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Failure Assessment of Cracked Pipes Based on Failure Assessment Diagram Considering Random and Fuzzy Uncertainties

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Abstract

Defects of pressure piping, especially cracks, are inevitable during the lifetime of pipe, which will result in the pipe failure. In order to ensure safe operation of pipe, failure assessment for the pressure pipe containing defects needs to be implemented. In fact, the assessment parameters would show two types of uncertainty: randomness and fuzziness, because of the errors of inspection and lack of precise information. In this paper, a fuzzy-probabilistic failure assessment method has been developed based on Failure Assessment Diagram (FAD) from API 579-1/ASME FFS-1 procedure. This method combines the probability theory with possibility theory to deal with fuzzy and random variables. Also, Monte Carlo simulation is used to calculate the failure probability of pipe. This method was applied to a pipe with a surface crack. The effects of variables and their uncertainties on failure probability were analyzed. The results show that the wall thickness and diameter of the pipe has the most important contribution to the pipe failure, followed by the fracture toughness, the yield strength, the operating pressure, the depth and length of the crack.

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Keywords: pressure piping; randomness and fuzziness; failure assessment diagram; Monte-Carlo method

1. Introduction

Pressure piping has been the major way to transport energy source due to its advantages of low cost and large transmission capacity. But defects, especially cracks, are inevitable during the lifetime of pipe, which affect the long-term reliability and integrity of the pipe. For ensuring safe operation of pipe, failure assessment of pressure

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pipe containing defects need to be implemented. There have been many assessment standards to assess the safe condition of in-service equipment, such as such as the API 579-1/ASME FFS-1 [1], the FITNET FFS [2], the R6 [3], the BS 7910 [4], the SINTAP [5-6].

In order to carry out failure assessment the above standards require parameters about defect geometry, wall thickness and diameter of pipe, mechanical properties of pipe materials, and actual operating pressure and temperature. These data are generally derived from design documents, operating, inspection and maintenance records. Unfortunately, some parameters are always inflicted with uncertainty (i.e. incompleteness and impreciseness) because of the random nature of the variables, the errors of inspection and the complexity of the operating conditions. Generally, the data show two types of uncertainty: randomness and fuzziness. There are many different approaches to uncertainty analysis: classical statistic, probabilistic, and sensitivity analysis [7-10]. However, these approaches cannot effectively solve fuzzy uncertainty, which will induce the unreasonable assessment results.

During the last few years researchers have introduced fuzzy set theory or other relevant theories into failure assessment. Zhou [11] proposed a method to compute the fuzzy failure probability of pressure piping based on fuzzy sets. Anoop et al. [12] assessed the safe condition of austenitic steel nuclear power plant pipeline against SCC in the presence of fuzzy and random uncertainties by using a new approach combining the vertex method with Monte Carlo simulation. Singh and Markeset [13] examined two different ways of combining probabilistic and possibilistic approaches for evaluating the integrity of corroded pipes.

In this study, a fuzzy-probabilistic failure assessment method based on API 579-1/ASME FFS-1 is presented to predict the failure probability of pipes with crack by using the Monte Carlo simulation. This method combines the probability theory with possibility theory to deal with fuzzy and random variables. The effects of variables and their uncertainties on failure probability were analyzed to identify the most important parameters affecting the safety of cracked pipe.

2. How to deal with hybrid uncertainties?

2.1. Random uncertainty and fuzzy uncertainty

In practice, two types of uncertainty, randomness and fuzziness, both exist in many cases. Some parameters have the nature of randomness, and others fuzziness. It is traditional to deal with random uncertainty through the use of probability theory, especially Monte Carlo simulation. Similarly, fuzzy uncertainty is handled by means of fuzzy set theory. If we could proposed a new method, combining the probabilistic and possibilistic approaches, would be better than either of the two above mentioned approaches. This work attempts to develop such a method according to the relationship between probability theory and possibility theory.

2.2. Brief introduction of possibility theory

Possibility theory, originally proposed by Zadeh in 1978 [14] and then further developed by Dubois and Prade [15-17], is considered as the extension and supplement of fuzzy set theory, and it can effectively deal with uncertain and imprecise information.

Probability theory uses a single probability measure (i.e. probability value, $P(A)$) to describe an event A, while possibility theory uses two measures: possibility measure (Π) and necessity measure (N), which are defined:

$$\Pi(A) = \sup_{\omega \in A} \pi(\omega) \tag{1}$$

$$N(A) = \inf_{\omega \in A} (1 - \pi(\omega)) \tag{2}$$

where $\pi(\omega)$ denotes possibility distribution function; it can be effectively represented by means of a fuzzy Q whose membership function is $\mu_Q(\omega) = \pi(\omega)$ [14].

2.3. The relationship between possibility variable and probability variable

Although probability theory and possibility theory differ in format, there is certain relation between them. Zadeh [14] pointed out that the probability measure of an event is less than or equal to its possibility measure, and greater than or equal to its necessity measure. Thus, an equivalence class P of probability measures P compatible with the available data can be defined:

$$\mathrm{P} = \{P \mid \forall A, N(A) \le P(A) \le \Pi(A)\} \tag{3}$$

According to the above formula, a procedure has been proposed by Savoia [18] to connect between fuzzy variable and random variable. Consider a fuzzy set Q with membership function $\mu_Q(x)$. Assuming $A = (-\infty, x]$ in Eq. (3), the following cumulative distribution functions (CFDs) are defined:

$$F_{fl}(x) = N((-\infty, x]) = N(A) \tag{4}$$

$$F_{fu}(x) = \Pi((-\infty, x]) = \Pi(A) \tag{5}$$

where $F_{fl}(x)$ and $F_{fu}(x)$ denotes the lower and upper bounds for all probability measures P belonging to class P, compatible with available data. That is,

$$F_{fl}(x) \le F(x) \le F_{fu}(x), \text{ where } F(x) = P((-\infty, x]) \tag{6}$$

Using Eqs. (4) and (5), the lower and upper bounds $F_{fl}(x)$ and $F_{fu}(x)$ may be rewritten in terms of the membership function of the fuzzy set Q as

$$F_{fu}(x) = \sup\{\mu_Q(\omega) \mid \omega \le x\} \tag{7}$$

$$F_{fl}(x) = \inf\{1 - \mu_Q(\omega) \mid \omega > x\} \tag{8}$$

According to Eqs. (7) and (8), the relationship amongst $F_{fl}(x)$, $F_{fu}(x)$ and $\mu_Q(\omega)$ is clearly shown in Fig. 1.

Based on the above analysis, we can convert one fuzzy variable to two random variables which obey $F_{fl}(x)$ and $F_{fu}(x)$, respectively. That means we can use probability theory to calculate failure possibility of cracked pipe considering random and fuzzy uncertainties.

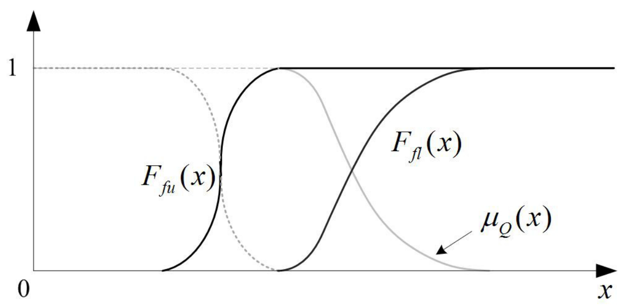

Fig. 1. The relationship amongst $F_{fl}(x)$, $F_{fu}(x)$ and $\mu_Q(\omega)$.

3. Proposed procedure of failure assessment of cracked pipe

In this section a fuzzy-probabilistic failure assessment method for cracked pipe was proposed based on Failure Assessment Diagram (FAD) from API 579-1/ASME FFS-1 procedure. This method combines the probability theory with possibility theory to deal with fuzzy and random uncertainties. The specific procedure is as follows.

A limit state function g(X) of a pressure pipe with cracks is defined as

$$g(X) = f(L_r) - K_r \tag{9}$$

$$f(L_r) = \begin{cases} \left(1 - 0.14L_r^2\right)\left[0.3 + 0.7\exp\left(-0.65L_r^6\right)\right], & L_r \le L_r^{\max} \\ 0, & L_r > L_r^{\max} \end{cases} \tag{10}$$

where $f(L_r)$ is the failure assessment curve (see Fig. 2). If the assessment point lies below the curve, that means $g(X)>0$, the pipe is safe. If the assessment point lies on or above the curve, $g(X)\le 0$, failure occurs. The failure probability of the pipe can be expressed as:

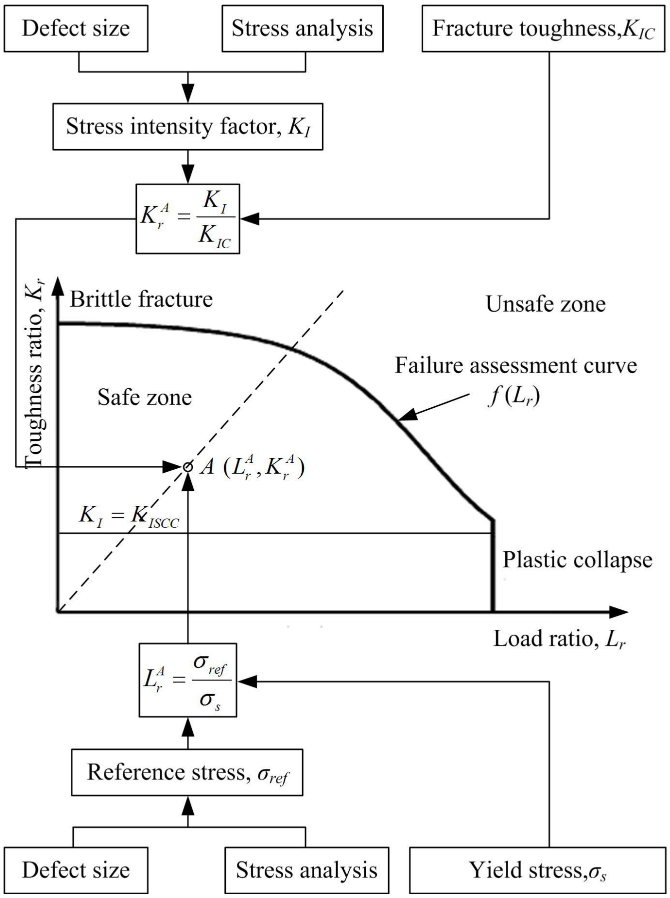

Fig. 2. Failure Assessment Diagram (FAD).

$$P_f = P(g(X) \le 0) \tag{11}$$

According to the assessment procedure of crack-like flaws in the API 579-1/ASME FFS-1 standard and the reliability theory, the failure probability can be written as:

$$P_f = \iint_{g(X)\le 0} P(L_r, K_r) dK_r dL_r \tag{12}$$

where $P(L_r, K_r)$ denotes the joint probability density function of L_r and K_r. Because $P(L_r, K_r)$ is too complicated to represent explicitly, Eq. (11) is hardly calculated by analytical methods. Therefore, the Monte Carlo simulation is used to compute the failure probability. A large number of random variables are generated based on the distributions they obey. Then, the random variables are input into the limit state function. According to the theory of large numbers [19], the failure probability can be calculated by the following relation:

$$P_f = \iint_{L_{SF}\le 0} P(L_r, K_r) dK_r dL_r = \frac{n}{N} = \frac{1}{N}\sum_{j=1}^{N} h_j \tag{13}$$

$$h_j = \begin{cases} 1, & g(X) \le 0 \\ 0, & g(X) > 0 \end{cases} \tag{14}$$

where N and n denote the total number of Monte Carlo simulation cycles and the number of cycles for $g(X)\le 0$, respectively.

The flowchart for calculating the failure probability of pipe containing crack is shown in Fig. 3. It is noteworthy that if there are one fuzzy variable, firstly convert it to two random variables which obey $F_{fl}(x)$ and $F_{fu}(x)$ by using Eqs. (7) and (8), and then calculate two failure probabilities corresponding to the two random variables, respectively. Similarly, if there are m fuzzy variables, 2^m failure probabilities can be obtained. We define $P_{f,\min}$ and $P_{f,\max}$ as the lower bound and upper bound probability, respectively. That is, we finally obtain a failure probability interval $[P_{f,\min}, P_{f,\max}]$.

4. Example

4.1. Case description

The above method was applied to a pipe with a semi-elliptical surface crack (see Fig. 4) to calculate the failure probability of the pipe. Except operating pressure P, the rest of parameters (i.e. the sizes of the crack, inner radius R_i, wall thickness t, fracture toughness K_{IC} and yield stress σ_s) are all treated as random variables. The specific values of the parameters and their probabilistic distributions are given in Table 1. The operating pressure P of the pipe is considered as a fuzzy variable represented triangular membership function. Since the authors could not obtain information on the variation of P, support for the fuzzy set of operating pressure P is determined assuming a variation of 10% of the design value (3.4 MPa). The specific membership function of the operating pressure is shown in Fig. 5.

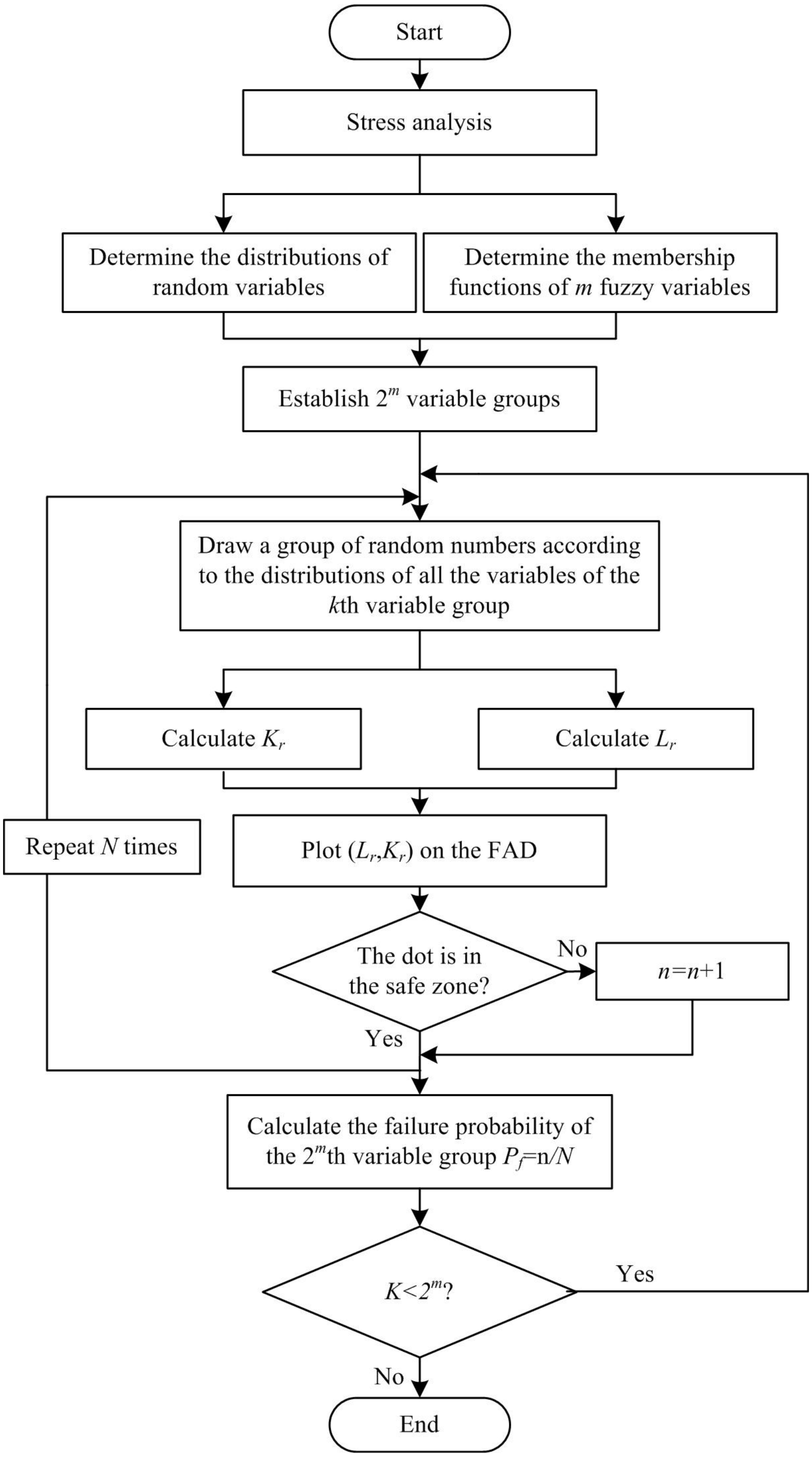

Fig. 3. The flowchart to calculate the failure probability.

4.2. Calculation of failure probability

The details and results of calculation of failure probability are given in Appendix A. The results show that the interval range of the failure probability of the pipe is from 0.0002 to 0.0012. The maximum failure probability is 0.0012, which is recommended for decision-making purpose.

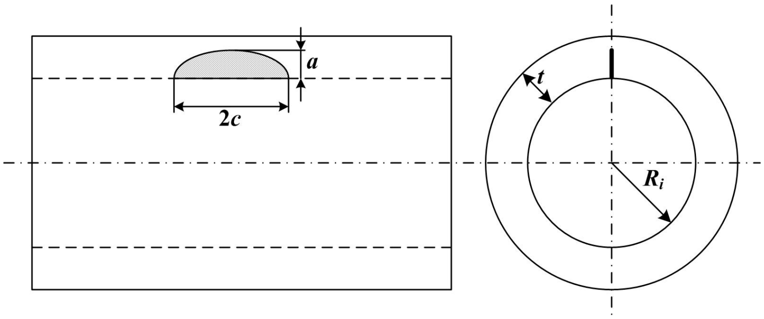

Fig. 4. A semi-elliptical surface crack in a straight pipe.

Table 1. Random parameters and their random distributions used in the assessment.

Parameters	Distribution	Mean value	Standard deviation
Inner radiu R_i /mm	Normal	1500	150
Wall thickness t /mm	Normal	25	2.5
Crack depth a /mm	Normal	5	0.5
Crack length 2c /mm	Normal	80	8
Fracture toughness KIC /MPa·m0.5	Lognormal	329.7	32.97
Yield stress σ_s /MPa	Lognormal	248	24.8

5. Results and discussion

5.1. Influence of uncertainty of fuzzy variables

The coefficient of variation of a fuzzy variable (*fcov*) is defined to be a measure of the degree of fuzzy uncertainty.

$$fcov(P) = \frac{\Delta P}{P} \tag{15}$$

where ΔP denotes the maximum deviation between the design pressure and actual operating pressure.

By changing the value of ΔP, the failure probabilities of the pipe at different *fcov*(*P*) were calculated (where *P* is constant). The results are shown in Fig. 6. It can be seen that the failure probability as well as the range of the fuzzy failure probability interval increases with increasing *fcov*(*P*).The reason is that with increasing ΔP, the fuzzy uncertainty of the variable increases, which can increase possibility to allow g(*X*) to fall within the unsafe zone. In addition, it can be noticed that the fuzzy failure probability is less than the probability calculated based on the maximum pressure $P+\Delta P$, which shows the estimated failure probability based on $P+\Delta P$ is more conservative than fuzzy failure probability.

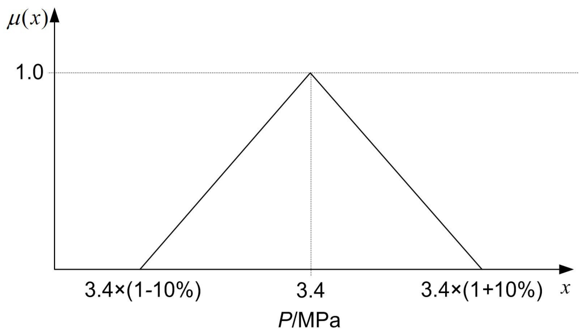

Fig. 5. Membership function for operating pressure in the example problem.

5.2. Influence of uncertainty of random variables

Similar to *fcov*, the coefficient of variation of a random variable (*rcov*) is defined to be a measure of the degree of random uncertainty.

$$rcov = \frac{\sigma}{\mu} \tag{16}$$

where σ and μ denotes the standard deviation and the mean value of a random variable, respectively. Through only changing the standard deviation, the failure probabilities of the pipe at different *rcov* (from 0 to 0.4) were calculated.

Figure 7 and Fig. 8 illustrate the influence of $rcov(R_i)$ and $rcov(t)$ on failure probability, respectively. The results shows that with the increasing $rcov(R_i)$ and $rcov(t)$, failure probability obviously increases. This is because that with increasing standard deviation of R_i or t, the random variable increases the possibility to allow g(X) to fall within the unsafe zone, thus increasing the failure probability. Besides, it can be seen that the upper-bound failure probability increases from 0.0012 to 0.0604 as the $rcov(R_i)$ increase from 0 to 0.4, whereas that increase from 0.0012 to 0.1684 as the $rcov(t)$ increase from 0 to 0.04. This implies $rcov(t)$ has a bigger influence on the failure probability of the pipe than $rcov(R_i)$.

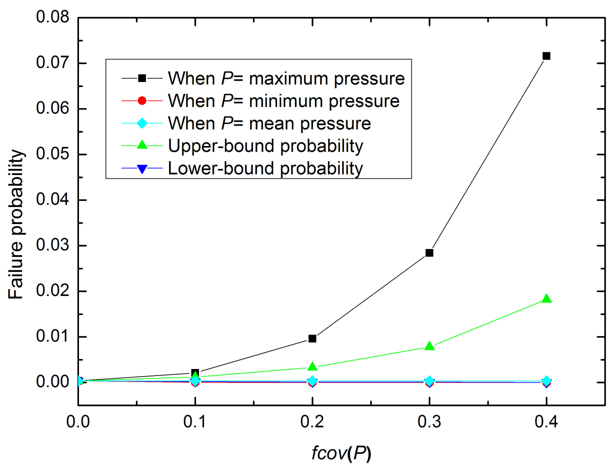

Fig. 6. Effect of the *fcov*(*P*) on the failure probability.

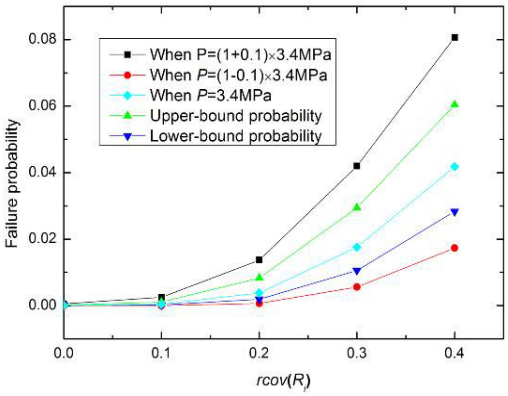

Fig. 7. Effect of the rcov(Ri) on the failure probability.

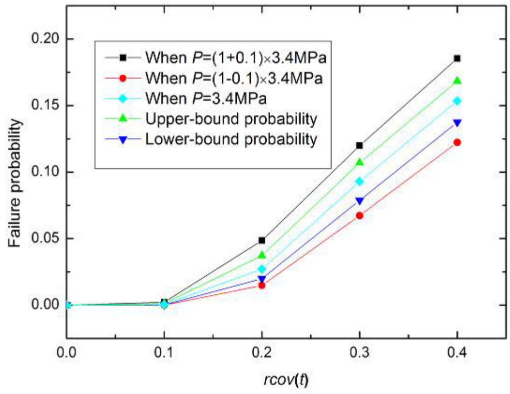

Fig. 8. Effect of the rcov(t) on the failure probability.

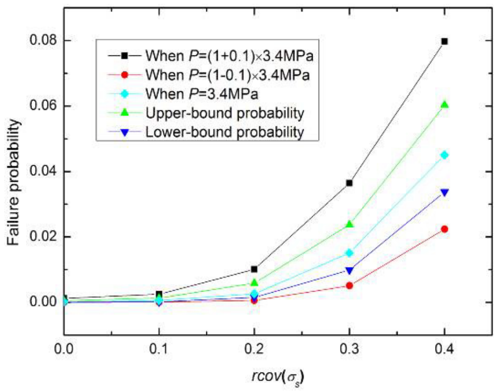

Fig. 9. Effect of the $rcov(\sigma_s)$ on the failure probability.

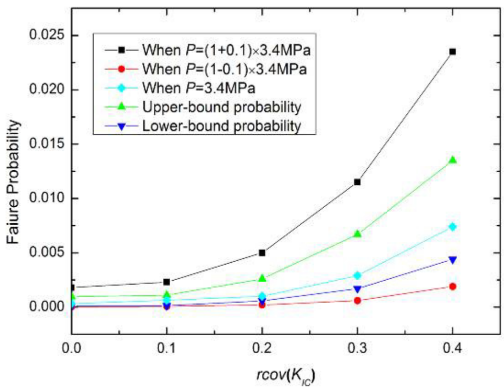

Fig. 10. Effect of the rcov(KIC) on the failure probability.

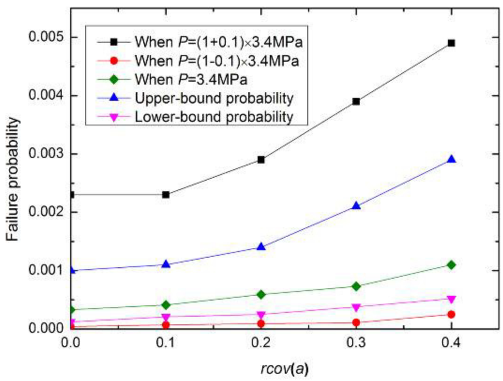

Fig. 11. Effect of the rcov(a) on the failure probability.

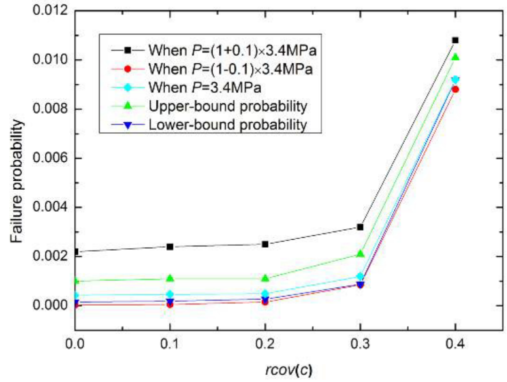

Fig. 12. Effect of the rcov(c) on the failure probability.

Figure 9 and Fig. 10 show the influence of *rcov*(σ_s) and *rcov*(K_{IC}) on failure probability, respectively. The similar tendencies are seen for the influence of *rcov*(σ_s) and *rcov*(K_{IC}). The failure probability is more sensitive to *rcov*(σ_s) than it is to *rcov*(K_{IC}).

Figure 11 and Fig. 12 illustrate the effect of *rcov*(*a*) and *rcov*(*c*) on failure probability, respectively. Although with the increasing *rcov*(*a*) and *rcov*(*c*) failure probability gradually increases, the increasing extent is relatively small.From the above analysis, it is shown that the failure probability of the pipe is more sensitive to standard deviations of *t*, R_i and σ_s than that of K_{IC}, , *a* and *c*.

5.3. Sensitivity Analysis

In this section the effect of the mean value of the different variables on the failure probability was analyzed. In the above case, the limit state function is influenced by variables *P*, R_i, *t*, σ_s, K_{IC}, *a* ,*c*. By changing the mean value of any of these variables from -30% to 30% and keeping the rest constant, the changing trend of the limit state function is obtained. The results are shown in Fig. 13. The slope of the curve reflects the influence of the variable on *g*(*X*). Obviously, the limit state function increases with increasing the mean value of σ_s, K_{IC}, *t*, whereas it decreases with increases increasing of *a* ,*c*, R_i, *P*.

The ranking of the variables is revealed more clearly in Fig. 14. If variables have a positive effect on the limit state function *g*(*X*), the upside of the bar is shown in black and the down side of the bar in red color. If variables have a reverse effect on *g*(*X*), the bars are shown in a reversed manner (see Fig. 14). It can be seen that the pipe wall thickness t has the biggest effect on pipe failure, followed by R_i, K_{IC}, σ_s *P*, *a* and *c*. Therefore, in order to reduce the failure probability of the pipe by the mean value of variables, priorities should be sequentially given to *t*, R_i, K_{IC}, σ_s *P*, *a* and *c*. It is obvious that for an in-service pipe only *P*, *a* and *c* can be focused on to control the failure probability.

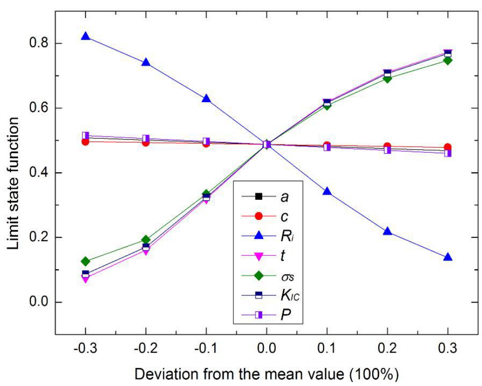

Fig. 13. Effect of variation of variables on limit state function.

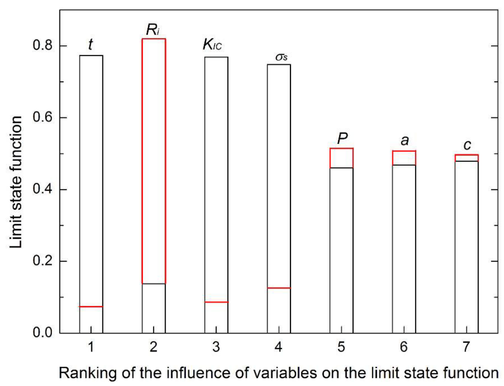

Fig. 14. Ranking of the effect of variation of variables on limit state function.

6. Conclusions

In this paper, a fuzzy-probabilistic failure assessment method has been developed based on API 579-1/ASME FFS-1 procedure. The proposed method combines probability theory and possibility theory to deal with hybrid uncertainties: randomness and fuzziness. The failure probability of a pipe containing a crack was calculated by using Monte Carlo simulation. The effect of parameters (*P*, *t*, R_i, σ_s, K_{IC}, , *a* and *c*) and their uncertainty on failure probability was systematically discussed and the following results are obtained.

The failure probability increases with the increasing *fcov*(*P*) and increasing standard deviations of the random variables (t, R_i, σ_s , K_{IC},, a and c). The failure probability of the pipe is more sensitive to standard deviations of t, R_i and σ_s than that of K_{IC}, , a and c. Sensitivity analyses by only changing the mean value of any of these involved variables shows that the wall thickness t and inner radius of the pipe R_i have the most important contribution to the pipe failure, followed by K_{IC}, σ_s P, a and c.

From the results obtained, it is noted that the fuzzy-probabilistic failure assessment method used in the present study is promising for failure assessment of cracked pipe.

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Appendix A. Calculation of fuzzy failure probability of the pipe

This appendix describe the detailed calculations of fuzzy failure probability of the pipe containing a crack (see Fig. 4). The specific process is as follows.

Step 1: Sample from random variables

First of all, convert each of fuzzy variable P to two random variables which obey $F_{fl}(x)$ and $F_{fu}(x)$ by using Eqs. (7) and (8), and then obtain a group of samples of random variables according to their distributions.

Step 2: Calculate toughness ratio K_r

Calculate toughness ratio K_r of the semi-elliptical surface crack in pipe by using the following equations [20].

$$K_r = \frac{K_I}{K_{IC}} \tag{A1}$$

$$K_I = \sqrt{\pi a}(\sigma_m f_m + \sigma_B f_b) \tag{A2}$$

$$f_m = \frac{1}{\left[1+1.464\left(\frac{a}{c}\right)^{1.65}\right]^{0.5}}\left\{1.13-0.09\frac{a}{c}+\left(-0.45+\frac{0.89}{0.2+\frac{a}{c}}\right)\left(\frac{a}{t}\right)^2+\left[0.5-\frac{1}{0.65+\frac{a}{c}}+14\left(1-\frac{a}{c}\right)^{24}\right]\left(\frac{a}{t}\right)^4\right\} \tag{A3}$$

$$f_b = \left\{1+\left(-1.22-0.12\frac{a}{c}\right)\frac{a}{t}+\left[0.55-1.05\left(\frac{a}{c}\right)^{0.75}+0.47\left(\frac{a}{c}\right)^{1.5}\right]\left(\frac{a}{t}\right)^2\right\}f_m \tag{A4}$$

$$\sigma_m = \frac{PR_i}{t} \tag{A5}$$

where σ_m and σ_B denote membrane stress and bending stress, respectively.
Substitute the group of samples from Step 1 into the Eqs. (A1-A5) to obtain the toughness ratio K_r.

Step 3: Calculate load ratio L_r

Firstly, calculate the reference stress (σ_{ref}) of the semi-elliptical surface crack in pipe by using the following equations [20]

$$L_r = \frac{\sigma_{ref}}{\sigma_s} \tag{A6}$$

$$\sigma_{ref} = 1.2 M_s P_m + \frac{2P_b}{3(1-\alpha)^2} \tag{A7}$$

$$M_s = \frac{1-[a/(tM_T)]}{1-(a/t)} \tag{A8}$$

$$\alpha = \left(\frac{a}{t}\right) \Big/ \left(1+\frac{t}{c}\right) \tag{A9}$$

$$M_T = \sqrt{1+1.6\left(\frac{c^2}{R_i t}\right)} \tag{A10}$$

$$P_m = \frac{PR_i}{t} \tag{A11}$$

where P_m and P_b denote primary membrane stress and primary bending stress, respectively.
Substitute the group of samples from Step 1 into the Eqs. (A6-A11) to obtain the load ratio L_r.

Step 4: Determine the *position of* (K_r, L_r)

Substitute K_r from Step 2 and L_r from Step 3 into Eq. 10. If $g(X) \leq 0$, the pipe is unsafe, that is, $n=n+1$, otherwise, n remains constant.

Step 5. Determine the failure probability

Repeat Steps 1-4 N times ($N=10^6$), and then determine the failure probability interval based on Eq.(13). The results show that the interval range of the failure probability of the pipe is from 0.0002 to 0.0012.

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