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Pushing the cycles out of multipartite tournaments

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Abstract

Let D be a digraph and $X \subseteq V(D)$. By *pushing* X we mean reversing each arc of D with exactly one end in X. Klostermeyer proved that it is NP-complete to decide if a given digraph can be made acyclic using the push operation. Here we characterize, in terms of forbidden subdigraphs, the multipartite tournaments which can be made acyclic (resp. ordinary, unidirectional) using the push operation. This implies that the problem of deciding if a given multipartite tournament can be made acyclic (resp. ordinary, unidirectional) using the push operation and, if so, finding a suitable subset of vertices to push, is solvable in polynomial time. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

Let D be a digraph and $X \subseteq V(D)$. We define D^X to be the digraph obtained from D by reversing the orientation of all arcs with exactly one endvertex in X. We say the vertices of X are *pushed* and that D^X is the result of *pushing* X in D. Note that, $D^X = D^{V(D)-X}$ and, when $X = \emptyset$ or $V(D)$, $D^X = D$.

This operation has been studied by Fisher and Ryan [2], Klostermeyer [3,4], Klostermeyer et al. [5,6], and MacGillivray and Wood [7]. Klostermeyer [4] proved that the problems of deciding whether a given digraph can be made acyclic, strongly connected, Hamiltonian, or semi-connected, using the push operation are NP-complete. Klostermeyer et al. [6] showed that any sufficiently large tournament can be pushed so as to have an exponential number of Hamilton cycles. By contrast, Klostermeyer [4] showed that every tournament on at least three vertices, except the two tournaments in Fig. 1, can be transformed into a Hamiltonian tournament by pushing. Thus, the problem of deciding if a tournament can be made Hamiltonian using the push operation is solvable in polynomial time. The authors in [5] generalized the result in [4] by characterizing

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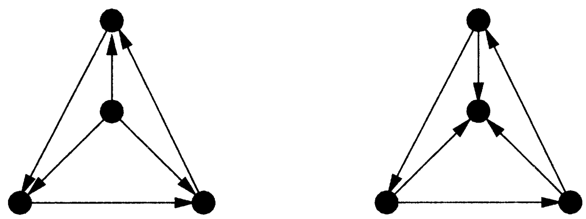

Fig. 1. The forbidden subtournaments for acyclically pushable tournaments.

which multipartite tournaments can be made Hamiltonian using the push operation. The problem of deciding whether a tournament can be made acyclic using the push operation is also solvable in polynomial time [7]. It turns out that the tournaments in Fig. 1 are precisely the obstructions; a tournament can be made acyclic using the push operation if and only if it contains neither of these as a subtournament.

In this paper, we characterize, in terms of forbidden subdigraphs, the multipartite tournaments which can be made acyclic (resp. ordinary, unidirectional) using the push operation. Our characterizations imply that the problems of deciding if a given multipartite tournament can be made acyclic (resp. ordinary, unidirectional) using the push operation and, if so, finding a suitable subset X of vertices to push, are solvable in polynomial time.

A digraph that contains no directed cycles is called *acyclic*. A digraph D is called *acyclically pushable* if there exists $X \subseteq V(D)$ such that D^X is acyclic. Terminology not defined in this paper follows [1] or [8].

Let x be a vertex of a digraph D. The *in-neighbourhood* $I(x)$ (resp. *out-neighbourhood* $O(x)$) of x is the set of vertices y such that there is an arc from y to x (resp. x to y). We use $\mathrm{id}_D(x)$ (resp. $\mathrm{od}_D(x)$) to denote the indegree (resp. outdegree) of x in D. The subscript will be omitted when D is clear from the context. We say vertex x *dominates* vertex y when there is an arc from x to y. For a subset $P \subseteq V(D)$, we shall use $O_P(x)$ (resp. $I_P(x)$) to denote the out-neighbourhood (resp. in-neighbourhood) of x contained in P. Again the subscript will be omitted when P is clear from the context.

A *tournament* is an oriented complete graph. A *multipartite tournament* is an oriented complete multipartite graph T; there is therefore a partition $P_1, P_2, \ldots, P_m$ of $V(T)$ such that there is an arc between two vertices $x \in P_i$ and $y \in P_j$ if and only if $i \neq j$. We refer to $P_1, P_2, \ldots, P_m$ as the *parts* of T. If $m = 2$, then T is called a *bipartite tournament* and (P_1, P_2) is called the *bipartition* of T. Clearly, every tournament is a multipartite tournament.

A multipartite tournament T is called *ordinary* if, for any pair of parts P and P' of T, all arcs between P and P' are directed either from P to P' or from P' to P. An ordinary multipartite tournament T is called *unidirectional* if its parts can be ordered $P_1, P_2, \ldots, P_m$ such that all arcs between P_i and P_j are directed from P_i to P_j whenever $i < j$. Clearly, every tournament is an ordinary multipartite tournament and every acyclic tournament is a unidirectional multipartite tournament.

Let T be a multipartite tournament. We define two vertices x and y in the same part of T to be *neighbourhood-comparable* if $O(x) \subseteq O(y)$, or $O(x) \subseteq I(y)$, or $I(x) \subseteq I(y)$, or $I(x) \subseteq O(y)$.

Preliminaries and the characterization of acyclically pushable bipartite tournaments are given in Section 2. Section 3 extends these results to multipartite tournaments. Finally, in Section 4 we characterize the multipartite tournaments which can be made ordinary and those which can be made unidirectional using the push operation.

2. Preliminaries

In this section, we give some preliminary results on general digraphs, multipartite tournaments, and bipartite tournaments. In particular, we shall characterize the bipartite tournaments which can be made acyclic using the push operation. The following is well known.

Lemma 2.1. *A digraph D is acyclic if and only if its vertices can be ordered as $v_1, v_2, \ldots, v_n$ such that the existence of an arc from v_i to v_j implies that $i < j$.*

We shall refer to the ordering $v_1, v_2, \ldots, v_n$ described in the above lemma as a *topological ordering* of an acyclic digraph D.

Corollary 2.2. *Let T be a bipartite tournament with bipartition (A, B). Then T is acyclic if and only if the vertices of A can be ordered as $a_1, a_2, \ldots, a_s$ such that $O(a_1) \supseteq O(a_2) \supseteq \cdots \supseteq O(a_s)$.*

When the neighbourhoods $O(a_1), O(a_2), \ldots, O(a_s)$ are such that $O(a_1) \supseteq O(a_2) \supseteq \cdots \supseteq O(a_s)$, we say the neighbourhoods are *nested*.

Lemma 2.3. *A digraph D is acyclically pushable if and only if every subdigraph of D is acyclically pushable.*

Proof. Clearly, if every subdigraph of D can be made acyclic using the push operation, then D can be made acyclic using the push operation as D is a subdigraph of itself. Conversely, suppose that D^X is acyclic for some $X \subseteq V(D)$. Then, for any subdigraph D' of D, $(D')^{X \cap V(D')}$ is a subdigraph of D^X and hence acyclic. □

Note that, for any $X, Y \subseteq V(D)$, $(D^X)^Y = (D^Y)^X = D^{X \triangle Y}$ where $X \triangle Y$ is the symmetric difference of X and Y.

Lemma 2.4. *Let D be acyclically pushable and let w be a vertex of D. Then there exists a set $Y \subseteq V(D)$ such that D^Y is acyclic and w is of indegree zero in D^Y.*

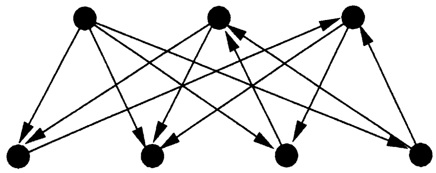

Fig. 2. The bipartite tournament T^*.

Proof. Since D is acyclically pushable, D^X is acyclic for some $X \subseteq V(D)$. Let $v_1, v_2, \ldots, v_n$ be a topological ordering of D^X. Thus $w = v_k$ with $1 \leqslant k \leqslant n$. Denote $Y = X \triangle \{v_1, v_2, \ldots, v_{k-1}\}$. Then D^Y is acyclic, as $w = v_k, v_{k+1}, \ldots, v_n, v_1, v_2, \ldots, v_{k-1}$ is a topological ordering of D^Y. $\square$

We use $D \cong D'$ to denote that the two digraphs D and D' are isomorphic. Observe that the relation $\equiv$ on all digraphs, defined by $D \equiv D'$ if and only if $D^X \cong D'$ for some $X \subseteq V(D)$, is an equivalence relation. The equivalence class that contains D shall be denoted by $[D]$. One can easily verify that the two tournaments in Fig. 1 form an equivalence class.

Fact 2.5. *Let D be a digraph and $D' \in [D]$. Then D is acyclically pushable if and only if D' is acyclically pushable.*

Using Lemma 2.4, it is easy to verify that the bipartite tournament T^* in Fig. 2 is not acyclically pushable. Thus by Lemma 2.3 and Fact 2.5 we have the following.

Lemma 2.6. *No acyclically pushable digraph contains any digraph in $[T^*]$ as a subdigraph.*

Lemma 2.7. *Let T be a multipartite tournament with P being a part of T and let $w \in P$ be of indegree zero. Suppose that T does not contain any digraph in $[T^*]$ as a subdigraph. Then the vertices of P can be linearly ordered as $w = a_1, a_2, \ldots, a_s$ such that $O(a_1) \supseteq H(a_2) \supseteq H(a_3) \supseteq \cdots \supseteq H(a_s)$, where $H(a_i) = O(a_i)$ or $I(a_i)$ for each $i = 2, 3, \ldots, s$.*

Proof. First, we show that every pair of vertices of P are neighbourhood-comparable. Suppose that there are two vertices $x, y \in A$ which are not neighbourhood-comparable. Since w is of indegree zero, $O(w)$ contains the in-neighbourhood as well as the out-neighbourhood of every vertex in P. Thus both x and y are in $P - \{w\}$. Since x and y are not neighbourhood-comparable, the definition implies that none of $O(x) \cap I(y)$, $O(x) \cap O(y)$, $I(x) \cap O(y)$, and $I(x) \cap I(y)$ are empty. Let $b_1 \in O(x) \cap I(y)$, $b_2 \in O(x) \cap O(y)$, $b_3 \in I(x) \cap O(y)$, and $b_4 \in I(x) \cap I(y)$. Then the subdigraph of T induced by $\{w, x, y, b_1, b_2, b_3, b_4\}$ contains T^* as a subdigraph, contradicting the assumption.

Next, we apply induction on s $(=|P|)$ to show that the vertices of P can be linearly ordered such that the desired properties are satisfied. Since every pair of vertice of

P are neighbourhood-comparable, the lemma is clearly true for $s = 1, 2$ and 3. So assume that $s > 3$ and that lemma is true when P contains fewer than s vertices. Suppose that $O(u) = \emptyset$ for some $u \in P - \{w\}$. Set $H(u) = O(u) = \emptyset$. By the induction hypothesis, the vertices in $P - \{w\}$ can be linearly ordered as $w = v_1, v_2, \dots, v_{s-1}$ such that $O(v_1) \supseteq H(v_2) \supseteq \cdots \supseteq H(v_{s-1})$. Then $v_1, v_2, \dots, v_{s-1}, u$ is a desired ordering of the vertices of P. Similarly, if $I(u)=\emptyset$, then we can obtain a desired ordering of the vertices of P. So we may assume that neither $O(u)$ nor $I(u)$ is empty for every $u \in P - \{w\}$, that is, $H(u) \neq \emptyset$ for any choice of $H(u)$.

Let $z \in P - \{w\}$ be an arbitrary vertex. By the induction hypothesis, the vertices of $P - \{z\}$ can be linearly ordered as $w = a_1, a_2, \dots, a_{s-1}$ such that $O(a_1) \supseteq H(a_2) \supseteq H(a_3) \supseteq \cdots \supseteq H(a_{s-1})$, where $H(a_i) = O(a_i)$ or $I(a_i)$ for each $i = 2, 3, \dots, s-1$. In the rest of the proof, we fix this choice of $H(a_i)$ for each $i = 2, 3, \dots, s-1$. Since z and a_{s-1} are neighbourhood-comparable, either $H(a_{s-1}) \supseteq H(z)$ or $H(z) \supseteq H_{s-1}$ for some choice of $H(z)$. If $H(a_{s-1}) \supseteq H(z)$ for some choice of $H(z)$, then we are done as we have $O(a_1) \supseteq H(a_2) \supseteq H(a_3) \supseteq \cdots \supseteq H(a_{s-1}) \supseteq H(z)$. Thus $H(z) \supseteq H(a_{s-1})$ for some choice of $H(z)$. We fix this choice of $H(z)$ so that $H(z) \supseteq H(a_{s-1})$. We claim that for each $i = 2, 3, \dots, s-2$, either $H(a_i) \supseteq H(z)$ or $H(z) \supseteq H(a_i)$. Indeed, suppose that there is some j, $2 \leqslant j \leqslant s-2$, such that $H(a_j) \not\supseteq H(z)$ and $H(z) \not\supseteq H(a_j)$. Let $c_1 \in H(z) - H(a_j)$, $c_2 \in H(a_j) - H(z)$ and $c_3 \in H(a_{s-1})$ (note that $H(a_{s-1})$ is not empty and is contained in $H(z) \cap H(a_j)$). Then it is easy to verify that the subdigraph induced by $\{w, z, a_j, a_{s-1}, c_1, c_2, c_3\}$ contains a digraph in $[T^*]$ as a subdigraph, a contradiction. Hence, for each $i = 2, 3, \dots, s-2$, either $H(a_i) \supseteq H(z)$ or $H(z) \supseteq H(a_i)$. Let k $(2 \leqslant k \leqslant s-1)$ be the least such that $H(z) \supseteq H(a_k)$. Then we have a desired ordering: $O(a_1) \supseteq \cdots \supseteq H(a_{k-1}) \supseteq H(z) \supseteq H(a_k) \supseteq \cdots \supseteq H(a_{s-1})$. $\square$

The following theorem characterizes acyclically pushable bipartite tournaments.

Theorem 2.8. *Let T be a bipartite tournament. Then T is acyclically pushable if and only if no digraph in $[T^*]$ is a subdigraph of T.*

Proof. By Lemma 2.6, we need only prove sufficiency. Suppose no digraph in $[T^*]$ is a subdigraph of T. Let (A, B) be the bipartition of T and let w be a vertex in A. Consider the bipartite tournament T' obtained from T by pushing $I(w)$. Clearly, w is of indegree zero in T' which does not contain any digraph in $[T^*]$ as a subdigraph. Hence, by Lemma 2.7, the vertices of A can be linearly ordered as $w = a_1, a_2, \dots, a_s$ such that $O(a_1) \supseteq H(a_2) \supseteq H(a_3) \supseteq \cdots \supseteq H(a_s)$, where $H(a_i) = O(a_i)$ or $I(a_i)$ for each $i = 2, 3, \dots, s$. Set $X = \{a_i \mid H(a_i) = I(a_i)\}$. Then the out-neighbourhoods of the vertices of A are nested in $(T')^X$. Hence, by Corollary 2.2, $(T')^X$ is acyclic. $\square$

3. Acyclic multipartite tournaments

According to Theorem 2.8, the obstructions for acyclically pushable bipartite tournaments consist of one equivalence class, namely $[T^*]$. As shown in [7], this is also

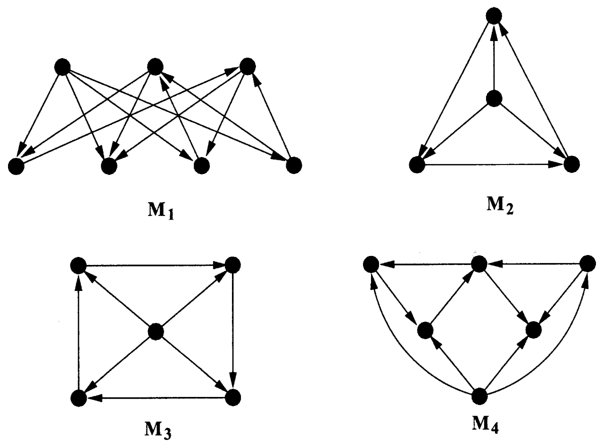

Fig. 3. The generators of the equivalence classes of forbidden subdigraphs for acyclically pushable multipartite tournaments.

true for acyclically pushable tournaments. For acyclically pushable multipartite tournaments, the situation is more complex. The reader should have little difficulty verifying that none of the digraphs in Fig. 3 are acyclically pushable (note that the bipartite tournament T^* in Fig. 2 is renamed as M_1 in Fig. 3).

Lemma 3.1. *None of* M_1, M_2, M_3, M_4 *in Fig.* 3 *are acyclically pushable.*

Theorem 3.2. *Let* T *be a multipartite tournament. Then* T *is acyclically pushable if and only if* T *contains no* $M \in \bigcup_{i=1}^{4} [M_i]$ *as a subdigraph.*

Proof. We only show sufficiency, as necessity follows immediately from Lemmas 3.1 and 2.3. Suppose that T contains no $M \in \bigcup_{i=1}^{4} [M_i]$ as a subdigraph. Let w be a vertex of T and let P be the part of T that contains w. We may assume without loss of generality that $id(w) = 0$, as otherwise we consider $T^{I(w)}$ instead of T. We will prove that T is acyclically pushable. By Lemma 2.4, it suffices to show that there exists a set $S \subset P$ such that T^S is acyclic.

If T contains no cycles, then let $S = \emptyset$ and we are done. So assume that T contains a cycle. Since T is a multipartite tournament, it is easy to see that T contains a cycle of length three or four. If there is a cycle (of length three or four) in $T - P$, then T contains either M_2 or M_3 as a subdigraph, contradicting the assumption. So every cycle of T must contain at least one vertex in P.

Since T contains no digraph in $[M_1]$ as a subdigraph, by Lemma 2.7 the vertices of P can be linearly ordered as $w = p_1, p_2, \ldots, p_s$ in such a way that $O(p_1) \supseteq H(p_2) \supseteq \cdots \supseteq H(p_s)$, where $H(p_i)$ is either $O(p_i)$ or $I(p_i)$, for $i = 2, 3, \ldots, s$. Let $X = \{p_i \mid H(p_i) = I(p_i)\}$ and consider T^X. Then the out-neighborhoods of the vertices in P are nested in T^X. If T^X contains no cycles, then we are done. So assume that T^X contains some cycles. Then T^X contains a cycle of length three or four. We claim that T^X must contain a cycle of length three. Suppose that $uxyzu$ with $u \in P$ is a cycle of length four in T^X. Since the neighborhoods of the vertices in P are nested in T^X, $P \cap \{u,x,y,z\} = \{u\}$. Thus, there is an arc between u and y and hence there is a cycle of length three contained in the subdigraph induced by $\{u,x,y,z\}$. Therefore T^X contains a cycle of length three.

Let $Y = P - \{w\}$ and we prove that $(T^X)^Y$ $(=T^{X\triangle Y})$ is acyclic. Suppose it is not acyclic. Then a similar argument as above shows that $T^{X\triangle Y}$ contains a cycle of length three. Hence there are two vertices $p_i, p_j \in P$ (not necessarily distinct) such that p_i is in a cycle of length three in T^X and p_j is in a cycle of length three in $T^{X\triangle Y}$. This means that there are vertices a, a', b, b' such that $p_ia, aa', a'p_i, p_jb, b'b, b'p_j$ are arcs in T^X. If $p_i = p_j$, then the subdigraph of T^X induced by $\{w, p_i, p_j, a, b, a', b'\}$ contains either M_4 or a digraph in $[M_3]$ as a subdigraph. This means that T contains a digraph in $[M_3] \cup [M_4]$ as a subdigraph, a contradiction. So $p_i \neq p_j$. Since neighbourhoods of p_i and p_j are nested in T^X, we have in T^X either $O(p_i) \subseteq O(p_j)$ or $O(p_i) \supseteq O(p_j)$.

We only consider the case when $O(p_i) \subseteq O(p_j)$, as the case when $O(p_i) \supseteq O(p_j)$ can be discussed in a similar way. Since $O(p_i) \subseteq O(p_j)$, we have $I(p_i) \supseteq I(p_j)$ and hence both p_ja and $b'p_i$ are arcs of T^X. There must be an arc between b and p_i as p_i is in the same part as p_j which is adjacent to b. If p_ib is an arc, then $\{w, p_i, a, a', b, b'\}$ induces in T^X a subdigraph containing M_4 or a digraph in $[M_3]$ as a subdigraph. This implies that T contains a digraph in $[M_3] \cup [M_4]$ as a subdigraph, contradicting the assumption. Thus bp_i is an arc of T^X. Similarly, p_ja' is an arc of T^X. Note that a' and b could be the same vertex. In the case when $a' = b$, $\{a, a', b', p_i, p_j\}$ induces a subdigraph containing a digraph in $[M_3]$, a contradiction. Hence $a' \neq b$. We claim that there is no arc between a and b'. Indeed, if ab' is an arc of T^X, then the subdigraph of T^X induced by $\{w, p_j, a, b, b'\}$ contains a digraph in $[M_3]$ as a subdigraph, a contradiction; if $b'a$ is an arc of T^X, then a digraph of $[M_3]$ is contained in the subdigraph of T^X induced by $\{w, p_i, a, a', b'\}$, a contradiction. Therefore a and b' are not adjacent in T^X. This implies that there must be an arc between a and b and an arc between a' and b'. If ba is an arc, then a digraph of $[M_3]$ is contained in the subdigraph of T^X induced by $\{w, p_i, a, a', b\}$, a contradiction. Thus ab is an arc of T^X. Now the subdigraph of T^X induced by $\{p_i, p_j, a, b, b'\}$ contains a digraph in $[M_3]$, a contradiction. □

The above theorem and proof imply a polynomial algorithm to decide if a given multipartite tournament is acyclically pushable and, if it is, to find a suitable set of vertices to push.

4. Ordinary and unidirectional multipartite tournaments

In this section, we characterize the multipartite tournaments which can be made ordinary as well as those which can be made unidirectional using the push operation. Note that a multipartite tournament T can be made ordinary (resp. unidirectional) if and only if every induced subdigraph of T can be made ordinary (resp. unidirectional).

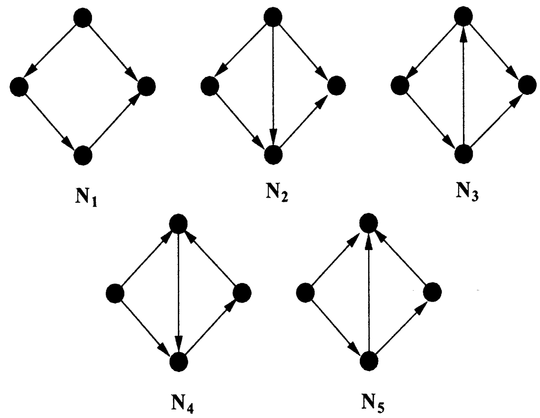

Fig. 4. The forbidden subdigraphs for the multipartite tournaments which can be made ordinary by pushing.

Lemma 4.1. *Let T be a multipartite tournament. Then T can be made ordinary by pushing if and only if for any two vertices x and y in the same part, either $O(x)=O(y)$ or $O(x)=I(y)$.*

Proof. Suppose that $O(x) \neq O(y)$ and $O(x) \neq I(y)$. By symmetry, we may assume that $O(x) \subset O(y)$. If $O(x) \neq \emptyset$, then let $a \in O(y) - O(x)$ and $b \in O(x) \cap O(y)$ and we see that $\{x, y, a, b\}$ induces a subdigraph which cannot be made ordinary by pushing, a contradiction. So assume that $O(x) = \emptyset$. Then $I(y) \neq \emptyset$ as $O(x) \neq I(y)$. Since $I(x) = O(y) \cup I(y)$, we must have $I(x) \cap O(y) \neq \emptyset$ and $I(x) \cap I(y) \neq \emptyset$. Let $c \in I(x) \cap O(y)$ and $d \in I(x) \cap I(y)$. We see that $\{x, y, c, d\}$ induces a subdigraph which cannot be made ordinary by pushing, a contradiction.

Conversely, suppose that, for any two vertices x and y in the same part of T, either $O(x) = O(y)$ or $O(x) = I(y)$. Let $P_1, P_2, \ldots, P_m$ be the parts of T. Thus each part P_i can be partitioned into P_i' and P_i'' such that the vertices of P_i' have the same out-neighbourhood, which is the in-neighbourhood of each vertex in P_i''. Define $X = P_1' \cup P_2' \cup \cdots \cup P_m'$. It is now easy to see that T^X is an ordinary multipartite tournament. □

Theorem 4.2. *Let T be a multipartite tournament. Then T can be made ordinary by pushing if and only if it contains none of the digraphs in Fig.* 4.

Proof. Clearly, none of the digraphs in Fig. 4 can be made ordinary by pushing. Thus if T contains any digraph in Fig. 4, then it cannot be made ordinary by pushing. Conversely, if T cannot be made ordinary by pushing, then by Lemma 4.1 there exist two vertices x and y in the same part of T such that $O(x) \neq O(y)$ and $O(x) \neq I(y)$. The digraphs in Fig. 4 can be easily derived from the proof of Lemma 4.1. □

Theorem 4.3. *A multipartite tournament T can be made unidirectional if and only if it contains none of the digraphs in Fig.* 1 *or Fig.* 4.

Proof. Clearly, none of the digraphs in Fig. 1 or in Fig. 4 can be made unidirectional. Thus if T can be made unidirectional then it contains none of the digraphs in Fig. 1 or Fig. 4. Conversely, suppose that T contains none of the digraphs in Fig. 1 or Fig. 4. Then by Theorem 4.2 T^X is ordinary for some $X \subseteq V(T)$. Let u be an arbitrary vertex in T^X and let $Y = I_{T^X}(u)$. Then it can be easily verified that $(T^X)^Y = T^{X \triangle Y}$ is unidirectional. $\square$

It is again easy to see that the problems of deciding if a given multipartite tournament can be made ordinary (resp. unidirectional) using the push operation and, if so, finding a suitable set of vertices to push are solvable in polynomial time.

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