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Nucleon–meson coupling constants and form factors in the quark model

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Abstract

We demonstrate the calculation of the coupling constants and form factors required by effective hadron Lagrangians using the quark model. These relations follow from equating expressions for strong transition amplitudes in the two approaches. As examples we derive the NNm nucleon–meson coupling constants and form factors for $m = \pi, \eta, \eta', \sigma, a_0, \omega$ and ρ, using harmonic oscillator quark model meson and baryon wavefunctions and the $^3\mathrm{P}_0$ decay model; these results are relevant to quark-based descriptions of the NN force. This technique should be useful in the application of effective Lagrangians to processes in which the lack of data precludes the direct determination of coupling constants and form factors from experiment.

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1. Introduction

Effective hadronic Lagrangians are widely used in the description of the interactions of hadrons. In this method a distinct quantum field is introduced for each relevant hadronic species, and interactions are assumed for these fields that are consistent with known symmetries and conservation laws. Although a more physically justified description of hadron scattering amplitudes would employ momentum-dependent coupling constants (form factors), it often suffices near threshold to assume pointlike “hard” hadronic vertices with fixed coupling constants. This approximation may be relaxed by incorporating hadronic form factors as a power series of gradient interactions. Unfortunately, this leads to non-renormalizable ultraviolet divergences, which requires the determination of many coupling constants from experiment.

When using effective Lagrangians one typically ignores the existence of hadronic substructure (quarks and gluons), and determines each coupling constant in the effective Lagrangian from experiment. Although this procedure is feasible in experimentally highly constrained problems such as the NN interaction, in other processes with little data the coupling constants are simply assigned plausible values. In such cases it would be very useful to have numerical estimates of the coupling constants. Since these coupling constants and form factors are determined by the underlying QCD degrees of freedom, one may evaluate them directly in terms of the interactions of quarks and gluons and the substructure of the hadronic bound states.

In this Letter we will investigate this relation between effective Lagrangian couplings and quark model bound state hadron wavefunctions. The specific cases we consider are the meson–nucleon couplings, which are usually fitted to data in meson exchange models of the NN force. These NNm vertices are chosen as our initial examples in part because they are among the most important hadronic couplings for nuclear physics applications, and also because the NNπ coupling constant $g_{\mathrm{NN}\pi} \approx 13.5$ is the best determined strong coupling in hadron physics. It is clearly of interest to determine whether this experimental cou-

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pling constant is consistent with the value predicted using quark model hadron wavefunctions.

The results presented here are also relevant to composite models of the NN force that are based on both quark and meson degrees of freedom. In this type of model the short ranged NN interactions are attributed to quark–gluon forces encountered by overlapping nucleon quark wavefunctions, whereas the long ranged NN interactions are described by t-channel meson exchange. The associated meson–nucleon couplings and form factors, which we compute here, are determined by the convolution of meson and nucleon wavefunctions with a quark pair creation operator.

2. Technique

Our method for defining effective hadronic coupling constants and form factors is to equate specific hadron emission amplitudes predicted by the effective hadron Lagrangian to the corresponding decay amplitude in the quark model. Applied to NNm couplings in particular, we require that

$$\langle Nm|H_{\mathrm{eff}}|N\rangle = \big\langle N\big(q^3\big)m(q\bar{q})\big|H_{\mathrm{decay}}\big|N\big(q^3\big)\big\rangle. \tag{1}$$

This general approach was used previously by the Orsay group of Le Yaouanc et al. [1], who considered $\rho\pi\pi$, NNπ and NNρ couplings; the relation between this earlier reference and our results will be discussed. A similar quark model approach was more recently applied to the determination of HQET electroweak form factors by Isgur et al. [2].

Ideally one would impose this relation using a relativistic quark model, in which case there would be no difficulty in identifying the effective Lagrangian matrix element with the quark model result. As the non-relativistic quark model formalism is much better established, we will instead use non-relativistic quark model wavefunctions, and apply our defining relation Eq. (1) between matrix elements near threshold. Since the quark model wavefunctions we use are non-relativistic, the form factors we extract by equating quark model and (relativistic) field theory matrix elements have an ambiguity in how we relate results in different frames. In this initial study we will simply assume a particular frame, the “decay frame” of a rest initial hadron, as in the earlier work of the Orsay group [1]. The complication of implicit frame dependence will be investigated in detail a subsequent study.

It is important to note that these frame ambiguities are resolved in the weak binding and non-relativistic limits, in which the quark model states approach the transformation properties specified by the Lorentz group. Thus our quark model predictions of form factors are best justified at zero three-momentum, where we define the coupling constants. Of course the model assumption is that the predicted form factors remain useful estimates at moderate non-zero momentum.

The specific quark model strong decay Hamiltonian we use is the $^3\mathrm{P}_0$ model [1,3,4], which has seen very wide application, and is known to give reasonably accurate numerical results for both meson [5–7] and baryon [8] strong decays.

2.1. NNπ, NNη and NNη'

As a first example we consider the NNπ coupling. We assume the standard pseudoscalar effective Lagrangian

$$\mathcal{L}_{\mathrm{NN}\pi} = -i g_{\mathrm{NN}\pi}\bar{\Psi}\gamma_5\vec{\tau}\Psi\cdot\vec{\phi}_\pi, \tag{2}$$

where $\Psi = [\psi_p, \psi_n]$ is an isodoublet of Dirac nucleon fields, the $\vec{\tau}$ are isospin Pauli matrices, and $\vec{\phi}_\pi$ is an isotriplet of pion fields, $[\phi_{\pi^+}, \phi_{\pi^0}, \phi_{\pi^-}]$. Specializing to the transition $p(+1/2)\to p(+1/2)\pi^0$ (an initial spin-up proton at rest going to a spin-up proton with momentum $-\vec{\mathrm{P}}$ and a recoiling π^0 with momentum $+\vec{\mathrm{P}}$), we find the matrix element

$$\begin{aligned}&\big\langle p\pi^0\big|H_{\mathrm{eff}}|p\big\rangle/\delta(\vec{\mathrm{P}}_f-\vec{\mathrm{P}}_i)\\ &\quad\equiv h_{fi}\\ &\quad= i g_{\mathrm{NN}\pi}\frac{M_p}{E_p}\frac{1}{\sqrt{(2\pi)^3 2E_\pi}}\left[\frac{\mathrm{P}\cos(\theta)}{\sqrt{2M_p(E_p+M_p)}}\right].\end{aligned} \tag{3}$$

We similarly evaluate the quark model matrix element of the $^3\mathrm{P}_0$ decay model Hamiltonian for $p(+1/2)\to p(+1/2)\pi^0$, using the techniques and Gaussian baryon and meson wavefunctions given in Refs. [4,9]. The result is

$$\begin{aligned}&\big\langle p\big(q^3\big)\pi^0(q\bar{q})\big|H_{\mathrm{decay}}\big|p\big(q^3\big)\big\rangle/\delta(\vec{\mathrm{P}}_f-\vec{\mathrm{P}}_i)\\ &\quad\equiv h_{fi}\\ &\quad= -\gamma\frac{10}{9\pi^{3/4}}\frac{(1+r^2/4)r^{3/2}}{(1+r^2/3)^{5/2}}\frac{\mathrm{P}\cos(\theta)}{\alpha^{3/2}}\\ &\qquad\times\exp\left\{-\frac{1}{6}\frac{(1+5r^2/12)}{(1+r^2/3)}\left(\frac{\mathrm{P}}{\alpha}\right)^2\right\},\end{aligned} \tag{4}$$

where α and $\beta=\alpha/r$ are the baryon and meson Gaussian quark wavefunction inverse length scales, and γ is the dimensionless $^3\mathrm{P}_0$ model $q\bar{q}$ pair production amplitude. This result is consistent with the earlier result of Le Yaouanc et al. [1], given the parameter relations between that reference and the current work, $R_{\mathrm{N}}=1/\alpha$, $R_\pi=1/\beta$ and $\gamma_{\mathrm{LeY}}=\sqrt{24\pi}\gamma$. Typical values for these parameters found in the literature are $\alpha=0.25$–0.4 GeV, $\beta=0.3$–0.4 GeV, and $\gamma=0.4$–0.5. Here we fix $\gamma=0.4$ (taken from our extensive studies of light and charmed meson strong decays [4–7]), and show numerical results for this range of α and β.

The NNπ coupling constant we find by equating these expressions at threshold is

$$g_{\mathrm{NN}\pi} = \gamma\frac{2^4\cdot 5}{3^2}\pi^{3/4}\frac{(1+r^2/4)r^{3/2}}{(1+r^2/3)^{5/2}}\frac{M_p m_\pi^{1/2}}{\alpha^{3/2}}. \tag{5}$$

(We have suppressed an overall phase factor of $+i$, which is normalization convention dependent.) The numerical value of this coupling constant as a function of α and β is shown in Fig. 1. Evidently the range of typical wavefunction length scale parameters α and β leads to a factor of two variation in the theoretical $g_{\mathrm{NN}\pi}$; it ranges between 7.0–12.2 for the parameters shown in the figure. The experimental value of ≈ 13.5 evidently corresponds to values of α and β near the lower end of their respective ranges, provided that we fix the $q\bar{q}$ pair production amplitude at the meson decay value of $\gamma=0.4$.

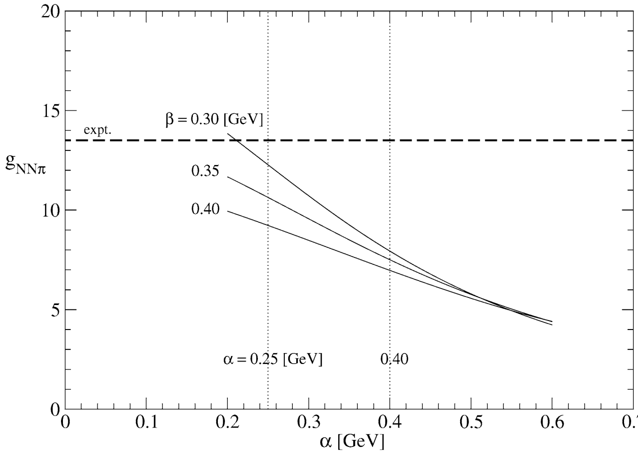

Fig. 1. The theoretical pion–nucleon coupling constant $g_{\mathrm{NN}\pi}$ (Eq. (5)) as a function of the quark model Gaussian wavefunction length scales α (baryons) and β (mesons). The range of values of α and β typically found in the quark model literature is indicated (see text). The $^3\mathrm{P}_0$ model $q\bar{q}$ pair production amplitude $\gamma = 0.4$ was taken from studies of meson decays. The experimental $g_{\mathrm{NN}\pi} \approx 13.5$ is also shown.

Since the "experimental" value of $g_{\mathrm{NN}\pi}$ is not actually based on a direct observation of pion emission, it is prudent to carry out an independent calculation of a closely related decay process that does involve a detected pion in the final state. The transitions $\Delta \to \mathrm{N}\pi$ are useful in this regard because the matrix elements are related to the $\mathrm{NN}\pi$ coupling by SU(6) symmetry, assuming identical spatial wavefunctions. Specializing to $\Delta^{++} \to p\pi^+$, the quark model result for the total width is

$$\begin{aligned}\Gamma(\Delta^{++} \to p\pi^+) \\ &= \gamma^2 \pi^{1/2} \frac{2^8}{3^3} r^3 \frac{(1+r^2/4)^2}{(1+r^2/3)^5} \frac{E_p E_\pi}{M_\Delta} \left(\frac{\mathrm{P}}{\alpha}\right)^3 \\ &\quad \times \exp\left\{-\frac{1}{3} \frac{(1+5r^2/12)}{(1+r^2/3)} \left(\frac{\mathrm{P}}{\alpha}\right)^2\right\}. \qquad (6)\end{aligned}$$

This theoretical quark model decay rate is shown in Fig. 2 for the same range of wavefunction parameters α, β as $g_{\mathrm{NN}\pi}$ in Fig. 1, and the experimental width of 110 MeV is also shown. Evidently the parameter constraints due to $g_{\mathrm{NN}\pi}$ and $\Gamma(\Delta \to \mathrm{N}\pi)$ are approximately consistent.

Alternatively, since the dimensionless pair production amplitude γ represents poorly understood non-perturbative physics, it may have a different strength in baryon decays than in meson decays. In Fig. 3 we show the ratio of theory to experiment for $\Gamma(\Delta^{++} \to p\pi^+)$ and $g_{\mathrm{NN}\pi}$ for our "standard" quark model baryon and meson wavefunction length scales $\alpha = \beta = 0.4$ GeV, varying the pair production amplitude γ. Evidently with these wavefunctions a value of $\gamma \approx 0.7$ is favored for pion emission from light baryons, whereas $\gamma \approx 0.4$–0.5 gives the best description of light meson decays.

Nucleon couplings to the other ground state pseudoscalars (η and η') are simply related to $g_{\mathrm{NN}\pi}$, provided that we assume identical spatial wavefunctions and pure $q\bar{q}$ states. Given the effective Lagrangian

$$\mathcal{L}_{\mathrm{NN}\eta(')} = -i g_{\mathrm{NN}\eta(')} \bar{\Psi} \gamma_5 \Psi \phi_{\eta(')}, \qquad (7)$$

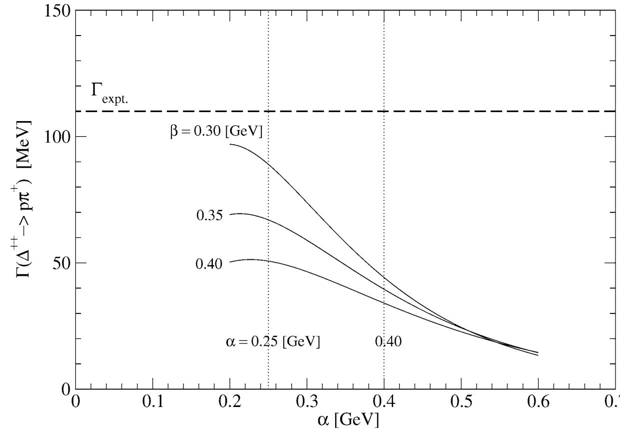

Fig. 2. The theoretical $^3\mathrm{P}_0$ quark model $\Gamma(\Delta^{++} \to p\pi^+)$ width (Eq. (6)) as a function of the wavefunction length scales α and β, for a pair production amplitude of $\gamma = 0.4$. The experimental width is also shown.

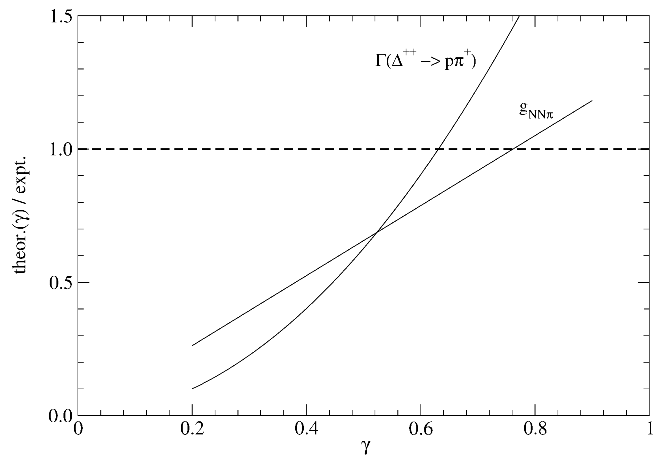

Fig. 3. The ratio of theory and experiment for $\Gamma(\Delta^{++} \to p\pi^+)$ and $g_{\mathrm{NN}\pi}$ given our standard quark model wavefunction length scales $\alpha = \beta = 0.4$ GeV, but with the pair production amplitude γ variable.

and taking the $|\eta\rangle$ and $|\eta'\rangle$ flavor states to be the maximally mixed linear combinations $(|n\bar{n}\rangle \pm |s\bar{s}\rangle)/\sqrt{2}$, the $\mathrm{NN}\eta(')$ coupling constants and form factors are related to the $\mathrm{NN}\pi$ results by

$$g_{\mathrm{NN}\eta(')} = \frac{3}{2^{1/2} \cdot 5} \left(\frac{m_{\eta(')}}{m_\pi}\right)^{1/2} g_{\mathrm{NN}\pi}. \qquad (8)$$

If we use $g_{\mathrm{NN}\pi} = 13.5$ as input, this gives $g_{\mathrm{NN}\eta} = 11.5$ and $g_{\mathrm{NN}\eta'} = 15.3$. Although these appear to be rather large $\mathrm{NN}\eta(')$ couplings, their effect on NN scattering is suppressed by the larger η and η' masses in propagators and in the $1/\sqrt{2E}$ external line normalizations.

2.2. $\mathrm{NN}\sigma$

The $\mathrm{NN}\sigma$ coupling may be the most important nucleon–meson coupling in all of nuclear physics. In meson exchange models the exchange of a light scalar $I = 0$ "sigma" meson is held to be the dominant mechanism underlying the intermediate ranged attraction, which is responsible for nuclear binding. The balance between this attraction and the short distance repulsion in the nuclear core determines the equilibrium density of $n_0 \approx 0.16$ nucleons/fm^3 in bulk nuclear matter. (Although pions are lighter and hence longer-ranged, and the $g_{\mathrm{NN}\pi}$ coupling constant is quite large, the fact that pions are emitted in a relative P-wave suppresses their contribution to the near-threshold interactions of nucleons.)

Although the intermediate ranged attraction plays a crucial role in nuclear physics, the σ meson itself is a dubious concept in meson spectroscopy. In $I=0$ $\pi\pi$ S-wave scattering one sees a very broad positive (attractive) phase shift, which if interpreted as an s-channel resonance would imply a mass of *ca.* 1 GeV and a comparably large width. There are arguments from the quark model against a $q\bar{q}$ state with these properties; for example, an $I=0$ 0^{++} $n\bar{n}$ resonance ($n=u,d$) at this mass would have a rather large two-photon width of $\Gamma_{\gamma\gamma} \approx 2$ keV, and no such state is evident in $\gamma\gamma \to \pi^0\pi^0$. There is instead evidence in this reaction for a moderately broad scalar enhancement near 1.3 GeV, with about the expected two-photon width [10][1]; this broad $f_0(\approx 1300)$ is often identified with the $I=0$ 0^{++} $n\bar{n}$ quark model state.

The explicit σ meson included in meson exchange models has been explained as a parametrization of "correlated two-pion exchange", so that its fitted strong coupling to NN and low mass need not correspond to the properties of a physical P-wave $n\bar{n}$ meson. The picture of the "sigma meson" as a strongly mixed $(n\bar{n})$–$\pi\pi$ state is supported by the large coupling predicted between these channels in the $^3\mathrm{P}_0$ model; the analogous S-wave kaon system is discussed in Ref. [11].

We can test the plausibility of sigma meson exchange models of the intermediate ranged NN attraction by calculating the NNσ coupling for a pure $n\bar{n}$ σ meson, using the same techniques we applied above to the NNπ system. If the sigma is dominantly a physical $n\bar{n}$ scalar meson, we would expect approximate agreement between the calculated and fitted $g_{\mathrm{NN}\sigma}$ coupling constants. If the sigma is instead a parametrization of two-pion exchange, agreement between the theoretical and fitted coupling constants would be fortuitous.

The calculation of the NNσ coupling differs from the NNπ case through the meson spin, space and isospin wavefunctions and the effective Lagrangian. We assume the form

$$\mathcal{L}_{\mathrm{NN}\sigma} = -g_{\mathrm{NN}\sigma} \bar{\Psi}\Psi\phi_\sigma. \tag{9}$$

In our quark model description the $|\sigma\rangle$ is the usual $I=0$ $|n\bar{n}\rangle$ flavor state $(|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}$ times the $|J,L,S\rangle = |0,1,1\rangle$ angular momentum state

$$\frac{1}{\sqrt{3}}\big(|1,+1\rangle|1,-1\rangle - |1,0\rangle|1,0\rangle + |1,-1\rangle|1,+1\rangle\big)$$

where the basis states are $|L,L_z\rangle|S,S_z\rangle$. The P-wave momentum space $q\bar{q}$ wavefunctions are given in Ref. [4]. On equating the effective Lagrangian and quark model $h_{fi} = \langle p\sigma|H|p\rangle$ matrix elements, we find

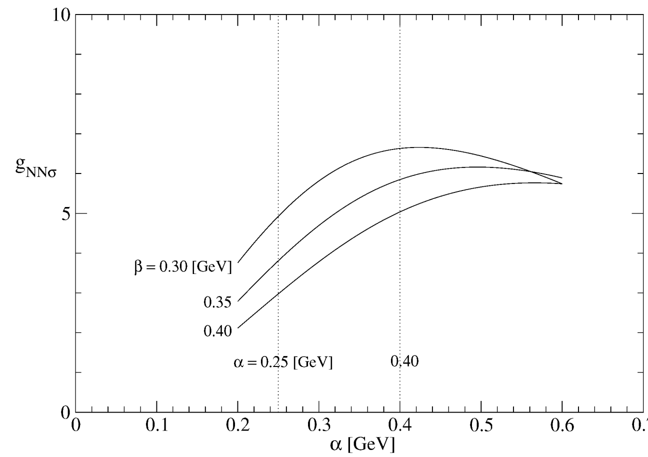

Fig. 4. The theoretical nucleon–sigma meson coupling constant $g_{\mathrm{NN}\sigma}$ (Eq. (10)).

$$g_{\mathrm{NN}\sigma} = \gamma 2^{5/2} 3^{1/2} \pi^{3/4} \frac{r^{5/2}}{(1+r^2/3)^{5/2}} \frac{m_\sigma^{1/2}}{\alpha^{1/2}}. \tag{10}$$

(A normalization convention dependent overall phase of (-1) in our result is suppressed here.) The NNσ form factor is the quadratic $(1 + [(1+r^2/4)/9(1+r^2/3)](\mathrm{P}/\alpha)^2)$ times the Gaussian found in the NNπ case in Eq. (4).

The numerical $g_{\mathrm{NN}\sigma}$ predicted by Eq. (10) is shown in Fig. 4 as a function of α and β (assuming $m_\sigma = 500$ MeV). Evidently a value in the range 3–7 is predicted by the quark model given this m_σ, with $g_{\mathrm{NN}\sigma} \approx 5$ preferred.

Although it is of great interest to compare our calculated $g_{\mathrm{NN}\sigma}$ coupling constant with the values reported in meson exchange model fits to NN scattering data, there is unfortunately no single consistent value reported for $g_{\mathrm{NN}\sigma}$ in these models. The three best known meson-exchange models of the NN force in the literature are the Paris [12,13], Nijmegen [14–16] and Bonn [17] models, and their NNm couplings are given in Table 1, together with our quark model results. The Paris model did not consider a σ meson. In the recent "CD-Bonn" model [17], different $g_{\mathrm{NN}\sigma}$ coupling constants and σ masses are assumed in different NN channels; in the $I=0$ $^3\mathrm{S}_1$ NN channel a σ mass and coupling constant of $m_\sigma = 350$ MeV and $g_{\mathrm{NN}\sigma} \approx 2.5$ are used, whereas in $I=1$ $^1\mathrm{S}_0$, $m_\sigma = 452$ MeV and $g_{\mathrm{NN}\sigma} \approx 7.3$ are used. The $g_{\mathrm{NN}\sigma}$ coupling is allowed to vary with L and I in the $L>0$ NN channels, and ranges from 1.9 to 9.9 (see Table XVI of Ref. [17]). In the Nijmegen model [15] a larger value of $g_{\mathrm{NN}\sigma} = 17.9$ is quoted. Thus, although our quark model result $g_{\mathrm{NN}\sigma} \approx 5$ is similar to the mean S-wave NN value quoted in the CD-Bonn model, the scatter in the fitted values of this parameter precludes an accurate comparison between theory and experiment at present.

One may similarly evaluate the quark model prediction for the NN coupling of the a_0 $I=1$ partner of the σ. Given the effective Lagrangian

$$\mathcal{L}_{\mathrm{NN}a_0} = -g_{\mathrm{NN}a_0} \bar{\Psi}\vec{\tau}\Psi \cdot \vec{\phi}_{a_0} \tag{11}$$

[1] Rather surprisingly, the PDG has chosen to report the $\gamma\gamma \to f_0(1300) \to \pi^0\pi^0$ coupling under the entry for the broad $f_0(600)$, despite the fact that the scalar part of the $\pi^0\pi^0$ invariant mass distribution peaks near 1250 MeV.

Table 1
A summary of NNm coupling constants. Our calculated values are shown in the middle columns, and fitted or assumed values in the meson exchange model literature are shown at right. Values in square brackets were fixed input

Coupling	This Ref.[a]	This Ref.[b]	Paris [12,13]	Nijmegen [15]	Bonn [17]
$g_{\mathrm{NN}\pi}$	7.1	[13.5]	[13.3]	13.3	[13.1]
$g_{\mathrm{NN}\eta}$	6.0	11.5	–	9.8	–
$g_{\mathrm{NN}\eta'}$	7.9	15.3	–	10.5	–
$g_{\mathrm{NN}\sigma(\sim 500)}$	5.0		–	17.9	(2.5; 7.3)[c]
$g_{\mathrm{NN}a_0(\sim 1300)}$	2.7		–	3.3	–
$g_{\mathrm{NN}\omega}$ (γ_μ)	30.2		12.2	12.5	15.9
$g_{\mathrm{NN}\rho}/g_{\mathrm{NN}\omega}$ (γ_μ)	$+1/3$[d]		0.43[e]	0.22	0.20
κ_ω $(\sigma_{\mu\nu}/\gamma_\mu)$	$-3/2$		-0.12	0.66	0
κ_ρ $(\sigma_{\mu\nu}/\gamma_\mu)$	$+3/2$		–	6.6	6.1

[a] Assumes "standard" quark model parameters $\alpha = \beta = 0.4$ GeV and $\gamma = 0.4$ (see text).
[b] Assumes $g_{\mathrm{NN}\pi} = 13.5$.
[c] This "CD-Bonn" model introduces different $g_{\mathrm{NN}\sigma}$ coupling constants for ($I = 0$; $I = 1$) NN channels, which would not be expected for an isosinglet σ. In addition these $g_{\mathrm{NN}\sigma}$ couplings and the σ mass are allowed to vary with L (S-wave is quoted here), and a higher-mass σ with large $g_{\mathrm{NN}\sigma}$ couplings is also assumed.
[d] Assumes $m_\rho = m_\omega$.
[e] This value is cited in Ref. [12] but is not actually used in the Paris model, which does not incorporate ρ exchange.

we find

$$g_{\mathrm{NN}a_0} = \frac{1}{3}\left(\frac{m_{a_0}}{m_\sigma}\right)^{1/2} g_{\mathrm{NN}\sigma}. \tag{12}$$

Although $I = 1$ scalar exchange contributes a somewhat smaller amplitude to NN scattering than $I = 0$ exchange, it may nonetheless be possible to test for the presence of both of these scalar meson exchange amplitudes through their interference, for example by comparing the $I = 0$ and $I = 1$ S-wave NN scattering amplitudes discussed above.

2.3. NNω and NNρ

The NNV couplings are interesting in that the short ranged repulsive core in the NN interaction has previously been attributed to vector meson exchange, and the existence of the ω meson was regarded as support for this picture. This mechanism now appears less plausible, since the short range of vector meson exchange ($R \sim 1/m_\omega \sim 0.25$ fm) implies extensive overlap of the NN quark wavefunctions.

Evaluation of the NNV couplings and form factors uses the same procedure as the scalar and pseudoscalar couplings discussed above, although there are complications due to the presence of two form factors and the non-transverse components of the vector field.

As above we assume a term in the effective Lagrangian for each coupling. For NNω this Lagrangian is

$$\mathcal{L}_{\mathrm{NN}\omega} = -g_{\mathrm{NN}\omega}\left(\bar{\Psi}\gamma_\mu\Psi\omega_\mu - \frac{\kappa_\omega}{4M_{\mathrm{N}}}\bar{\Psi}\sigma_{\mu\nu}\Psi \mathrm{F}_{\mu\nu}\right), \tag{13}$$

where $\mathrm{F}_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$. We then equate near-threshold Hamiltonian matrix elements h_{fi} found from this effective Lagrangian to the corresponding $^3\mathrm{P}_0$ decay model matrix elements. There is a complication in relating the relativistic effective Lagrangian and non-relativistic quark model matrix elements; we find that one must assume a vector meson polarization vector and four-momentum of the form $\epsilon_\mu = (0, \hat{\epsilon})$ and $q_\mu = (0, \vec{q})$ to equate these expressions. The NNω γ_μ and $\sigma_{\mu\nu}$ form factors may be separated by equating h_{fi} matrix elements with different spin states. The transitions $p(+1/2) \to p(+1/2)\omega(0)$ and $p(+1/2) \to p(+1/2)\omega(-1)$ are useful in this regard, since they receive contributions from only the γ_μ and $\sigma_{\mu\nu}$ terms, respectively. The resulting form factors are proportional to the NNπ result Eq. (5), since they involve the same Gaussian overlap integrals. The NNω coupling constants (and form factors) satisfy the relations

$$g_{\mathrm{NN}\omega} = \frac{9}{5}\left(\frac{m_\omega}{m_\pi}\right)^{1/2} g_{\mathrm{NN}\pi} \tag{14}$$

and

$$\kappa_\omega = -\frac{3}{2}. \tag{15}$$

The NNρ form factors, defined through the generalization of the NNω effective Lagrangian to an $I = 1$ ρ meson,

$$\mathcal{L}_{\mathrm{NN}\rho} = -g_{\mathrm{NN}\rho}\left(\bar{\Psi}\gamma_\mu\vec{\tau}\Psi \cdot \vec{\rho}_\mu - \frac{\kappa_\rho}{4M_{\mathrm{N}}}\bar{\Psi}\sigma_{\mu\nu}\vec{\tau}\Psi \cdot \vec{\mathrm{F}}_{\mu\nu}\right) \tag{16}$$

(where $\vec{\mathrm{F}}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$) are related to the NN$\omega$ results by

$$g_{\mathrm{NN}\rho} = \frac{1}{3}\left(\frac{m_\rho}{m_\omega}\right)^{1/2} g_{\mathrm{NN}\omega} \tag{17}$$

and

$$\kappa_\rho = +\frac{3}{2}. \tag{18}$$

The NNρ vector (γ_μ) coupling constant was previously evaluated by Le Yaouanc et al. [1]. Their Eq. (3.13) for $g_{\mathrm{NN}\rho}$ is consistent with our Eqs. (5), (17), provided that (i) their factor of $3R_{\mathrm{N}}^2R_\rho^2$ is actually $3R_{\mathrm{N}}^2R_\rho$ (their result as written is dimensionally incorrect), (ii) their factor of $m_\rho^{3/2}$ should instead be $m_\rho^{1/2}m_{\mathrm{N}}$, analogous to their $g_{\mathrm{NN}\pi}$ result, and (iii) the factor of $\frac{1}{2}\vec{\tau}$ in their ρ effective Lagrangian Eq. (2.17) should be $\vec{\tau}$, as in their π effective Lagrangian Eq. (2.12). Le Yaouanc et al. did not evaluate the $\sigma_{\mu\nu}$ term, and did not consider the NNω case.

It is interesting to compare our theoretical NNV couplings with the fitted values found in meson exchange models of NN scattering. If the $^3P_0$ model is reasonably accurate in describing the coupling between non-strange baryons and vector mesons (which is currently being tested at TJNAF in their search for missing baryon resonances decaying to $N\omega$ and $N\rho$), and the meson exchange models are correct in assuming that vector meson exchange is the dominant mechanism underlying the short ranged NN force, we would expect to find approximate agreement between these couplings.

The fitted NNV couplings found in the three well-known meson exchange models are given in Table 1, together with our quark model results. Evidently we do not find good agreement. Note in particular that the fitted strength of the dominant $NN\omega$ γ_μ coupling in the meson exchange models is about a factor of 2 smaller than the quark model prediction. The ratio of the $NN\rho$ to $NN\omega$ γ_μ couplings is rather similar in the two approaches; the meson exchange models quote a ratio of ≈ 0.2–0.4, whereas the theoretical ratio (an SU(6) symmetry factor rather than a detailed test of the quark model predictions) is $+1/3$. Although the ratio "κ" of magnetic ($\sigma_{\mu\nu}$) to vector (γ_μ) couplings does not yet appear to be well determined for both vectors in the meson exchange model fits, there does appear to be agreement that $|\kappa_\rho| \gg |\kappa_\omega|$. This is inconsistent with our quark model prediction of equal magnitudes for these NNV "strong magnetic" couplings, $\kappa_{(\omega,\rho)} = (-,+)3/2$. Since these ratios are simple SU(6) factors and do not involve uncertainties in the spatial wavefunctions, this disagreement may imply that vector meson exchange is not the dominant short ranged NN interaction mechanism. This will be addressed in detail in a future study of the NN scattering amplitudes and phase shifts due to meson exchange, augmented by quark model constraints on the nucleon–meson vertices.

3. Summary and conclusions

In this Letter we have developed a formalism for determining hadron strong vertices and form factors, "three-point functions", in the context of the quark model. We apply this approach to the evaluation of meson–nucleon vertices, several of which are important in meson exchange models of nuclear forces. The quark model expression we find for the $NN\pi$ coupling confirms an earlier Orsay result. With standard quark model parameters, this $g_{NN\pi}$ is about half the experimental value. Our quark model expression for the theoretical $g_{NN\sigma}$ strong coupling of nucleons to scalar mesons is a new result, and is numerically similar to the isospin-mean fitted NN S-wave value in the CD-Bonn model. Our quark model result for the $NN\rho$ vector (γ_μ) coupling is consistent with an earlier Orsay result (after correcting typographical errors), although we find a non-zero magnetic ($\sigma_{\mu\nu}$) coupling. The strengths of the fitted NNV γ_μ couplings in meson exchange models are rather smaller than our quark model predictions. The NNV $\sigma_{\mu\nu}$ couplings are also not in good agreement with quark model predictions, although they may not be well determined in the current fits to NN scattering data.

In future we plan to carry out calculations of the NN phase shifts predicted by meson exchange models, assuming quark model constraints on the NNm couplings and form factors as derived here. This should allow a determination of the sensitivity of the data to parameters such as the $g_{NN\omega}/g_{NN\rho}$ and $\kappa_\omega/\kappa_\rho$ ratios, for which we have definite quark model predictions.

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