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## Short-range force between two Higgs bosons

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## ABSTRACT

The  $S$ -wave scattering length and the effective range of the Higgs boson in Standard Model are studied using effective-field-theory approach. After incorporating the first-order electroweak correction, the short-range force between two Higgs bosons remains weakly attractive for  $M_H = 126$  GeV. It is interesting to find that the force range is about two order-of-magnitude larger than the Compton wavelength of the Higgs boson, almost comparable with the typical length scale of the strong interaction.

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## 1. Introduction

The ground-breaking discovery of a new particle with mass around 126 GeV by the ATLAS and CMS Collaborations at CERN Large Hadron Collider (LHC) in July 2012 [1,2], heralds an exciting new era of particle physics. Undoubtedly, the top priority in the coming years is to pin down the detailed property of this new boson as precisely as possible, e.g., its quantum number, decay and production patterns [3]. Hopefully, one will finally be able to determine whether this new boson is the long-sought Brout–Englert–Higgs boson of Standard Model (SM)<sup>2</sup> or of some exotic origin.

The SM Higgs boson is an elementary scalar particle carrying  $J^{PC} = 0^{++}$ . An enormous amount of work has been devoted to exploring the physics involving an individual Higgs boson, while the respective studies concerning the multi-Higgs-boson dynamics, such as double- or triple-Higgs productions at LHC experiments, are still in the infancy stage [3]. Nevertheless, a thorough investigation of the latter is crucial in unraveling the nature of the Higgs potential since it directly probes the self-coupling of the Higgs bosons.

It is of fundamental curiosity to inquire the short-range force which two Higgs bosons would experience. A few decades ago,

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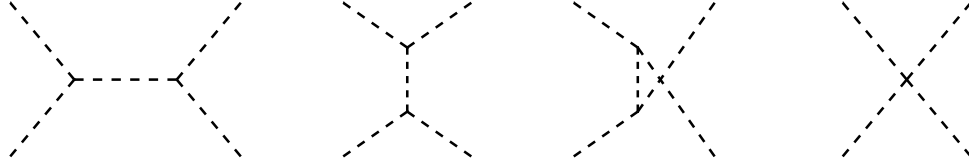
<sup>2</sup> For simplicity, hereafter we shall simply call it Higgs boson.

Cahn and Suzuki [5], as well as Rupp [6], studied the interaction between two Higgs bosons by utilizing some nonperturbative methods, only including the Higgs self-coupling. They claimed that the attraction would become strong enough as  $M_H > 1.3$  TeV to bind them together into a Higgs–Higgs bound state (Higgsium), albeit highly unstable. Such a large Higgs mass violates the perturbative unitarity bound [4]. If the new particle discovered in LHC is indeed the SM Higgs boson, the Higgsium seems unlikely to be formed in the first place. This expectation is supported by the recent lattice simulation of the electroweak gauge model [7]. Nevertheless, Grinstein and Trott recently suggested that the possibility for the existence of the light Higgsium might be still open due to some new physics scenario at TeV scale [8].

The model-independent parameters that characterize any short-range force are *scattering length* and *effective range*. The effective-field-theory (EFT) approach provides a systematic framework to expedite inferring these parameters. The aim of this paper is to decipher the short-range force experienced by two God particles following this modern doctrine. In particular, we will investigate the influence of the  $W$ ,  $Z$  and top quark on the inter-Higgs force. It will be interesting if our predictions can be confronted by the future lattice simulation, or even by the double Higgs production experiments.

## 2. The Higgs sector in SM

After the spontaneous electroweak symmetry breaking, the Higgs sector in SM Lagrangian reads (in unitary gauge):

Fig. 1. The tree-level diagrams for  $HH \rightarrow HH$  in SM.

$$\begin{aligned} \mathcal{L}_H = & \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}M_H^2 H^2 - \frac{\lambda v}{4}H^3 - \frac{\lambda}{16}H^4 \\ & + \frac{2M_W^2}{v}W^{+\mu}W_\mu^- H + \frac{M_W^2}{v^2}W^{+\mu}W_\mu^- H^2 \\ & + \frac{M_Z^2}{v}Z^\mu Z_\mu H + \frac{M_Z^2}{2v^2}Z^\mu Z_\mu H^2 - \frac{m_t}{v}\bar{t}tH + \dots, \end{aligned} \quad (1)$$

where  $v \approx 246$  GeV is the vacuum expectation value of the Higgs field,  $M_H$ ,  $M_W$ ,  $M_Z$ ,  $m_t$  signify the masses of the Higgs boson,  $W^\pm$ ,  $Z$ , and the top quark, respectively. All the other fermions are neglected due to much weaker Yukawa coupling. We follow the convention of parameterizing the Higgs potential as in Refs. [17,18] such that  $M_H = \sqrt{\frac{\lambda}{2}}v$ . For a light 126 GeV Higgs boson, the self-coupling  $\lambda \approx 0.52$ , and we have an entirely weakly-coupled Higgs sector.

### 3. Nonrelativistic EFT for Higgs boson

We are interested in the near-threshold elastic scattering between two Higgs bosons, thereby only the  $S$ -wave channel needs to be retained. Since the momentum of each Higgs boson is much lower than the remaining mass scales  $M_H \sim M_W \sim M_Z \sim m_t \sim v$ , it seems legitimate to integrate out the contribution from all the relativistic (hard) modes, and construct the following low-energy EFT which only involves the nonrelativized Higgs field [12]:

$$\begin{aligned} \mathcal{L}_{\text{NREFT}} = & \Psi^* \left( i\partial_t + \frac{\nabla^2}{2M_H} - \frac{\partial_t^2}{2M_H} \right) \Psi - \frac{C_0}{4} (\Psi^* \Psi)^2 \\ & - \frac{C_2}{8} \nabla(\Psi^* \Psi) \cdot \nabla(\Psi^* \Psi) + \dots, \end{aligned} \quad (2)$$

where  $\Psi^{(*)}$  field annihilates (creates) a Higgs boson. The 126 GeV Higgs boson appears to have a narrow width ( $< 10$  MeV) [9] so that we treat it as a stable particle. This EFT is organized by a velocity expansion, and remains valid as long as the momentum carried by the Higgs boson is smaller than the UV cutoff of this NREFT,  $\Lambda$ , which *a priori* is expected to be of the same size as the inverse of the force range,  $\sim 1/r < M_H$ . The  $S$ -wave scattering is mediated by the two 4-boson operators with the Wilson coefficients  $C_0$  and  $C_2$ . By naturalness one assumes  $C_0 \sim \frac{4\pi}{M_H \Lambda}$ ,  $C_2 \sim \frac{4\pi}{M_H \Lambda^3}$ . The two-body operator containing  $\partial_t^2$  in the parenthesis of (2) signals the relativistic correction. With this specific form, the Higgs state in our NREFT is understood to tacitly obey the relativistic normalization condition, *i.e.*,  $\langle H(\mathbf{k})|H(\mathbf{p}) \rangle = \sqrt{\mathbf{k}^2 + M_H^2}/M_H(2\pi)^3 \delta^3(\mathbf{p} - \mathbf{k})$ .

It is worth emphasizing the legitimacy of integrating out  $W$ ,  $Z$  and  $t$  in our NREFT. Suppose in a fictitious world in which  $M_W(M_Z, m_t)$  were very close to  $M_H$ , or  $M_W(M_Z, m_t)$  were roughly half of  $M_H$ , one would be forced to retain the nonrelativistic  $W(Z, t)$  fields as active degree of freedom in (2), in order to properly account for the near-threshold reactions such as  $H \rightarrow WW(ZZ, t\bar{t})$ ,  $HH \rightarrow WW(ZZ, t\bar{t})$ . Fortunately, in the real world, none of the above coincidence arises, so we are justified to only keep the nonrelativistic Higgs field in the NREFT.

The  $S$ -wave amplitude can be calculated order by order in velocity (loop) expansion from (2), with the UV divergence conveniently subtracted in the  $\overline{\text{MS}}$  scheme. In a NREFT that only contains contact interactions, an all-order result is available by summing the infinite number of bubble diagrams as a geometric series [10,11]. Remarkably, that nonperturbative result is only subject to slight change when the relativistic correction is included [12]:

$$\mathcal{A}_{\text{NREFT}}^{S\text{-wave}} = - \left[ \frac{1}{C_0 + C_2 k^2 + \dots} + \frac{iM_H}{8\pi} \gamma^{-1} k \right]^{-1}, \quad (3)$$

where  $k$  denotes the momentum in the center-of-mass frame, and  $\gamma \equiv \sqrt{1 + k^2/M_H^2}$  is a Lorentz dilation factor, which embodies the full relativistic correction.

A traditional way of parameterizing the  $S$ -wave elastic amplitude mediated by a short-range interaction is through the  $S$ -wave phase shift:

$$\begin{aligned} \mathcal{A}_{\text{ERE}}^{S\text{-wave}} = & \frac{8\pi}{M_H} [k \cot \delta_0 - i\gamma^{-1} k]^{-1} \\ = & \frac{8\pi}{M_H} \left[ -\frac{1}{a_0} + \frac{r_0}{2} k^2 + \dots - i\gamma^{-1} k \right]^{-1}. \end{aligned} \quad (4)$$

$\delta_0$  implies the  $S$ -wave phase shift, while  $a_0$  and  $r_0$  signify the  $S$ -wave scattering length and effective range, each of which is physical observable. The second line specifies the so-called effective range expansion, valid only for small  $k$ . Again, a factor of  $\gamma^{-1}$  is included to recover the Lorentz invariance [12].

Equating (3) and (4), one determines  $a_0$  and  $r_0$  via

$$a_0 = \frac{M_H}{8\pi} C_0, \quad r_0 = \frac{16\pi}{M_H} \frac{C_2}{C_0^2}. \quad (5)$$

Our central goal is then to deduce the coefficients  $C_0$  and  $C_2$  to next-to-leading order (NLO) in electroweak couplings. This can be achieved through matching the  $S$ -wave amplitude of  $HH \rightarrow HH$  in SM onto that in NREFT to one-loop order.

### 4. LO results for $a_0$ and $r_0$

At tree level, there arise only 4 tree diagrams for the Higgs-Higgs elastic scattering (in any gauge), as shown in Fig. 1. Only the physical Higgs field is involved. The corresponding amplitude is [4]

$$\begin{aligned} \mathcal{A}_{\text{SM}}^{(0)} = & -\frac{3M_H^2}{v^2} \left( 1 + \frac{3M_H^2}{s - M_H^2} + \frac{3M_H^2}{t - M_H^2} + \frac{3M_H^2}{u - M_H^2} \right) \\ \approx & \frac{12M_H^2}{v^2} \left( 1 - \frac{8}{3} \frac{k^2}{M_H^2} \right) + O(k^4), \end{aligned} \quad (6)$$

where  $s, t, u$  are Mandelstam variables. In the second line, we have carried out the threshold expansion by treating  $k^2/M_H^2$ ,  $t/M_H^2$ ,  $u/M_H^2$  as small perturbations. Near the threshold, the above expansion automatically projects out the  $S$ -wave contribution, up to  $O(k^4)$ .

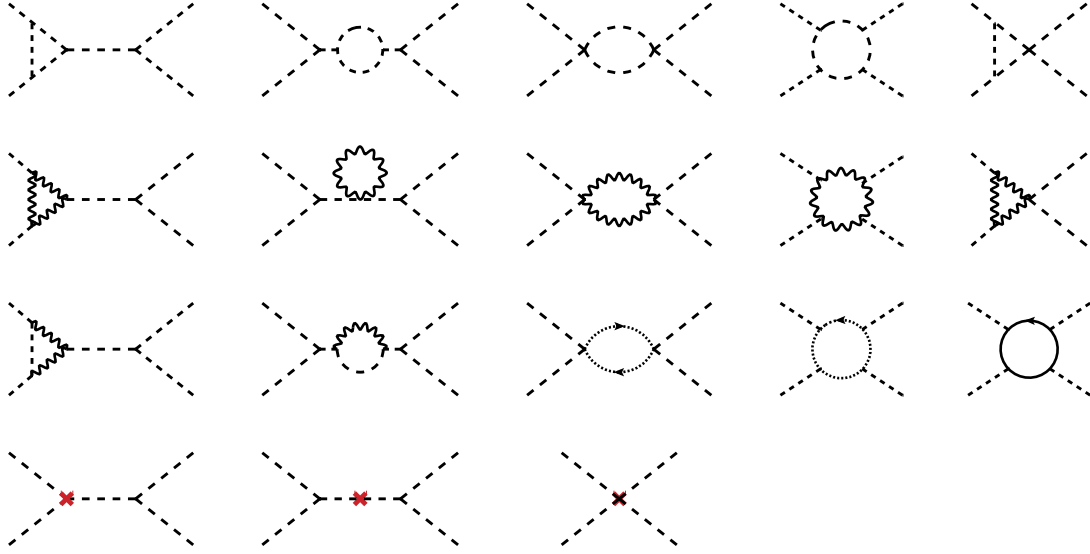


Fig. 2. Some sample NLO diagrams for  $HH \rightarrow HH$  in Feynman gauge. The dashed line stands for the Higgs boson (or the Goldstone bosons inside the loop), the wavy lines for the  $W/Z$  bosons, the dotted curve for the ghosts, and the solid line for the  $t$  quark. The crosses represent the counterterms for the  $H^3$ ,  $H^2$ , and  $H^4$  vertices, respectively.

The tree-level  $S$ -wave amplitude in the NREFT side can be obtained by expanding (3) accordingly:  $\mathcal{A}_{S\text{-wave,NREFT}}^{(0)} = -C_0 - C_2 k^2 + \dots$ . Comparing it with (6), one extracts the Wilson coefficients at LO:  $C_0^{(0)} = -\frac{3}{v^2}$ ,  $C_2^{(0)} = \frac{8}{M_H^2 v^2}$ . Following (5), we then determine the LO  $S$ -wave scattering length and the effective range as

$$a_0^{(0)} = -\frac{3}{8\pi} \frac{M_H}{v^2} = -\frac{3}{16\pi} \frac{\lambda}{M_H}, \quad (7a)$$

$$r_0^{(0)} = \frac{128\pi}{9} \frac{v^2}{M_H^3} = \frac{256\pi}{9} \left(\frac{1}{\lambda}\right) \frac{1}{M_H}. \quad (7b)$$

We observe that the short-range inter-Higgs force is *weakly attractive*. The magnitude of the scattering length is much smaller than the Compton wavelength of Higgs boson, while the effective range is much larger, and  $\frac{|a_0|}{r_0} = \frac{27}{1024\pi^2} \frac{M_H^4}{v^4} = \frac{27\lambda^2}{4096\pi^2} \ll 1$ . It is somewhat surprising that the effective range is considerably ( $\approx 170$  times) larger than the Compton wavelength of the Higgs boson, the typical force range of weak interaction. Note that this situation is drastically opposite to that for the nuclear force, where the shallow (virtual) bound state arises in the  $pn(^3S_1)$  and  $pn(^1S_0)$  channels due to  $|a_0| \gg r_0$  [10,11].

For such an unnatural case, we might infer the valid range of our NREFT by enforcing the convergence of the effective range expansion:

$$\frac{1}{k} \tan \delta_0 = -a_0 \left( 1 + \frac{a_0 r_0}{2} k^2 + O(k^4) \right). \quad (8)$$

Since  $\frac{1}{2}|a_0|r_0 = \frac{8}{3M_H^2}$ , the NREFT seems to apply provided that  $k$  is smaller than the UV cutoff  $\Lambda = \sqrt{\frac{3}{8}} M_H \approx 77$  GeV. This cutoff value is considerably greater than the naive expectation of  $\Lambda \sim 1/r_0 \approx 1$  GeV. As a consequence of the wide valid range of this NREFT, it appears to be a virtue to explicitly retain the effect of relativistic correction as in (3).

It is instructive to contrast the Higgs model with the simplistic  $\lambda\phi^4$  theory containing a scalar field with mass  $m$ . There the interparticle short-range force is of course repulsive, and the effective range is about the same order as the Compton wavelength, quantitatively,  $a_0 = \frac{3}{16\pi} \frac{\lambda}{m}$ ,  $r_0 = \frac{16}{3\pi m} + O(\lambda)$  [12]. We thus infer that, in the Higgs model, it is the triple Higgs interaction in (1) that yields

the attractive force and ultimately wins the competition against the repulsive quartic interaction. It is also the nontrivial pattern of spontaneous symmetry breaking that generates the unnaturally large force range.

## 5. NLO results for $a_0$ and $r_0$

We wish to assess the impact of the  $W$ ,  $Z$  and top quark on the inter-Higgs force. It is then necessary to incorporate the first-order electroweak correction to  $HH \rightarrow HH$ . Because the intermediate  $WW$ ,  $ZZ$  states are permissible to go on-shell, the coefficients  $C_0^{(1)}$  and  $C_2^{(1)}$  would become complex, so are  $a_0$  and  $r_0$ . This situation is analogous to the nucleon–antinucleon system which can annihilate into multiple pions [13].

We first look at the NREFT side. Expanding the nonperturbative expression in (3) to one-loop order, one finds the  $S$ -wave amplitude now becomes

$$\mathcal{A}_{S\text{-wave,NREFT}}^{(1)} = -C_0 - C_2 k^2 + i \frac{M_H k}{8\pi} \left[ C_0^2 + 2C_0 \left( C_2 - \frac{C_0}{4M_H^2} \right) k^2 + \dots \right] + \dots. \quad (9)$$

The last term stems from the one-loop integration, with the first-order relativistic correction incorporated. It is odd in powers of  $k$ , which is characteristic of the nonrelativistic loop integration.

We then proceed to compute the first-order electroweak correction to the near-threshold scattering between two Higgs bosons in the SM side. There have existed some NLO calculations for  $HH \rightarrow HH$  with arbitrary Higgs momentum. However, the results appear to be either incomplete [14] or approached in an unrealistic limit  $M_W, M_Z \rightarrow 0$  [15].

We choose to work in the Feynman gauge, at a cost of including many diagrams containing unphysical particles such as the Goldstone bosons and ghost particles. We use the MATHEMATICA package FEYNARTS [19] to generate all the Feynman diagrams and the corresponding amplitudes. For clarity, some sample diagrams out of the total 603 NLO diagrams are illustrated in Fig. 2. We use dimensional regularization to regularize UV divergence. The package FEYNCALC [20] is employed to perform the tensor reduction.

We choose the standard on-shell renormalization scheme [16–18] to fix the counterterms for quadratic, triple, quartic Higgs

boson vertices. Apart from the apparent Higgs wavefunction renormalization constant  $\delta Z_H$  and mass counterterm  $\delta M_H^2$ , we still need 4 additional renormalization constants,  $\delta t$ ,  $\delta s_W$ ,  $\delta M_W^2$ ,  $\delta Z_e$ , representing the counterterms for Higgs tadpole, Weinberg angle ( $s_W \equiv \sin\theta_W$ ,  $c_W \equiv \cos\theta_W$ ),  $W$  boson mass, and the electric charge, respectively [18]. Some of their analytic expressions are rather cumbersome.

Fortunately, it is the following specific combination of the renormalization constants that always enters the expressions for the  $H^3$  and  $H^4$  counterterms, which can be recast as

$$\delta Z_e - \frac{\delta s_W}{s_W} - \frac{1}{2} \frac{\delta M_W^2}{M_W^2} = \frac{\Delta r + \Delta\rho}{2} - \frac{\alpha}{8\pi s_W^2} \left( 6 + \frac{7 - 4s_W^2}{2s_W^2} \ln c_W^2 \right) - \frac{1}{2} \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{c_W}{s_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \quad (10)$$

where  $\alpha$  is the fine structure constant,  $\Sigma_T^{ZZ}(0)$  and  $\Sigma_T^{AZ}(0)$  are the  $Z$  boson self energy and the photon- $Z$  two-point function evaluated at zero momentum, which are much simpler than  $\delta M_W^2$  and  $\delta s_W$  [18].  $\Delta r$  and  $\Delta\rho$ , as constructed out of the meticulous combination of the various gauge boson self energies, are some familiar UV-finite parameters that can be directly fixed from the data [16–18].

The analytic NLO expression for  $HH \rightarrow HH$  would be extremely involved for general kinematics. Fortunately, we are only interested in its near-threshold behavior. For most diagrams, particularly with  $W$ ,  $Z$ ,  $t$  circulating in the loop, one can simply expand the integrand in powers of the external momentum  $k$ , prior to carrying out the loop integration. This leads to great simplification, since all the encountered loop integrals then reduce into a set of 2-point (or less) scalar integrals.

The  $s$ -channel loop diagrams composed entirely of the Higgs field, e.g., the ones in the first row of Fig. 2, deserve some special attention. Unlike all other diagrams solely dictated by the *hard* region, the nonrelativistic Higgs fields can propagate almost on-shell in the loop, i.e., they also receive the contribution from the *potential* region, which should be fully mimicked by the one-loop diagrams from the low-energy NREFT. For these diagrams, we employ the method of region [21] to extract the contributions from the hard and potential regions separately. The resulting master integrals are also simple 2-point scalar integrals.

Upon summing all the expanded one-loop diagrams and the counterterm diagrams, the UV divergences are canceled as expected, and one automatically projects out the  $S$ -wave contribution through  $O(k^2)$ . Comparing with (9), we find its last term is fully reproduced by the contribution from the potential regions of the aforementioned  $s$ -channel diagrams. This can be viewed as a nontrivial check of our calculation. It is then straightforward to deduce the NLO coefficients  $C_0^{(1)}$  and  $C_2^{(1)}$ , subsequently convert into  $a_0^{(1)}$  and  $r_0^{(1)}$  in line with (5), i.e.,  $a_0^{(1)}/a_0^{(0)} = C_0^{(1)}/C_0^{(0)}$ , and  $r_0^{(1)}/r_0^{(0)} = C_2^{(1)}/C_2^{(0)} - 2C_0^{(1)}/C_0^{(0)}$ . Conceivably, their analytic expressions are too lengthy to be reproduced here.

For  $M_H = 126$  GeV, Eq. (7) then implies that the LO scattering length and effective range are  $a_0^{(0)} = -4.90 \times 10^{-5}$  fm,  $r_0^{(0)} = 0.267$  fm.

To estimate the NLO contribution, we choose  $\alpha = 1/137.036$ ,  $G_F = 1.166 \times 10^{-5}$  GeV $^{-2}$ ,  $M_W = 80.39$  GeV,  $M_Z = 91.188$  GeV,  $m_t = 173.1$  GeV,  $\Delta r = 0.0357$ ,  $\Delta\rho \approx \frac{3G_F m_t^2}{8\pi^2 \sqrt{2}} = 0.0094$  [22]. And the NLO corrections turn out to be

$$a_0^{(1)}/a_0^{(0)} = -0.0355 + 0.0063i, \quad (11a)$$

$$r_0^{(1)}/r_0^{(0)} = 0.0245 - 0.0145i. \quad (11b)$$

The electroweak radiative correction has a modest effect, only modifying the tree-level result by a few percent. However, the imaginary parts for both  $a_0$  and  $r_0$  arise due to the opening of the inelastic channels. Incorporating the NLO correction, we then predict  $a_0 = (-4.73 \times 10^{-5} - 3.10 \times 10^{-7}i)$  fm, and  $r_0 = (0.273 - 3.87 \times 10^{-3}i)$  fm. These are by far the most precise predictions for the basic parameters that characterize the inter-Higgs force.

To extract the  $a_0$  and  $r_0$ , one needs a very accurate knowledge of the  $S$ -wave phase shift  $\delta_0$  for Higgs–Higgs elastic scattering near threshold. The rather weak inter-Higgs force implies a nearly vanishing  $\delta_0$  over a large momentum range. To determine a (tiny) phase of the  $S$ -matrix is always a very challenging task. For instance, notwithstanding tremendous efforts, it takes several decades to unambiguously pin down the  $\pi$ - $\pi$   $S$ -wave scattering lengths via the interference method [23].

The double Higgs boson production is one of the important missions in the next phase of LHC experiment, whose dominant production mechanism is through the parton process  $gg \rightarrow HH$  and  $W^+W^-/ZZ \rightarrow HH$ . Due to the low production rates, it perhaps needs a decade to finally observe the double Higgs signals. Even though we could accumulate sufficient amount of double Higgs events near threshold, it would still be difficult to envisage how to extract the phase of  $S$ -wave scattering amplitude via its interference with the  $D$ -wave amplitude from this *inclusive* production process, not mention the intrinsic uncertainty of parton distribution functions of gluons and  $W/Z$  inside the proton. On the other hand, the prospective high-luminosity electron-positron collider may offer much cleaner place to measure the Higgs–Higgs scattering information via the *exclusive* processes  $e^+e^- \rightarrow ZHH$ ,  $\nu\bar{\nu}HH$ , provided that the sufficient statistics could be achieved.

## 6. Inter-Higgs force in the large $M_W$ , $M_Z$ , $m_t$ limit

It is curious to assess the influence of  $W$ ,  $Z$  and  $t$  on the profile of the Higgs–Higgs interaction. Taking the limit  $M_W(M_Z) \rightarrow \infty$  (while keep  $v$  and  $s_W$  intact) and  $m_t \rightarrow \infty$ ,<sup>3</sup> we find asymptotically,

$$a_0^{(1)} \rightarrow -\frac{9(2M_W^4 + M_Z^4)}{64\pi^3 M_H v^4} + \frac{9m_t^4}{16\pi^3 M_H v^4}, \quad (12a)$$

$$r_0^{(1)} \rightarrow -\frac{32(2M_W^4 + M_Z^4)}{9\pi M_H^5} + \frac{128m_t^4}{9\pi M_H^5}, \quad (12b)$$

where the subleading terms of order  $M_Z^2$  and  $m_t^2$  are neglected. The fourth-power mass dependence indicates that the NLO corrections would rapidly dominate over the LO results as the gauge boson masses or top quark masses keep increasing, which defies the decoupling theorem. As  $M_W$  ( $M_Z$ ) grow, both  $a_0$  and  $r_0$  decrease. On the contrary, both  $a_0$  and  $r_0$  increase with increasing  $m_t$ . When  $m_t$  crosses around 300 GeV,  $a_0$  would even reverse the sign, so the Higgs–Higgs force would become even repulsive.

## 7. Summary

For the first time, we have thoroughly investigated the profile of the short-range force between two SM Higgs bosons within the

<sup>3</sup> These limits correspond to setting the couplings  $g_1, g_2, y_t \rightarrow \infty$ , in which situation the EW theory ceases to be perturbative. Nevertheless, our goal is to assess the leading behavior with the large gauge boson and top quark masses, so we are not too rigorous here.

modern EFT context, deducing the  $S$ -wave scattering length and the effective range by including the first-order electroweak correction. The impact of  $W$ ,  $Z$  and  $t$  on these parameters is addressed. The inter-Higgs force is extremely weak, yet attractive. But the force range is as large as 0.3 fermi, comparable with the typical range of the QCD force. It will be interesting, albeit challenging, if the future lattice simulation can test our predictions. It might also be of phenomenological incentive to transplant our analysis to some classes of new physics models.

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