor are modelled using the activated sludge model number 1 (ASM1) that was developed by Henze et al. [1]. This model is an internationally accepted standard for activated sludge modeling. It describes nitrogen and chemical oxygen demand within suspended-growth treatment processes, including mechanisms for nitrification and denitrification. The model has been found to give a good description of the activated sludge process provided that the wastewater has been characterised in detail and is of domestic or municipal, but not industrial, in origin.

The ASM1 model includes eight processes that are fundamental to the activated sludge process. These are: aerobic and anoxic growth of heterotrophic biomass, death of heterotrophic biomass, aerobic growth of autotrophic biomass, decay of autotrophic biomass, ammonification of soluble organic nitrogen and hydrolysis of both entrapped particulate organic matter and entrapped organic nitrogen.

Earlier studies have only investigated the model using direct integration of the governing equations. Such an approach is time-consuming as parameter regions of interest (in terms of the effluent quality leaving the bioreactor) can only be determined through laborious and repetitive simulations. In this work, we utilise continuation methods to determine how the control parameters, such as the residence time ( $\tau$ ) and the oxygen transfer coefficient ( $K_{L,A}$ ), affect the performance of the reactor.

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## 2. Model equations

Following Henze et al. [1] we assume that the reactor is well mixed and that the hydrolysis rate of soluble biomass obtained from the wastewater is the same as the hydrolysis rate of biomass obtained from degradation of biomass. The ASM1 model contains 13 differential equations. However, four equations, pertaining to inert soluble organic material, particulate inert organic material, non-biodegradable particulate products arising from biomass decay and alkalinity, can be shown to be uncoupled from the remaining nine equations, and therefore do not affect the dynamics of the system. Therefore, we consider a reduced system of nine ordinary differential equations, similar to the systems studied by Fikar et al. [2] and Yoon and Lee [3]:

Readily biodegradable soluble substrate

$$\frac{dS_S}{dt} = \frac{q}{V} (S_{S,in} - S_S) - \frac{1}{Y_H} \cdot \mu_{\max,H} \cdot M_2 \cdot (M_{8h} + I_8 \cdot M_9 \cdot \eta_g) \cdot X_{B,H} + k_h \cdot k_{\text{sat}} (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H}. \quad (1)$$

Slowly biodegradable particulate substrate

$$\frac{dX_S}{dt} = \frac{q}{V} (X_{S,in} - X_S) + \frac{q_r}{V} (b - 1) X_S + (1 - f_p) (b_H X_{B,H} + b_A X_{B,A}) - k_h \cdot k_{\text{sat}} (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H}. \quad (2)$$

Active heterotrophic particulate biomass

$$\begin{aligned} \frac{dX_{B,H}}{dt} = & \frac{q}{V} (X_{B,H,in} - X_{B,H}) + \frac{q_r}{V} (b - 1) X_{B,H} + \mu_{\max,H} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & + \mu_{\max,H} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} - b_H \cdot X_{B,H}. \end{aligned} \quad (3)$$

Active autotrophic particulate biomass

$$\frac{dX_{B,A}}{dt} = \frac{q}{V} (X_{B,A,in} - X_{B,A}) + \frac{q_r}{V} (b - 1) X_{B,A} + \mu_{\max,A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A} - b_A \cdot X_{B,A}. \quad (4)$$

Soluble oxygen

$$\begin{aligned} \frac{dS_O}{dt} = & \frac{q}{V} (S_{O,in} - S_O) + K_{L,A} (S_{O,\max} - S_O) - \frac{(1 - Y_H)}{Y_H} \cdot \mu_{\max,H} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & - \frac{(4.57 - Y_A)}{Y_A} \cdot \mu_{\max,A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A}. \end{aligned} \quad (5)$$

Soluble nitrate and nitrite nitrogen

$$\frac{dS_{NO}}{dt} = \frac{q}{V} (S_{NO,in} - S_{NO}) - \frac{(1 - Y_H)}{2.86 Y_H} \cdot \mu_{\max,H} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} + \frac{1}{Y_A} \cdot \mu_{\max,A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A}. \quad (6)$$

Soluble ammonium ( $\text{NH}_4^+$  and NH) nitrogen

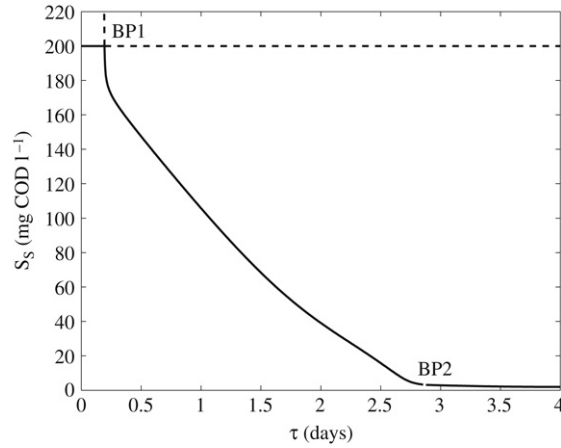
$$\begin{aligned} \frac{dS_{NH}}{dt} = & \frac{q}{V} (S_{NH,in} - S_{NH}) - i_{XB} \mu_{\max,H} \cdot M_2 \cdot (M_{8h} + I_8 \cdot M_9 \cdot \eta_g) X_{B,H} \\ & - \left( i_{XB} + \frac{1}{Y_A} \right) \mu_{\max,A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A} + k_A \cdot S_{ND} \cdot X_{B,H}. \end{aligned} \quad (7)$$

Soluble biodegradable organic nitrogen

$$\frac{dS_{ND}}{dt} = \frac{q}{V} (S_{ND,in} - S_{ND}) - k_A \cdot S_{ND} \cdot X_{B,H} + k_h \cdot k_{\text{sat}} (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H} \frac{X_{ND}}{X_S}. \quad (8)$$

Particulate biodegradable organic nitrogen

$$\begin{aligned} \frac{dX_{ND}}{dt} = & \frac{q}{V} (X_{ND,in} - X_{ND}) + \frac{q_r}{V} (b - 1) X_{ND} + (i_{XB} - f_p \cdot i_{XP}) (b_H \cdot X_{B,H} + b_A \cdot X_{B,A}) \\ & - k_h \cdot k_{\text{sat}} \cdot (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H} \cdot \frac{X_{ND}}{X_S}. \end{aligned} \quad (9)$$



**Fig. 1.** The dependence of the biodegradable soluble substrate upon the residence time in the reactor. The points labelled BP1 and BP2 represent the branch points of the system.

The corresponding reaction rates are

$$M_2 = \frac{S_S}{K_S + S_S}, \quad (10)$$

$$M_{8a} = \frac{S_O}{K_{O,A} + S_O}, \quad (11)$$

$$M_{8h} = \frac{S_O}{K_{O,H} + S_O}, \quad (12)$$

$$M_9 = \frac{S_{NO}}{K_{NO} + S_{NO}}, \quad (13)$$

$$M_{10} = \frac{S_{NH}}{K_{NH} + S_{NH}}, \quad (14)$$

$$I_8 = \frac{K_{O,H}}{K_{O,H} + S_O}, \quad (15)$$

$$k_{\text{sat}} = \frac{X_S}{K_X X_{B,H} + X_S}. \quad (16)$$

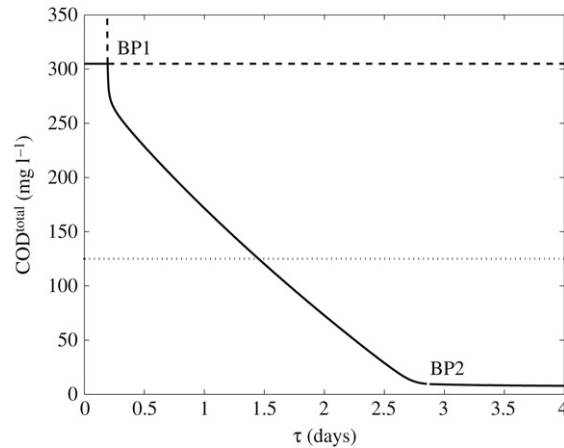
The terms appearing in Eqs. (1)–(16) are defined in the nomenclature. The numerical constants, 2.86 and 4.57, are required to convert the concentrations of nitrate nitrogen and ammonia nitrogen into units of COD. The parameter values used in this study are based on those given by Henze et al. [1], Fikar et al. [2] and Yoon & Lee [3]. In the following, we take the residence time ( $\tau$ ) as the primary bifurcation parameter.

### 3. Results

The path following software program Auto 97 [4] was used to obtain steady-state diagrams. In these the standard representation is used: solid lines are stable steady states; dotted lines are unstable steady states.

#### 3.1. Steady-state analysis

Fig. 1 shows a steady-state diagram for the readily biodegradable soluble substrate, determined for the parameter values given in the nomenclature. (Steady-state diagrams were obtained for all nine components of the model; for brevity only one is shown). Unstable steady-state solutions other than the washout solution are not shown. (Washout occurs when the influent and effluent are identical. This happens when the biological reactor cannot support any microorganisms). Two features of this diagram are of immediate practical interest: the branch points, labelled BP1 and BP2. The first branch point BP1, at a residence time of 0.19 days, determines the condition under which the washout solution loses stability. Thus the residence time must be greater than this value to prevent washout occurring in the system. Between the first and second branch points, at a residence time of 3.29 days, the value of the biodegradable soluble substrate decreases steadily with increasing residence time: from 200 mg COD l<sup>-1</sup> along the washout solution to 3.15 mg COD l<sup>-1</sup> at the second branch point.



**Fig. 2.** The dependence of the total chemical oxygen demand in the reactor ( $\text{COD}_{\text{total}}$ ) upon the residence time ( $\tau$ ). The dotted line  $\text{COD}_{\text{total}} = 125 \text{ mg l}^{-1}$  is the legislative performance requirement.

This represents around only 1.6% of the concentration entering the reactor. After the second branch point BP2, the soluble substrate concentration decreases marginally with increasing values of residence times. For example, if the residence time is doubled, from the value at the second branch point, to 6.58 days the soluble substrate concentrations decreases only to  $1.38 \text{ mg COD l}^{-1}$ .

### 3.2. Performance

The main aim of wastewater treatment plants is to improve the effluent quality. The allowable concentrations of organic, nitrogen, phosphorus and other pollutants leaving a wastewater treatment plant are increasingly specified by legislation. For example, European Union directive 91/271 ('Urban wastewater) specified the maximum concentrations of chemical oxygen demand (COD), biological oxygen demand (BOD), suspended solids (SS), and total nitrogen (TN) leaving a small sized wastewater treatment plant as:  $\text{COD}_{\text{max}} = 125 \text{ mg l}^{-1}$ ,  $\text{BOD}_{\text{max}} = 25 \text{ mg l}^{-1}$ ,  $S_{s,\text{max}} = 35 \text{ mg l}^{-1}$  and  $\text{TN}_{\text{max}} = 10 \text{ mg l}^{-1}$  [2]. In this study, we quantify effluent quality using chemical oxygen demand (COD) as the performance index. In other words, we want to determine the residence time such that the total chemical oxygen demand  $\text{COD}_{\text{total}} \leq 125 \text{ mg l}^{-1}$ . The  $\text{COD}_{\text{total}}$  is given by [1]

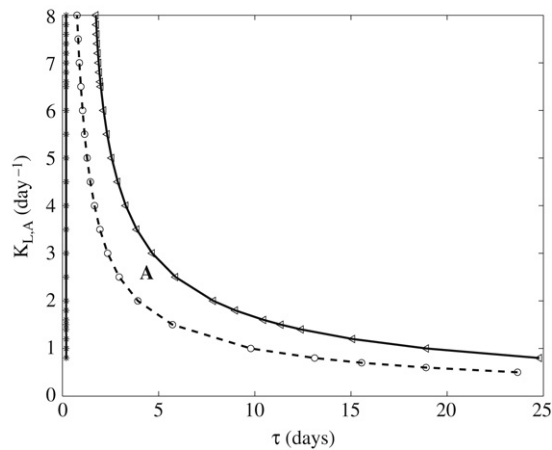
$$\text{COD}_{\text{total}} = S_s + S_i + X_s + X_i \quad (17)$$

where the steady-state values for the inert soluble organic material ( $S_i$ ) and the particulate inert organic matter ( $X_i$ ) are given by the corresponding values of the influent, i.e.  $S_{i,\text{in}}$  and  $X_{i,\text{in}}$  respectively. These values are given in the Appendix.

Fig. 2 shows the steady-state value of the chemical oxygen demand as a function of the residence time. The COD decreases steadily as the residence time is increased from the value at the first branch point, 0.19 days, towards the value at the second branch point, 3.29 days. However, the decrease is only marginal for values of the residence time greater than that at the second branch point. The dotted horizontal line gives  $\text{COD}_{\text{total}} = 125 \text{ mg l}^{-1}$ . Hence to satisfy the legislative requirement the residence time must be chosen such that total COD is below this dotted line. The residence time at which  $\text{COD}_{\text{total}} = 125 \text{ mg l}^{-1}$  is 1.67 days. Thus, in order to satisfy the relevant legislation for COD, the biological reactor must be operated at a residence time higher than 1.67 days. At the second branch point BP2 the value of COD is  $9.38 \text{ mg l}^{-1}$ . Thus operating the reactor at a residence time of 3.29 days ensures that the value of COD in the effluent is only around 7.5% of the legislative requirement. If the residence time is doubled from the value at BP2 to 6.58 days, then the COD decreases only to  $6.92 \text{ mg l}^{-1}$ .

From our previous discussion, it is clear that the locations of the two branch points BP1 and BP2 are of considerable practical importance. Firstly, the reactor must be operated above the residence time at which the washout solution loses stability (BP1). Secondly, although the system performance always increases with residence time, the improvement in reactor performance with residence time is much more marginal after the second branch point (BP2). If the reactor performance is to be significantly improved over that obtained at the second branch point then either very large residence times or a different reactor configuration, such as a cascade, must be used.

The control of activated sludge reactors is often based on maintaining a constant concentration of dissolved oxygen within the reactor. Therefore it is sometimes assumed that the dissolved oxygen concentration can be treated as an adjustable parameter. This reduces the complexity of the model as the differential equation for oxygen, Eq. (5), can be removed from the model and the reaction rates in Eqs. (11), (12) and (15) reduce to constants. This approach was used in, for example, [3]. However, Henze [5] considers that the assumption that there is a constant oxygen concentration simplifies the model so much that the simultaneous nitrification–denitrification processes are no longer adequately modelled. Henze comments



**Fig. 3.** Classification of reactor performance as a function of the oxygen transfer coefficient. The asterisks denotes branch points at which the washout solution changes stability, whereas the line through the triangles represents branch points at which increasing the residence times only increases the performance marginally. The dashed line through the circles is the value of the residence time at which legislative requirements are met. Ideal residence time in terms of performance is denoted by region A.

that “the  $K_{L,A}$  model is recommended for professional modeling” [5, page 419]. We now investigate how the performance of the reactor depends upon the choice of the oxygen transfer coefficient ( $K_{L,A}$ ).

The value of the residence time corresponding to the branch points (BP1 and BP2) and the value at which  $\text{COD}^{\text{total}} = 125 \text{ mg l}^{-1}$  are shown in Fig. 3, as a function of the oxygen transfer coefficient. The residence time must be chosen in the region to the right of the dashed line through the circles. However our earlier analyses show that after the second branch point, denoted by the line through the triangles, the reduction is marginal for increasing residence times. Hence from the operational point of view, residence times in region A would be ideal.

#### 4. Conclusions

We have investigated the behaviour of the ASM1 model in a biological reactor without recycle. We have shown that the steady-state diagram contains two branch points. The branch point at the lower residence time marks a transition at which the washout solution becomes unstable. The branch point at the higher residence time marks a transition above which the improvement in performance with increasing residence time dramatically slows down.

We are currently completing our analysis of the single reactor activated sludge process by including recycle. In future work we intend investigating the improvements in effluent quality that result from using a cascade of reactors. The results obtained in this work provide a benchmark for evaluating the performance of both a single reactor with recycle and a cascade of reactors.

Two extensions of the ASM1 model have been developed. The ASM2 model incorporates biological uptake of phosphorus [6]. The ASM3 model provides an alternative description for heterotrophic bacteria that is appropriate for reactors operating near starvation conditions, under which heterotrophic bacteria provide an alternative source of organic material [7]. The results of the present investigation suggest that, in order to fully understand the dynamics of these systems, as well as to determine optimal operating conditions, it will be useful to apply continuation methods.

#### Acknowledgment

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#### Appendix. Nomenclature

The notation  $X_{\text{in}}$  is used to denote the concentration of component  $X$  in the influent.

BOD	Biological oxygen demand	( $\text{mg l}^{-1}$ )
COD	Chemical oxygen demand	( $\text{mg l}^{-1}$ )
$I_g$	Inhibition kinetics for soluble oxygen	(–)
$K_{L,A}$	Oxygen transfer coefficient	( $\text{day}^{-1}$ )
$K_{\text{NH}}$	Ammonia half-saturation coefficient for autotrophic biomass	( $\text{mg N l}^{-1}$ )
$K_{\text{NO}}$	Nitrate half-saturation coefficient for denitrifying heterotrophic biomass	( $\text{mg N l}^{-1}$ )
$K_{\text{O,A}}$	Oxygen half-saturation coefficient for autotrophic biomass	( $\text{mg O}_2 \text{ l}^{-1}$ )

$K_{O,H}$	Oxygen half-saturation coefficient for heterotrophic biomass	(mg O <sub>2</sub> l <sup>-1</sup> )
$K_S$	Substrate half-saturation coefficient for heterotrophic biomass	(mg COD l <sup>-1</sup> )
$K_X$	Half-saturation coefficient for hydrolysis of particulate biodegradable substrate	(-)
$M_2$	Monod kinetics for readily biodegradable soluble substrate	(-)
$M_{8a}$	Monod kinetics for the component $S_O$ with respect to autotrophic biomass	(-)
$M_{8h}$	Monod kinetics for the component $S_O$ with respect to heterotrophic biomass	(-)
$M_9$	Monod kinetics for soluble nitrate and nitrite nitrogen	(-)
$M_{10}$	Monod kinetics for soluble ammonium nitrogen	(-)
$S_I$	Concentration of inert soluble organic material	(mg COD l <sup>-1</sup> )
$S_{ND}$	Concentration of soluble biodegradable organic material	(mg NI l <sup>-1</sup> )
$S_{NH}$	Concentration of soluble ammonium nitrogen	(mg NI l <sup>-1</sup> )
$S_{NO}$	Concentration of soluble nitrate and nitrite nitrogen	(mg NI l <sup>-1</sup> )
$S_O$	Concentration of soluble oxygen	(mg l <sup>-1</sup> )
$S_{O,max}$	Maximum concentration of soluble oxygen	(mg l <sup>-1</sup> )
$S_S$	Concentration of readily biodegradable soluble substrate	(mg COD l <sup>-1</sup> )
SS	Suspended solids	(mg l <sup>-1</sup> )
TN	Total nitrogen	(mg l <sup>-1</sup> )
V	Reactor volume	(l)
$X_{B,A}$	Concentration of active autotrophic particulate mass	(mg COD l <sup>-1</sup> )
$X_{B,H}$	Concentration of active heterotrophic particulate mass	(mg COD l <sup>-1</sup> )
$X_{ND}$	Concentration of particulate biodegradable organic nitrogen	(mg COD l <sup>-1</sup> )
$X_I$	Concentration of particulate inert organic matter	(mg l <sup>-1</sup> )
$X_P$	Concentration of non-biodegradable particulate product arising from biomass decay	(mg COD l <sup>-1</sup> )
$X_S$	Concentration of slowly biodegradable particulate substrate	(mg COD l <sup>-1</sup> )
$Y_A$	Autotrophic yield coefficient	(g COD (g N) <sup>-1</sup> )
$Y_H$	Heterotrophic yield coefficient	(g COD (g COD) <sup>-1</sup> )
$b_A$	Autotrophic decay coefficient	(day <sup>-1</sup> )
$b_H$	Heterotrophic decay coefficient	(day <sup>-1</sup> )
$f_p$	Fraction of biomass yielding particulate products	(-)
$i_{XB}$	Nitrogen content in biomass	(mg N (mg SS) <sup>-1</sup> )
$i_{XP}$	Nitrogen content in inert particulate	(mg N (mg SS) <sup>-1</sup> )
$k_A$	Ammonification coefficient	(l mg (COD day) <sup>-1</sup> )
$k_h$	Hydrolysis coefficient	(day <sup>-1</sup> )
$q$	Feed flow rate	(day <sup>-1</sup> )
$k_{sat}$	Saturation kinetics	(-)
$t$	Time	(day)
$\eta_g$	Correction factor for anoxic growth of heterotrophs	(-)
$\eta_h$	Correction factor for anoxic hydrolysis	(-)
$\mu_{max,A}$	Maximum specific growth rate for autotrophs	(day <sup>-1</sup> )
$\mu_{max,H}$	Maximum specific growth rate for heterotrophs	(day <sup>-1</sup> )
$\tau$	Residence time ( $\tau = \frac{V}{q}$ )	(day)

Unless otherwise specified, we take the following typical parameter values. These values are based upon those used in [3], except for the values denoted by †.  $K_{L,A}^\dagger = 4 \text{ day}^{-1}$ ,  $K_{NH} = 1 \text{ mg N l}^{-1}$ ,  $K_{NO} = 0.5 \text{ mg NI l}^{-1}$ ,  $K_{O,A} = 0.4 \text{ mg O}_2 \text{ l}^{-1}$ ,  $K_{O,H} = 0.2 \text{ mg O}_2 \text{ l}^{-1}$ ,  $K_S = 20 \text{ mg COD l}^{-1}$ ,  $K_X = 0.03 (-)$ ,  $S_{I,in}^\dagger = 2 \text{ mg COD l}^{-1}$ ,  $S_{ND,in}^\dagger = 9 \text{ mg NI l}^{-1}$ ,  $S_{NH,in} = 15 \text{ mg NI l}^{-1}$ ,  $S_{NO,in} = 1 \text{ mg NI l}^{-1}$ ,  $S_{O,in}^\dagger = 2 \text{ mg l}^{-1}$ ,  $S_{O,max} = 10 \text{ mg l}^{-1}$ ,  $S_{S,in} = 200 \text{ mg COD l}^{-1}$ ,  $X_{B,A,in} = 0 \text{ mg COD l}^{-1}$ ,  $X_{B,H,in} = 0 \text{ mg COD l}^{-1}$ ,  $X_{ND,in}^\dagger = 0 \text{ mg COD l}^{-1}$ ,  $X_{I,in} = 3.0 \text{ mg l}^{-1}$ ,  $X_{P,in} = 0 \text{ mg COD l}^{-1}$ ,  $S_{S,in} = 100 \text{ mg COD l}^{-1}$ ,  $Y_A = 0.24 \text{ g COD (g N)}^{-1}$ ,  $Y_H = 0.67 \text{ g COD (g COD)}^{-1}$ ,  $b_A = 0.05 \text{ day}^{-1}$ ,  $b_H = 0.22 \text{ day}^{-1}$ ,  $f_p = 0.08 (-)$ ,  $i_{XB} = 0.086 \text{ mg N (mg SS)}^{-1}$ ,  $i_{XP} = 0.06 \text{ mg N (mg SS)}^{-1}$ ,  $k_A = 0.08 \text{ l mg (COD day)}^{-1}$ ,  $k_h = 3.0 \text{ day}^{-1}$ ,  $\eta_g = 0.8 (-)$ ,  $\eta_h = 0.4 (-)$ ,  $\mu_{max,A} = 0.8 \text{ day}^{-1}$ ,  $\mu_{max,H} = 6 \text{ day}^{-1}$ .

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