Physics Letters B 701 (2011) 388–392

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Charged gravastars admitting conformal motion

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ARTICLE INFO

Article history:
Received 9 February 2011
Accepted 1 June 2011
Available online 6 June 2011
Editor: M. Trodden

Keywords:
Gravitation
Equation of state
Gravastar

ABSTRACT

We propose a new model of a *gravastar* admitting conformal motion. While retaining the framework of the Mazur–Mottola model, the gravastar is assumed to be internally charged, with an exterior defined by a Reissner–Nordström instead of a Schwarzschild line element. The solutions, obtained by exploiting an assumed conformal Killing vector, involve (i) the interior region, (ii) the shell, and (iii) the exterior region of the sphere. Of these three cases the first one is of primary interest since the total gravitational mass here turns out to be an *electromagnetic mass* under some specific conditions. This suggests that the interior de Sitter vacuum of a charged gravastar is essentially an electromagnetic mass model that must generate gravitational mass which provides a stable configuration by balancing the repulsive pressure arising from charge with its attractive gravity to avert a singularity. Therefore the present model, like the Mazur–Mottola model, results in the construction of a compact astrophysical object, as an alternative to a black hole. We have also analyzed various other aspects such as the stress energy tensor in the thin shell and the entropy of the system.

1. Introduction

By extending the concept of Bose–Einstein condensate to gravitational systems, Mazur and Mottola [1,2] have proposed a new solution for the endpoint of a gravitational collapse in the form of cold, dark, compact objects known as *gravastars*, the gravitational vacuum star. These contain an isotropic de Sitter vacuum in the interior, while the exterior is defined by a Schwarzschild geometry, separated by a thin shell of stiff matter of arbitrary total mass M. This model implies that the space of a gravastar has three different regions with different equations of state (EOS) [3–15], as defined as follows:

I. Interior: $0 \leqslant r < r_1$, $p = -\rho$,
II. Shell: $r_1 < r < r_2$, $p = +\rho$,
III. Exterior: $r_2 < r$, $p = \rho = 0$.

Here $r_2 - r_1 = \delta$ is the thickness of the shell. The presence of matter on the thin shell is required to achieve the crucial stability of such systems under expansion by exerting an inward force to balance the repulsion from within.

Based on this background, we propose a new model of a gravastar admitting conformal motion by assuming a charged interior but with an exterior defined by a Reissner–Nordström line element instead of Schwarzschild. The basic motivation for this model is that, *in general*, compact stars tend to have a net charge on the surface [16–20]. This is an essential consideration in the study of the stability of a fluid sphere; in fact, it has been argued [16,17] that a spherical fluid distribution of uniform density with a net surface charge is more stable than a surface without charge. According to de Felice et al. [21], the inclusion of charge inhibits the growth of spacetime curvature and which therefore plays a key role in avoiding singularities. It has also been argued that gravitational collapse can be averted in the presence of a charge since the gravitational attraction may be counter-balanced by the electrical repulsion (in addition to the pressure gradient [22,23]). As a *special case*, we would like to mention that several interesting charged models are available in the literature [13,24–31]. Out of these works some were considered directly in connection to the charged sphere with conformal motion/symmetry [24,29] and some with a direct treatment of a gravastar where the analysis was carried out within Israel's thin shell formalism and the continuous profile approach [13].

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doi:10.1016/j.physletb.2011.06.001

In searching for a natural relationship between geometry and matter for such stars through the Einstein field equations, we take into account the well-known inheritance symmetry. This symmetry is contained in the set of conformal killing vectors (CKV)

$$L_\xi g_{ij} = \psi g_{ij}. \tag{1}$$

Here L is the Lie derivative operator and ψ is the conformal factor. It is supposed that the vector ξ generates the conformal symmetry and the metric g is conformally mapped onto itself along ξ. Neither ξ nor ψ need to be static, even in the case of a static metric [32,33]. Due to this and other properties, CKVs have provided a deeper insight into the spacetime geometry connected to the astrophysical and cosmological realm [34–41].

It has been shown by Ray et al. [42] that, under the conformal killing-vectors approach, charged fluid spheres provide *electromagnetic mass* (*EMM*) models in which gravitational mass and other physical parameters originate solely from the electromagnetic field. So one of the motivations in the present investigation is to include CKVs and see to what extent conformal motion admits EMM models along with other relevant physical features. This Letter is organized as follows: in Section 2 the Einstein–Maxwell field equations are provided, along with the CKVs for a charged gravastar. The solutions are obtained in Section 3 for a charged gravastar with conformal motion in connection with (A) the interior region, (B) the shell, and (C) the exterior region of the sphere. Sections 4, 5, and 6 deal with the stress energy tensor in the thin shell, entropy within the shell, and unknown constants, respectively. In Section 7 we conclude.

2. The Einstein–Maxwell field equations

The Einstein field equations for the case of a charged perfect fluid source are

$$G^i{}_j = R^i{}_j - \frac{1}{2}g^i{}_j R = -\kappa\left[T^i{}_j{}^{(m)} + T^i{}_j{}^{(em)}\right], \tag{2}$$

where the energy–momentum tensor components for the matter source and electromagnetic field, respectively, are given by

$$T^i{}_j{}^{(m)} = (\rho + p)u^i u_j + p g^i{}_j, \tag{3}$$

$$T^i{}_j{}^{(em)} = -\frac{1}{4\pi}\left[F_{jk}F^{ik} - \frac{1}{4}g^i{}_j F_{kl}F^{kl}\right]. \tag{4}$$

Here ρ, p and u^i are the matter–energy density, the fluid pressure, and the velocity four-vector of a fluid element (with $u_i u^i = 1$), respectively. The corresponding Maxwell electromagnetic field equations are

$$\left[(-g)^{1/2}F^{ij}\right]_{,j} = 4\pi J^i(-g)^{1/2}, \tag{5}$$

$$F_{[ij,k]} = 0, \tag{6}$$

where the electromagnetic field tensor F_{ij} is related to the electromagnetic potentials through the relation $F_{ij} = A_{i,j} - A_{j,i}$ and is equivalent to Eq. (6). In the above equations, J^i is the current four-vector satisfying $J^i = \sigma u^i$, where σ is the charge density and $\kappa = 8\pi$, using relativistic units $G = c = 1$. Here and in what follows a comma denotes the partial derivative with respect to the coordinates.

Next, given the static spherically symmetric spacetime

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{7}$$

the Einstein–Maxwell field equations may be written as

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi\rho + E^2, \tag{8}$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi p - E^2, \tag{9}$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi p + E^2, \tag{10}$$

and

$$\left[r^2 E\right]' = 4\pi r^2\sigma e^{\lambda/2}. \tag{11}$$

Eq. (11) may be expressed for the electric field E in the following equivalent form:

$$E(r) = \frac{1}{r^2}\int_0^r 4\pi r^2\sigma e^{\lambda/2}\,dr = \frac{q(r)}{r^2}, \tag{12}$$

where $q(r)$ is the total charge of the sphere.

3. The charged gravastar with conformal motion

Eq. (1) implies the following:

$$L_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik} \tag{13}$$

with $\xi_i = g_{ik}\xi^k$. Here 1 and 4 stand for the spatial and temporal coordinates r and t, respectively.

Eqs. (13) yield the following expressions [42]:

$$\xi^1\nu' = \psi,$$

$$\xi^4 = C_1,$$

$$\xi^1 = \frac{\psi r}{2},$$

$$\xi^1\lambda' + 2\xi^1_{,1} = \psi,$$

which imply

$$e^\nu = C_2^2 r^2, \tag{14}$$

$$e^\lambda = \left[\frac{C_3}{\psi}\right]^2, \tag{15}$$

$$\xi^i = C_1\delta^i_4 + \left[\frac{\psi r}{2}\right]\delta^i_1, \tag{16}$$

where C_1, C_2, and C_3 are integration constants.

Given solutions (14) and (15), Eqs. (8), (9), and (10) take the following form [42]:

$$\frac{1}{r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{2\psi\psi'}{rC_3^2} = 8\pi\rho + E^2, \tag{17}$$

$$\frac{1}{r^2}\left[1 - \frac{3\psi^2}{C_3^2}\right] = -8\pi p + E^2, \tag{18}$$

$$\left[\frac{\psi^2}{C_3^2 r^2}\right] + \frac{2\psi\psi'}{rC_3^2} = 8\pi p + E^2. \tag{19}$$

From the above equations, one may easily obtain values for E, ρ and p [42]:

$$E^2 = \frac{1}{2}\left[\frac{1}{r^2}\left(1 - \frac{2\psi^2}{C_3^2}\right) + \frac{2\psi\psi'}{rC_3^2}\right], \tag{20}$$

$$8\pi\rho = \frac{1}{2r^2} - \frac{3\psi\psi'}{rC_3^2}, \tag{21}$$

$$8\pi p = \frac{\psi\psi'}{rC_3^2} - \frac{1}{2r^2}\left[1 - \frac{4\psi^2}{C_3^2}\right]. \tag{22}$$

3.1. Interior region of the charged gravastar

Observe next that Eqs. (21) and (22) provide an essential relationship between the metric potentials and the physical parameters ρ and p:

$$\frac{2\psi}{r^2 C_3^2}(\psi - r\psi') = 8\pi r(\rho + p). \tag{23}$$

So given the *ansatz* $\rho + p = 0$, it follows from Eq. (23) that the value of ψ turns out to be either $\psi = 0$ or $\psi = \psi_0 r$, where ψ_0 is a dimensionless integration constant. This leads to the following exact analytical forms for all of the parameters:

$$8\pi\rho = \frac{1}{2r^2} - 3\tilde{\psi}_0^2 = -8\pi p, \tag{24}$$

$$E^2 = \frac{1}{2r^2}, \tag{25}$$

$$e^{\nu} = e^{-\lambda} = \tilde{\psi}_0^2 r^2, \tag{26}$$

$$\sigma = \frac{\tilde{\psi}_0}{4\pi\sqrt{2}r}. \tag{27}$$

Here $\tilde{\psi}_0 = \psi_0/C_3$ is a constant, which has the inverse dimension of r. The reason is that under the condition $\rho + p = 0$, $\tilde{\psi}_0 = C_2$, so that $C_2 C_3 = \psi_0$.

It is clear from Eq. (24) that $1/2r^2 - 3\tilde{\psi}_0^2 > 0$ (or $\tilde{\psi}_0 < 1/\sqrt{6}r$) illustrates a case of positive density and negative pressure, resulting in an outward push from the interior region, which is consistent with the physics of a gravastar. On the other hand, $1/2r^2 - 3\tilde{\psi}_0^2 < 0$ (or $\tilde{\psi}_0^2 < 1/6r^2$) represents a collapsing case with negative density and positive pressure, which is not the subject of concern here. Thus for the purpose of gravastar physics, our above solutions are assumed to obey the condition $0 < \tilde{\psi}_0^2 < 1/6r^2$.

For $\tilde{\psi}_0^2 = 0$, we find that both p and ρ are inversely proportional to r^2 but with opposite signs for their proportionality constants. A nonzero value of $\tilde{\psi}_0^2$ leads to a translational shift of this form with magnitude $3\tilde{\psi}_0^2$, as can be seen from Eq. (24). The electric field E is found to be inversely proportional to r and is independent of $\tilde{\psi}_0$. On the other hand, σ is inversely proportional to r. Its value is zero for $\tilde{\psi}_0 = 0$, which suggests that the above power law behavior of p and ρ is independent of σ. Both e^{ν} and $e^{-\lambda}$ are proportional to r^2 and hence equal with equal proportionality constants.

The active gravitational mass $M(r)$, by virtue of the field equation (8), may be expressed in the following form:

$$M(r) = 4\pi\int_0^r \left[\rho + \frac{E^2}{8\pi}\right] r^2\,dr = \frac{1}{2}r\left(1 - \tilde{\psi}_0^2 r^2\right). \tag{28}$$

Here, as evident from Eq. (24), the pressure and density fail to be regular at the origin, but the effective gravitational mass is always positive and regular since $\tilde{\psi}_0 < 1/\sqrt{6}r$ and will vanish as $r \to 0$. In other words, the expression for $M(r)$ does not lead to a singularity.

Let us now match the interior solution to the exterior Reissner–Nordström solution at the boundary in the customary manner. To do so, we have to keep in mind that instead of a solid sphere, we are dealing here with a bubble-like hollow sphere of radius $r_2 = r_1 + \delta$, so that for the limit $\delta \to 0$, we have the de Sitter spherical void of radius $r_2 \to r_1$. (According to Ref. [1], δ does not exceed Planck length by more than a few orders of magnitude.) For convenience of notation, let us denote the radius r_2 by a, the radius of the junction surface. Then following Ray et al. [42], the total gravitational mass $m(r = a)$, which is obtained after matching the solution interior to $r = a$ to the exterior Reissner–Nordström solution at the boundary, can be expressed as

$$m(a) = M(a) + \frac{q(a)^2}{2a} = \frac{1}{2\sqrt{2}}\left(3 - 8\tilde{\psi}_0^2 q^2\right)q, \tag{29}$$

where $M(a)$ is the total active gravitational mass and $q(a)^2/2a$ is the mass equivalence of the electromagnetic field.

It is interesting to observe that the mathematical expressions and physics of the interior region resemble the EMM model of Ray et al. [42]. The apparent reason for this is the use of the Reissner–Nordström line element. In other words, the interior de Sitter void ($p = -\rho$) in a charged gravastar is the same as in the case already addressed by Ray et al. [42]. This implies that the interior de Sitter void of a charged gravastar must, in analogous fashion, generate the gravitational mass. This particular feature is a new one and was not possible to obtain in the non-charged case of Mazur and Mottola [1,2]. This mass provides the attractive force resulting from the collapse of the sphere and counter-balances the repulsive force due to electromagnetic field.

However, in this connection it is worth noting that the equation of state $p = -\rho$ (known in the literature as a false vacuum, degenerate vacuum, or ρ-vacuum [43–46]) represents a repulsive pressure which in the context of an accelerating Universe may be related to the *Λ-dark energy*, an agent responsible for the second phase of the inflation [47–51]. So the *charged gravastar* seems to be connected to the *dark star* [52–54].

3.2. Shell of the charged gravastar

Using the EOS $p = \rho$, we get the solution

$$\psi^2 = \frac{C_2^2}{2} - \frac{\psi_1}{r}, \tag{30}$$

where $\psi_1 > 0$ is an integration constant.

Other parameters are

$$8\pi\rho = \frac{1}{2r^2}\left(1 - \frac{3\tilde{\psi}_1}{r}\right) = 8\pi p, \tag{31}$$

$$E^2 = \frac{1}{2r^2} - 8\pi\rho, \tag{32}$$

$$e^{\nu} = C_1^2 r^2, \tag{33}$$

$$e^{-\lambda} = 1/2 - \tilde{\psi}_1/r, \tag{34}$$

$$\sigma = \frac{\sqrt{3}\tilde{\psi}_1}{16\pi r^3}\sqrt{\frac{r}{\tilde{\psi}_1} - 2}, \tag{35}$$

where $\tilde{\psi}_1 = \psi_1/C_2^2$ has the same dimension as r. Although the electric field E is inversely proportional to r, it depends on the integration constant $\tilde{\psi}_1$, unlike the previous case. Eq. (35) suggests that the requirement of a real-valued σ may only be achieved with the condition $\tilde{\psi}_1 < r/2$. Combining this with the previous condition, $\psi_1 > 0$ (or $\tilde{\psi}_1 > 0$), we observe that the above solutions for the shell of the gravastar are valid within the range $0 < \tilde{\psi}_1 < r/2$. It is obvious from Eq. (31) that the EOS $p = \rho = 0$ for the exterior de Sitter region corresponds to $\tilde{\psi}_1 = r/3$, which is within the upper limit of the above condition $\tilde{\psi}_1 < r/2$.

The proper thickness of the shell is obtained next:

$$\ell = \int_{r_1}^{r_2} \sqrt{e^{\lambda}}\,dr$$

$$= \sqrt{2}\left[R + \tilde{\psi}_1 \ln(R + r - \tilde{\psi}_1)\right]_a^{a+\epsilon}, \tag{36}$$

where $R = r\sqrt{1-2\tilde{\psi}_1/r}$. Thus a real value for the thickness confirms that the integration constant $\tilde{\psi}_1$ must be less than $r/2$. This condition also suggests a real σ.

Finally, using the symbol $\tilde{E}$ for the energy, we get within the shell

$$\tilde{E} = 4\pi \int_{r_1}^{r_2} \left[\rho + \frac{E^2}{8\pi}\right] r^2\,dr = \frac{1}{4}[r_2 - r_1]. \tag{37}$$

Thus $\tilde{E}$ is exactly proportional to the coordinate thickness of the shell (as opposed to the proper thickness).

3.3. *Exterior region of the charged gravastar*

For the exterior region ($p = \rho = 0$), the Reissner–Nordström spacetime is

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \tag{38}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \tag{39}$$

4. The stress energy tensor in the thin shell

The second fundamental forms associated with the two sides of a thin shell (junction surface) are [16,17]

$$K_{ij}^{\pm} = -n_{\nu}^{\pm}\left[\frac{\partial^2 X^{\nu}}{\partial\xi^i\partial\xi^j} + \Gamma^{\nu}_{\alpha\beta}\frac{\partial X^{\alpha}}{\partial\xi^i}\frac{\partial X^{\beta}}{\partial\xi^j}\right]\Bigg|_S, \tag{40}$$

where $n_{\nu}^{\pm}$ are the unit normals to S:

$$n_{\nu}^{\pm} = \pm\left|g^{\alpha\beta}\frac{\partial f}{\partial X^{\alpha}}\frac{\partial f}{\partial X^{\beta}}\right|^{-\frac{1}{2}}\frac{\partial f}{\partial X^{\nu}} \tag{41}$$

with $n^{\mu}n_{\mu} = 1$. Here ξ^i are the intrinsic coordinates on the shell, $f = 0$ is the parametric equation of the shell S, and $-$ and $+$ corresponds to the interior and exterior regions, respectively.

Using the Lanczos equations [55–59], one can find the surface stress energy Σ and the surface tangential pressures $p_\theta = p_\phi \equiv p_t$:

$$\begin{aligned}\Sigma &= -\frac{1}{4\pi a}\left[\sqrt{e^{-\lambda}}\right]_{-}^{+}\\ &= -\frac{1}{4\pi a}\left[\sqrt{1 - \frac{2M}{a} + \frac{Q^2}{a^2}} - \tilde{\psi}_0 a\right],\end{aligned} \tag{42}$$

$$\begin{aligned}p_t &= \frac{1}{8\pi a}\left[\left(1 + \frac{a\nu'}{2}\right)\sqrt{e^{-\lambda}}\right]_{-}^{+}\\ &= \frac{1}{4\pi a}\left[\frac{1 - M/a}{2\sqrt{1-2M/a+Q^2/a^2}} - \tilde{\psi}_0 a\right].\end{aligned} \tag{43}$$

The surface mass M_{shell} of this thin shell may be defined as

$$\begin{aligned}M_{shell} &= 4\pi a^2\Sigma\\ &= -a\left[\sqrt{1 - \frac{2M}{a} + \frac{Q^2}{a^2}} - \tilde{\psi}_0 a\right].\end{aligned} \tag{44}$$

Here M can be interpreted as the total mass of the Reissner–Nordström gravastar. It takes the following form:

$$M = \frac{1}{2a}\left[a^2 + Q^2 + 2a^2\tilde{\psi}_0 M_{shell} - M_{shell}^2 - \tilde{\psi}_0^2 a^4\right]. \tag{45}$$

Now let $p_\theta = p_\phi = -\nu_\theta = -\nu_\phi = -\nu$, where ν_θ and ν_ϕ are the surface tensions. Then if Eqs. (42) and (43) are substituted in the form

$$\nu = \omega(a)\Sigma, \tag{46}$$

the EOS becomes

$$\omega(a) = \frac{(1/2\tilde{\psi}_0 a)(1-M/a) - \sqrt{1-2M/a+Q^2/a^2}}{(1/\tilde{\psi}_0 a)(1-2M/a+Q^2/a^2) - \sqrt{1-2M/a+Q^2/a^2}}. \tag{47}$$

With the requirement of a positive density and positive pressure, the equation of state parameter $\omega(a)$ is always positive. The position of the thin shell (junction surface) plays a crucial role: if a is sufficiently large, then $\omega(a) \approx 1$. For some value of a in Eq. (43), we may get $p_t = 0$, yielding a dust shell.

5. Entropy within the shell

Following Mazur and Mottola [1,2], we now calculate the entropy by letting $r_1 = b$ and $r_2 = b + \epsilon$:

$$S = 4\pi \int_b^{b+\epsilon} s r^2 \sqrt{e^{\lambda}}\,dr. \tag{48}$$

Here s is the entropy density, which may be written as

$$\frac{\alpha^2 k_B^2 T(r)}{4\pi\hbar^2 G} = \alpha\left(\frac{k_B}{\hbar}\right)\sqrt{\frac{p}{2\pi G}}, \tag{49}$$

where α^2 is a dimensionless constant and T is the radially dependent temperature of the system.

Thus the entropy of the fluid within the shell is

$$\begin{aligned}S &= 4\pi \int_b^{b+\epsilon} r^2\alpha\left(\frac{k_B}{\hbar}\right)\sqrt{\frac{\frac{1}{2r^2} - \frac{3\tilde{\psi}_1}{2r^3}}{16\pi^2 G^2}}\sqrt{\frac{1}{1/2 - \tilde{\psi}_1/r}}\,dr\\ &= \frac{\alpha k_B}{\hbar G}\int_b^{b+\epsilon} r\sqrt{\frac{r - 3\tilde{\psi}_1}{r - 2\tilde{\psi}_1}}\,dr\\ &= \frac{\alpha k_B}{\hbar G}\left[\frac{2r + 3\tilde{\psi}_1}{4}x - \frac{9\tilde{\psi}_1^2}{8}\ln(2x + 2r - 5\tilde{\psi}_1)\right]_b^{b+\epsilon}\end{aligned} \tag{50}$$

where $x = (r^2 - 5r\tilde{\psi}_1 + 6\tilde{\psi}_1^2)^{1/2}$. If $\tilde{\psi}_1 \to 0$ and the thickness of the shell is negligibly small compared to its position from the center of the gravastar (i.e., if $\epsilon \ll b$), then the entropy is given by $S \approx \frac{\alpha k_B}{\hbar G} b\epsilon$.

6. The unknown constants $\tilde{\psi}_0$ and $\tilde{\psi}_1$

In this section we determine the approximate values of the constants $\tilde{\psi}_0$ and $\tilde{\psi}_1$. To this end we recall that the thin shell of a gravastar consists of a perfect stiff fluid [1]. Earlier we denoted the outer radius by $r = a$, the junction surface. So by Eq. (31), $\rho = p = p_t$. Furthermore, for a sufficiently large, $p_t = -\nu \approx \Sigma$ by Eq. (46). So we have

$$\begin{aligned}&-\frac{1}{4\pi a}\left[\sqrt{1 - \frac{2M}{a} + \frac{Q^2}{a^2}} - \tilde{\psi}_0 a\right]\\ &\quad\approx \frac{1}{4\pi a}\left[\frac{1 - M/a}{2\sqrt{1-2M/a+Q^2/a^2}} - \tilde{\psi}_0 a\right]\\ &\quad= \frac{1}{8\pi}\left(\frac{1}{2a^2} - \frac{3\tilde{\psi}_1}{2a^3}\right).\end{aligned} \tag{51}$$

These two equations then yield the approximate values of the unknown constants:

$$\tilde{\psi}_0 \approx \frac{3 - 5M/a + 2Q^2/a^2}{4a\sqrt{1 - 2M/a + Q^2/a^2}}, \tag{52}$$

$$\tilde{\psi}_1 \approx \frac{a}{3}\left(1 + \frac{a - 3M + 2Q^2/a^2}{\sqrt{1 - 2M/a + Q^2/a^2}}\right). \tag{53}$$

Since $\Sigma > 0$, Eq. (42) also implies that $\tilde{\psi}_0 > \frac{1}{a} \times \sqrt{1 - 2M/a + Q^2/a^2}$.

7. Conclusions

This Letter discusses a new model of a gravastar admitting conformal motion, within the framework of the Mazur–Mottola model. The gravastar is assumed to be internally charged with an exterior defined by a Reissner–Nordström rather than a Schwarzschild metric. The solutions obtained cover (i) the interior region, (ii) the shell, and (iii) the exterior region of the sphere. Of these three cases, the first case is of particular interest because the total gravitational mass $m(r = a) = \frac{1}{2\sqrt{2}}(3 - 8\tilde{\psi}_0^2 q^2)q$, obtained by matching the interior solution to the exterior Reissner–Nordström solution at the boundary, becomes an EMM under the constraint $\tilde{\psi}_0 < \sqrt{3}/(2\sqrt{2})q$. This, in turn, suggests that the interior de Sitter void of a charged gravastar, having the same form as the EMM model, must generate the gravitational mass that provides the attractive force resulting from the collapse of the sphere and which counter-balances the repulsive force due to the charge. Moreover, the equation of state $p = -\rho$, known in the literature as a false vacuum or a ρ-vacuum, suggests that the *charged gravastar* is connected with the *dark star*.

An analysis of the stress energy tensor of the thin shell has shown that given the requirement of a positive density and positive pressure, the equation of state parameter $\omega(a)$ is always positive. Moreover, as the radius of the thin shell increases, $w(a) \to 1$. For some value of $r = a$, we may have $p_t = 0$, yielding a dust shell.

In calculating the entropy, it was found that if $\tilde{\psi}_1 \to 0$ and if, in addition, the thickness of the shell is small compared to the radius, then the entropy is given by $S \approx \frac{\alpha k_B}{\hbar G} b\epsilon$, where b is the inner radius of the gravastar shell. The final calculations determined the approximate values of the constants $\tilde{\psi}_0$ and $\tilde{\psi}_1$ with $\tilde{\psi}_0 > \frac{1}{a}\sqrt{1 - 2M/a + Q^2/a^2}$.

Acknowledgements

A.A.U., F.R., S.R. and K.K.N. wish to thank the authorities of the Inter-University Centre for Astronomy and Astrophysics, Pune, India, for providing the Visiting Associateship under which a part of this work was carried out. F.R. and Z.H. are also grateful to PURSE, Govt. of India, and D.S. Kothari Fellowship, for financial support. Thanks are also due to A. DeBenedictis, Department of Physics, Simon Fraser University, Canada, for helpful discussion.

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