HISTORIA MATHEMATICA 17 (1990), 313–338

The Varieties of Mechanics by 1800

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After an exploration of some of the basic categories attending mechanics, Newtonianism, and physics, a survey is given of the range of subjects covered by mechanics at the end of the 18th century, and the three main traditions operative in the subject are stressed. A few major French figures of that time are then noted, and also some views evident in certain other countries. The appendix of the paper is concerned with bibliographical questions; and an extensive bibliography, mostly of secondary literature, is appended. © 1990 Academic Press, Inc.

Après avoir présenté quelques catégories fondamentales de la mécanique, du newtonianisme et de la physique, je présente un aperçu des sujets afférents à la mécanique en en soulignant, en particulier, trois traditions prépondérantes à la fin du dix-huitième siècle. Je considère aussi quelques savants français et j'évoque les positions adoptées dans certains autres pays. Enfin, en appendice, je me penche sur des questions de bibliographie; je fournis, notamment, une bibliographie étoffée fondée principalement sur les sources secondaires. © 1990 Academic Press, Inc.

Nach der Darlegung der grundlegenden Kategorien in der Mechanik, im Newtonianismus und in der Physik gibt der Aufsatz eine Übersicht über die Themen der Mechanik Ende des 18. Jahrhunderts. Insbesondere werden die drei Haupttraditionen hervorgehoben. Der Aufsatz geht auf einige wichtigere französische Autoren dieser Zeit ein und einige Ansichten in bestimmten anderen Ländern. Der Anhang des Aufsatzes betrifft bibliographische Fragen. Dazu gehört eine ausführliche Bibliographie vor allem der Sekundärliteratur. © 1990 Academic Press, Inc.

AMS 1980 subject classifications: 01A50, 70-03, 73-03, 76-03, 80-03.
KEY WORDS: mechanics, history of mechanics.

1. INTRODUCTION

1.1. Scope

Commonly used words in the history of the physical sciences include "Newtonianism" and "mechanics," and the impression is sometimes given that the former is included within the latter, so that mechanics is the same as Newtonian mechanics. However, even a cursory reading of the original literature will reveal the falsity of such a view: much mechanics of the 18th century, especially on the Continent where most of the main research was being conducted, did not necessarily grant Newton's laws or the principles a particularly high place, and often pursued quite other lines.

This paper briefly surveys the state of affairs around 1800, concerning the main traditions (Section 2) and the range of the subject (Section 3). In Section 4 a few examples of philosophical issues attendant upon mechanics are given. Some

0315-0860/90 $3.00

principal texts of the late century are appraised in Section 5, followed in Section 6 by comparative remarks about certain more minor sources. The "concluding comments" of Section 7 stress the limits as well as the scope of our historical knowledge of the period. Section 8, the appendicial section, contains some general comments about the available primary and (especially) secondary literature, before giving way to the bibliography. The purposes of the paper are (a) to sketch a portrait of the situation in mechanics at that time; (b) to outline a classification by means of which this large subject can be divided; (c) to indicate the strengths and also weaknesses of the available secondary literature; (d) to indicate some un(der)studied questions; and (e) to provide a reasonably comprehensive secondary bibliography, and to indicate the existence of some other sources.

1.2. Limitations

I have chosen the time around 1800 for two reasons: first because enough work had been done by then to show the range of mechanics and especially the variety of approaches to which it was susceptible; and second, because after 1800 the subject was substantially influenced by the inauguration of mathematical physics, with the new mathematicizations of heat theory, physical optics, and electricity and magnetism. These changes brought in a new era, with a different range of influences both on and within mechanics, and they constitute a separate story; the essential French elements are outlined in [Grattan-Guinness 1990, Chaps. 9, 12–16].

I emphasize from the title that the chief concerns lie with the *late* 18th century, especially the last twenty years. I do not attempt to trace developments *during* the century; earlier stages are mentioned, but the details involved are not discussed. However, several of the items cited in the bibliography have a wider temporal coverage, and indeed the citations made in this paper are understood to apply only to the parts or chapters of those items appropriate to our period.

Throughout I shall of course take as known the basic notions and topics within mechanics itself; and I take for granted the development through to the end of the 18th century of the calculus and related mathematical topics such as the theory of differential equations, differential (and other parts of) geometry, functions, series, and the theory of equations. I also assume as known, at least in general terms, that these branches of mathematics provided staples for the formation and solution of problems in mechanics, and also for many of the principles upon which it drew. On the history of the calculus and related topics, see especially [Bos 1974; Engelsmann 1984; Fraser 1985b; Guicciardini 1989], the tables of contents of [Lacroix 1797–1800], the early parts of [Burkhardt 1908], and several articles in [Cantor 1908; Euler 1983]; a brief survey is contained in [Grattan-Guinness 1990, Chap. 3], where many other items of secondary literature are cited.

In this context I should mention that the sharp modern distinction between the calculus as pure mathematics and mechanics as applied mathematics was then mercifully not in operation: applications held the center of the mathematical stage, especially mechanics, and subjects like the calculus were developed very much with empirical interpretations in mind. While the adjectives "pure" and "applied" mathematics were then used, the preferable word "mixed" was often employed to refer to the second category; some philosophers argue that a distinction can be made between it and applied mathematics.

Finally, the bibliography consists only of those works which have been cited in the text. Further, I have chosen to cite only those items which either have a reasonably wide range, are rich in additional references, and/or deal with a specific point which has to be mentioned. Thus a number of respectable works are omitted, as they treat of matters too "local" to gain entry here. As some compensation, Section 8 indicates various kinds of other pertinent sources (for instance, editions and/or translations of the work of major figures, and certain encyclopaedias).

1.3. Mechanics, Physics, and Newtonianism

Some housework is needed on the main terms to be employed—"mechanics," "Newtonian," "Newtonian mechanics," and "physics." This will not be easy, as the words were rather carelessly used at the time, and with the recent rise of the history of science the situation has become a good deal worse. I shall explain my own distinctions, without attempting to trace all the variants.

"Newtonianism" has been especially abused; for to be Newtonian was a Good Thing, so that anybody labeled his theory "Newtonian" if some bit of it subscribed to one of the Newtonian canons. I shall use the word to refer only to a conception of the world in which matter was regarded as composed of elementary corpuscles held together by central forces of some repulsive and/or attractive kind. Its main concerns included mass and inertia, gravitation and weight, rest and motion, and a variety of "forces" acting in and on bodies; for the more philosophically minded, such as Newton himself, space and time themselves were under scrutiny. The mediation of these forces was a delicate question, especially if the ubiquitous ether was held to be involved. Schofield [1970] maps out some parts of this historical territory well, in connection with the British developments; Weiss [1988] is also very sharp, in the important context of Genevan science.

Unfortunately the term "Newtonianism" came also to involve anything that Uncle Isaac thought or did, and this went beyond physics into chemistry and the life sciences; and here is where the unclarities begin to mount. Even the "hypotheses non fingo" tag could make one a Newtonian, whatever the scientific subject studied! The phrase "Newtonian philosophy" was quite current; it included his optics as well as his mechanics and his physics, although the constitutive connections between them are not strong. Normally (and mercifully) Newton's law of cooling in heat theory was excluded from this treatment. For a typical and also well-known tract of this kind, see [Pemberton 1728].

"Mechanics" was another Good Thing, and so suffered (as it has done to our day) for the same reason. I take it to refer to the study of the rest and motion of bodies (which themselves are construed widely to include point-masses, extended solid and fluid objects, instruments, machines, frameworks, and constructs) under the action of normal physical forces (and so excluding, for example, heat, electrical, and magnetic actions). Principal categories include space and time, equilibrium and disequilibrium, inertia, mass, force, energy, momentum, impact, elasticity, flow, vibration, rotation, and oscillation.

Obviously mechanics and Newtonianism overlap considerably, but there are major differences also. For example, as will be clarified below, optics is definitely excluded from mechanics. Again, mechanics was a heavily mathematicized subject, whereas Newtonianism was largely prosodic; this affected substantially the kinds of studies undertaken.

In addition (and here is a main theme of this paper), much mechanics was not Newtonian mechanics; therefore this category needs specification also. By "Newtonian mechanics" I restrict myself to mechanics in which not only the inverse square law of central attraction but also Newton's three laws of motion were taken as the underlying bases, or at least served the principal role, with the conceptions of matter and force just mentioned in attendance. The range of concerns in Newtonian mechanics was basically the same as that in the other two traditions (to be described in Section 2), and in all traditions the role of mathematics was major; as we shall soon see, the differences concerned generality of application and certain mathematical and philosophical matters. Some examples of these points will be given *passim* in Sections 2–4.

Finally comes physics. Around 1800 it was a heavily experimental but largely nonmathematical discipline. Its principal subjects were the constitution of matter (thereby causing overlaps with our other categories); properties of air, with links to the propagation of sound and to barometry; heat theory, including gases and vapors; electrostatics (to use a term not then current) and magnetism; and physical optics, this being the largest single branch. (See [Fischer 1805] for a typical and influential textbook on physics of the time.) Physics overlapped with mechanics in some topics involving matter (for example, elasticity and gases), optics, and sound; but even in these situations the (non)role of mathematics led to significant differences in concern. These differences relate to the division of physical sciences in general into the so-called "classical" and "Baconian" kinds [Kuhn 1976]: we are entirely concerned with those of the first kind here.

Physics also overlapped with "natural philosophy" in English-speaking countries and "Naturphilosophie" in German ones, and these two traditions differed from each other: luckily I do not have to pursue those relationships here. Thus when I speak of mechanics, physics is explicitly excluded unless points of contact and overlap are mentioned. Hence, there is no discussion here of (physical) optics, electricity, and magnetism, which were but little mathematicized before 1800. Further, I have not cited general histories of physics in the bibliography, as (irrespective of their quality) they do not contain sufficient discussions of mechanics for my purpose. Similarly, I have excluded general histories of technology, as being rather too slight on the mathematical aspects of mechanics.

1.4. Some Boundary Lines

The following two examples should clarify by illustration the distinctions between the categories. First, the range of forces treated in mechanics was not fully clear, though as a rule heat, electricity, and magnetism were excluded, as belonging to physics. Now in the 1770s and 1780s Coulomb claimed to establish inverse square laws for electrostatics and magnetism, which made these sciences Newtonian [Gillmore 1971, Chap. 6]. However, these subjects did not thereby fall under mechanics: on the contrary, Coulomb even held them to be distinct from each other.

Second, we are so accustomed to regarding physics as a major, even dominating, science among the sciences that it is necessary to stress that the subject had a low status throughout the 18th century in many countries—in great contrast to the lofty position enjoyed by mechanics. But in the early years of the 19th century (and thus just outside our period of concern) physics at last began its steady rise to prominence and eventually dominance in science. One major stimulus came from a program initiated in France (then the main scientific country) by Laplace and Berthollet, involving central forces of attraction and repulsion acting between the molecules of matter [Fox 1974; Grattan–Guinness 1990, Chap. 7]. This sounds very Newtonianist; but it was not Newtonian, for the forces were explicitly assumed *not* to be inverse square, and the ensuing mathematization of (aspects of) optics, heat theory, electrostatics and magnetism brought the program into the domain of physics. While the pretensions of the program were not to be achieved, it was a crucial link between mechanics in general and classical mathematical physics (see [Grattan–Guinness 1987] on this transition). Indeed, the name "mechanical physics" was sometimes used by its practitioners; for example, the phrase was the title of the translation/edition into French of [Fischer 1805] (cited above) made by Biot, an important member of the group around Laplace and Berthollet.

2. THREE MAIN TRADITIONS

I begin with an outline of the three main traditions in mechanics which had emerged by the end of the century. Foundational aspects are the main concern: examples and special cases are mentioned in the next two sections. All traditions claimed some generality in their range, which meant that they were in competition with each other; indeed, to some extent, given the principles underlying one tradition, the principles of the other two should be derivable as theorems. However, generality was an issue under dispute; and in all cases adjoint concepts had to be introduced in specific areas in order to bolster the armoury. Hankins [1985] provides a general survey of the century, with valuable references to the secondary literature.

2.1. The Newtonian Tradition in Mechanics

The reception of Newton's *Principia* (1687) was dependent upon a variety of philosophical traditions and other lines of work within mechanics from earlier times. I shall not attempt to record this complicated story here (much of which concerns Newtonianism rather than mechanics), but note that by the end of the 18th century Newtonian mechanics had achieved a prominent position, although not a dominating one. The second law was frequently deployed as a starting point in dynamics, although the form which we now adopt, force equals mass $\times$ acceleration, in fact was popularized from mid-century on by figures such as Euler:

Newton himself had stated a law more concerned with minute changes in impulse [Cohen 1971], although he also used it in the second form [Shea 1986]. The third law gave, among other things, a basis for the primary properties of forces. The emphasis on equilibrium owed much to the first law, although the question of whether statical and/or dynamical equilibrium was covered in a given situation was not always treated with full clarity. In the same spirit, statics was usually presented before dynamics.

2.2. *Variational Mechanics and Its Cousins*

Euler also played a role in the emergence of a second tradition, which gained much currency during the second half of the century, especially because of d'Alembert and Lagrange [Pulte 1989; Fraser 1983]. A collection of methods expressible in terms of purely algebraic versions of the calculus (including the calculus of variations) was put forward: the principle of least action [Fleckenstein 1957], which gave the approach a strongly teleological character (to the discomfort of its critics), in that the "global" path of least action along which the action was held to take place was apparently predetermined along its course; d'Alembert's principle, which was asserted to reduce dynamics to statics [Fraser 1985a]; and the principle of virtual velocities [Lindt 1904], where I use the noun employed by d'Alembert and Lagrange (its significance will be explained in Subsection 5.1).

Equilibrium, and continuity in change of situation, were particularly prominent in this approach. For example, the work term was usually assumed always to take a potential: indeed, potential theory itself gained much of its early development within this approach [Todhunter 1873 I; Bacharach 1883]. A philosophical point thereby arose, although for some reason it was not much discussed. Force could be taken as a secondary concept; but then, ontologically speaking, what is a potential? Perhaps the very form of the question caused the silence!

2.3. *Energy Mechanics*

Finally, much attention was given to considerations of energy: its conservation and exchange into other forms. The noun "energy" did not gain popularity until the early 19th century; and the adjectives "kinetic" and "potential" energy were mid-century adornments introduced respectively by Thomson and Tait and by Rankine. The jargon of the 18th century spoke (in French, the principal scientific language of the time) of *forces vives*, mv^2 rather than our later $1/2\ mv^2$ (which is due to Coriolis in 1829). These *forces* were transmitted into other concepts, which we (again following Coriolis) call "work" but which before then took a wide range of terms.

Stimulus for this approach to mechanics often came in connection with engineering [Grattan–Guinness 1984]. Some writers of this kind, especially Lazare Carnot from the 1780s [Gillispie 1971], were impressed by the role in mechanics of percussion and impact, and therefore they stressed disequilibrium, and dynamical as well as statical equilibrium; they also advocated that dynamics be given priority over statics, conversely to the normal view adopted in the other traditions. Aware that in contexts of interest the force functions could be discontinuous, they did *not* permit work terms always to take a potential. This whole movement formed an important part of the prehistory of the energy physics of the mid-19th century, where "physics" encompassed both the realm of mechanics and the branches of physics itself mentioned in Subsection 1.3, in the extended forms that obtained by then.

2.4. Competition

The scale of the change envisioned by Carnot and effected by his successors is worth emphasizing. Prior to Carnot energy mechanics was usually an adjunct of the other two traditions, with the conservation equation arising as the first integral of Newton's second law or obtained via the principle of least action; but in this new view it was elevated (largely after 1800), in its extended conceptual form, to a true competitor with those other two.

As was mentioned at the head of Section 1, the competition arose principally in the claims that the principles of one tradition were held to be sufficient to derive those of the others, and also in the range of generality of which each tradition was capable. Whether such derivations could be satisfactorily achieved was one of the points of dispute, and whether such generality could be justified was another.

Each theory did have its adherents; other figures were more pragmatic. Euler is an important example of the latter: although he asserted that the principle of least action was quite general, he used the other traditions in many areas of his concerns in mechanics (for example, hydrodynamics, machines, and most celestial mechanics). To this range we now turn.

3. THE RANGE OF MECHANICS

It is (and was) normal to divide mechanics into its terrestrial and celestial branches; but from the middle of the century at the latest a finer classification is needed. I propose the following division into five branches, which I call "corporeal," "celestial," "planetary," "engineering," and "molecular," and which I shall treat in that order. The boundary lines between them are not razor-sharp, and indeed one could work simultaneously in more than one (a few examples will be given).

3.1. Corporeal Mechanics

"Corporeal mechanics" refers to the mechanics of "ordinary-sized" objects; here I put also the basic principles discussed in the last section, since they were partly conceived in "ordinary" contexts and the possibility of their extension into other ranges of phenomena, especially the other branches of mechanics, was a major question concerning generality. One element of uncertainty is evident concerning the status of the principle of angular momentum, which does not follow from Newton's laws without additional assumptions about the non-addition of torque to the system [Truesdell 1968, Chap. 5].

Within statics, there was a strange failure: nobody before Poinsot [1803] realized that the theory of the composition of forces had to be enriched by a comparable theory of the couple (his word as well as idea, in his wonderfully clear presentation). Prior to him couples had been used in some specific contexts, especially elasticity theory and certain engineering artefacts; but only Poinsot recognized that the notion of a couple involved *a fundamental component of statics* and not just a case of exception to the composition of forces.

Within the history of dynamics, there has been a strange failure of attention to the introduction of fixed and moving frames of reference, although a related idea has been noted: the "Coriolis force." The naming is reasonable, since he was the first (in 1835) to understand its full generality; however, it had been present, implicitly or in special cases, in Clairaut, Euler, and Laplace.

Central topics in this branch included the (dis)equilibrium of point masses (itself an important conceptual innovation, due largely to Euler), of systems of such masses, and of extended bodies. Numerous special cases were studied, especially in dynamics: motions under resistance of various kinds, special problems in rotation, and so on.

Among particular subbranches, hydrostatics and hydrodynamics were granted especial attention, and the notion of pressure (mainly due to Euler) was recognized to be of major importance [Truesdell 1954, 1955]. Euler was also a (perhaps the) main contributor to elasticity theory, and some of the principal properties of elastic and flexible bodies of various kinds were studied in detail, both theoretically [Truesdell 1960] and experimentally [Bell 1973]. In addition, acoustics gained a good press, especially concerning the mode and velocity of the propagation of sound, where Newton's model was known to be very inaccurate [Dyment 1931]; vibration theory in general gained some prominence [Cannon & Dostrovsky 1981]. Finally, capillarity grew in importance; one motivation was the need for precise barometric readings [Bikerman 1978].

Within this branch of mechanics falls the operation of certain instruments. An important case was the pendulum, which even in its so-called "simple" form was a pretty complicated object when fine details of motion were considered [Wolf 1889–1891]. A main motivation for this use was the analysis of the shape of the earth; this is an example of where the corporeal and planetary branches of mechanics worked together. We now turn to the other branches.

3.2. Celestial Mechanics

"Celestial mechanics" is restricted to those parts of mathematical astronomy in which the heavenly bodies were construed as point masses. Major questions included the fine details of the orbital motions and rotations, usually based upon Newton's second law, with extensive study made of the perturbation effects [Wilson 1980; 1987]. A major issue was the three-body problem, on which some partial success was attained [Gautier 1817]. In addition, in contrast to Newton's position admitting possible instability, Lagrange and Laplace tried to prove mathematically that the planetary system was stable. In a similar spirit great attention was paid to the near-resonance between Saturn and Jupiter, which seemed in danger of upsetting the system [Wilson 1985]. One of the successors of the 18th-century astronomers surveyed their collective achievements [Delambre 1827].

3.3. Planetary Mechanics

"Planetary mechanics" takes the shape of the heavenly body into account—indeed, the shape itself was a major question. The earth was, of course, the most frequently studied case, and after the vindication of Newton's prediction of oblateness in the 1740s [Greenberg Geodesy], much effort was put into developing potential theory and the Legendre functions (to use the modern name) to analyze attraction [Todhunter 1873 I] and to study stable profiles of stationary and rotating bodies. In addition, aspects of cartography, topography, and navigation involved planetary considerations [Berthaut 1902].

The next most popular subject was the moon (partly in connection with the three-body problem just mentioned); of course the term "planetary" is improper in such cases (although in the 18th century the phrase "planet of the second order" was sometimes used for satellites). A variety of methods were developed to study its numerous wobbles [Forbes 1971; Waff 1976]. Other topics falling into this branch included precession and nutation of the heavenly bodies, and the theory of the tides [Aiton 1953].

3.4. Engineering Mechanics

"Engineering mechanics" gives technological concerns a prime position. A number of topics can be grouped together under the rubric of "friction studies," where Coulomb made notable and influential contributions in the 1770s [Gillmore 1971; Heyman 1972]. Particular topics included embankments [Kötter 1892], the construction of arches [Poncelet 1852] and bridges [Gauthey 1809], and indeed of a number of similar architectural objects; the stability of frameworks; the building and steerage of ships; and a variety of matters concerning the workings and efficiency of instruments, and of machines and their parts. This last interest led to early traces of ergonomics, in which the work rates of man and animals were also tackled.

Within hydraulics a major question was the appraisal of the flows of water in large quantities [Mouret 1921], with particular interest taken in these and other questions involving canals and locks [Gauthey 1816], and dams and rivers [Bernard 1787]. Flow of water out of orifices, and cavitation and contraction, were also noteworthy themes. Machines such as waterwheels and turbines were extensively studied, as was the design of pumps, valves, and pistons. The same sort of ideas was applied to the behavior of gases, vapors, and explosions, although at a rather primitive level.

The preference often shown by engineers for energy mechanics was noted in

Subsection 2.3. Rühlmann's work [1881–1885] is an excellent but little-known history of engineering mechanics from ancient times to his own, with a good coverage of the 18th century and a nice treatment also of the development of energy mechanics.

Military concerns were present in a number of these areas. The best known was Euler's annotated translation into German, published in 1745, of Robins's treatise on gunnery; but a variety of other studies was undertaken by a number of figures. Some of the topics mentioned in the last paragraph and the next subsection include military material; but the ensemble is intermingled with engineering in general in a manner which unfortunately no historian has yet unraveled. Some small hints from the German context are indicated in Subsection 6.1.

The simple machines (such as pulleys and wedges) were part of the standard fare in presenting mechanics in general; however, the more advanced topics, such as those mentioned above, were often rather separate from the other branches (for example, hydraulics belonged here whereas hydrodynamics pertained to corporeal and planetary mechanics). One feature which distinguished this branch from the others was an especial desire to generate numbers-in–numbers-out mechanics: some links were made with numerical methods (examples are given in Subsection 5.2). This difference of strategy led to quite radical changes concerning the type of question asked and the content of the required answers to them: in particular, the engineer would take a theoretician's answer and develop and (over?)simplify it in order to convert it into his preferred number-friendly forms (see [Grattan-Guinness 1989] for examples of these differences).

3.5. *Molecular Mechanics*

In "molecular mechanics" the action of the supposed intimate structure of matter was a main concern. This was the smallest of the five branches, but it arose at times in connection with the other four. For example, elasticity and friction studies would entail certain assumptions about the constitution of the bodies under examination; indeed, one aspect of the development of elasticity theory was (gradually) to recognize the differences between hard, elastic, inelastic, and flexible bodies (although molecular models were not necessarily involved). Again, tidal theory was sometimes examined in terms of the motions of the "molecules" of water. Corporeal mechanics would involve molecularist issues when the laws under adoption were extended from point masses (and systems of them) to "continuous" extended bodies: indeed, the relationship between these types of bodies was itself a concern of the century [Körner 1904]. The extension was not always accommodated with ease, especially for the tradition using energy mechanics recorded in Subsection 2.3 [Scott 1970].

Another context involving molecularist considerations was ether theory, when such a medium was presumed both to exist and to possess a "molecular" structure. In such cases the etherian molecules were held to be far smaller in size even than those of ordinary matter.

4. SOME PHILOSOPHICAL ISSUES

Mechanics contains many philosophical questions in addition to the technical details, and the 18th-century workers were well aware of them. I conclude this survey with brief indications of three issues. Dühring [1873] gives a nice general survey of foundational aspects of mechanics through to the time of his writing (Part 3 is the most pertinent here); Hankins [1970] provides a good overview of many of them in and around the important case of d'Alembert; Pulte [1989] examines several in depth in connection with the principle of least action. The primary literature is often unclear or incoherent; it seems as though the authors did not want to discuss the issues! (Examples were given in Subsection 2.2, above.) Understandably, the secondary literature in general is rather poor in these areas, and no repair can be attempted here.

4.1. Truths or Hypotheses?

By and large, whichever approach of Section 2 was advocated, the foundations were held to be true in some (not necessarily clearly stated) sense. Newton's "hypotheses non fingo" haunted the upper reaches of the thinkers' minds, even if they did not adopt his approach. Sometimes the framework of space and time was set up, and basic notions (rest, motion, velocity, acceleration, and so on) defined within it. Appeals to sensory experience were invoked to supply certainty, but then a balance between sense and reason had to be found (or, alternatively, one or the other category minimized in significance).

Religion was deployed on occasion in the advocacy of truthhood; and in this context I record a curious difference, which has been but little commented upon. On the Continent the principle of least action (Subsection 2.2) was advocated in midcentury with considerable religious fervor; mechanics, especially the corporeal branch, was thereby given a holy ring. However, the attempts to prove the stability of the planetary system (Subsection 3.2) removed such an element from celestial mechanics. But this element had been *urged* by Newton, and maintained its place in Britain, where however corporeal mechanics was a purely secular exercise. Thus the division between heaven and marketplace was made in contrary directions on the two sides of the Channel.

Talk of hypotheses was much more ready among those who concentrated upon engineering mechanics. It would take a bold thinker to claim to have truths in his hand concerning the motion of a river, for example, or the structure of a bank of earth. Some areas of planetary mechanics were tackled in the same spirit, for the motion of the seas (to take a particularly important example) was similarly set in mystery.

The issue of truths or hypotheses is complicated by the (to us, metatheoretic) attitude adopted toward hypothesishood: that is, whether a hypothesis was a candidate for truthhood, and indeed successfully could gain this status if confirmation of its predictions was obtained from experiment and observation; or whether it had to remain at a guess level even if such a confirmation was obtained. In a variant version, the principles of the basic approach or tradition adopted were supposed to be true, but hypotheses were permitted to be adjoined to them when context demanded, and even allowed to remain indefinitely suppositional in status.

4.2. The Concepts of Force

There was much disputation on the status of force: whether it was an entity of some kind (the "rational" or "metaphysical" position); whether it was known only by its effects (a more empiricist attitude in the sense of the last subsection, and one in which the philosophy of space and time had to be granted especial attention); or whether it was *defined* via Newton's second law (d'Alembert's position, by means of which he hoped to eliminate an obscure concept, and in consequence of which the "principle of virtual *velocities*" gained its name in the 18th century). In addition, some confusion is manifest in the (non)discussion between force as such, the effects of (for some, a distinct category known as) force, and change of force. The situation was made still worse by the plethora of terms containing the word "force": *forces vives* of Subsection 2.3 is only one example, for it was joined by dead forces, solicited ones, moving ones, latent ones, and so on and on (see, for example, the opening of the influential treatise of Hermann [1716]). The terms referred, not necessarily cleanly, to concepts which the 19th century was either to sort out into force, kinetic energy, potential energy, work, and power, or else to abandon as special categories (for example, wind).

Gravitational force constituted a special quandary. This was more an issue for the physicist than for the mathematician; but it concerned, for example, Laplace, who includes a short and rather nervous chapter on its supposedly "successive transmission" in the fourth volume of his *Mécanique céleste*. This was one context where etherian mechanics (Subsection 3.5), probably "molecular" in character, might be involved, though reluctance so to act in Laplace's case is doubtless a main cause of his nervousness. Let us note now his and other major works published around 1800.

5. THREE MAJOR TEXTS, A HISTORY, AND A REPORT

In the early 1780s, three figures died who had contributed substantially to mechanics (and many other parts of mathematics): Euler, Daniel Bernoulli, and d'Alembert. Thereafter, the center of gravity of the subject switched very strongly to France (a major country already, of course); Laplace, Lagrange (there from 1787), Legendre, Monge, Carnot, and Coulomb were active, soon to be joined by younger figures such as Delambre, de Prony, and Girard. (In [Grattan-Guinness 1990], see Chapter 5 for a general survey of French views on corporeal, celestial, and planetary mechanics around 1800, Chapter 6 for some of the immediately succeeding developments, and Chapter 8 on the engineering mechanics.) Three of these figures produced comprehensive works in mechanics, which we can use as initial appraisals of the state of (parts of) the subject at the time. Then I note the pertinent parts of the major history of mathematics of the time, Montucla's, which also appeared at the turn of the century; and finally a general report by Delambre, which came out in 1810.

5.1. Lagrange

I start with Lagrange's *Méchanique analitique*, published in Paris in 1788 but apparently written by 1782 during his time at the Berlin Academy [Lagrange 1788]. This volume, of 512 pages, was a reductionist exercise in two parts, on statics and dynamics (itself held to be reduced to statics via d'Alembert's principle). Lagrange followed the methods described in Subsection 2.2 above and conveyed the impression that "all" of mechanics thereby fell under their scope. In fact, as usual with an overambitious exercise, it failed to meet its norms, covering quite a lot of corporeal mechanics but very little celestial or planetary, and no engineering or molecular mechanics. The principle of virtual velocities was given a lot of work to do (as it were); but no proof was offered, so that a considerable exercise in proving was stimulated around 1800, especially by Lagrange himself, Laplace, Fourier, de Prony, Ampère, and Poinsot [Bailhache 1975], and for some of these men and certain other authors for several decades afterward [Lindt 1904].

The contrast between the approach which Lagrange advocated and the alternatives is particularly clear in his treatise, where the historical passages are astoundingly quiet about Euler. Lagrange was always reserved in print about Euler's work, although on his deathbed he confessed to his great admiration of Euler [Grattan-Guinness 1985].

Much post-Lagrangian mechanics was also anti-Lagrangian mechanics, in the sense that the restrictions of his framework were too tight for effective developments to be made. In particular, there were very few new results in his book: his methods showed their strength best in reformulating and systematizing results already found by other means. His approach bears some similarity with modern axiomatization, although the analogy should not be stressed strongly.

5.2. De Prony

A different impression of mechanics comes over in de Prony's *Nouvelle architecture hydraulique*, which appeared in two volumes in the 1790s [de Prony 1790–1796]. The title was rather modest, in that in the first volume of 575 large quarto pages and 72 pages of tables he covered many parts of corporeal and engineering mechanics as well as the hydraulics of his title. The five parts of this volume, covering mechanics "for artists in general," treated statics, dynamics, hydrostatics, hydrodynamics, and "machines and motors" (with many of the friction studies mentioned in Subsection 2.3 given prominence). The second volume, just short of 200 pages, was concerned with steam engines, where he treated various well-known cases: he also handled the conversion of circular motions into rectilinear ones, and essayed a speculation on the behavior of gases.

De Prony's chief father figure was not Lagrange but Coulomb, whose work of the 1770s on friction was noted in Subsection 3.4. De Prony's mathematics was, wherever possible, the algebra of trigonometry and difference equations rather than the partial differential and variational calculi of Lagrange's empire. For approaches he usually preferred Newtonian principles or energy equations rather than the Lagrangian menu.

The contrast between de Prony and Lagrange is marked. One was literally visual: whereas Lagrange explicitly shunned the provision of diagrams, de Prony supplied plenty. The topic of de Prony's second volume was not treated by Lagrange; in reverse, the principle of virtual velocities was a minor topic for de Prony. Lagrange described no machines or motors; de Prony devoted the largest part of his first volume to them. Even in his next major book on mechanics, the (incomplete) *Mécanique philosophique* [de Prony 1800], written in connection with his teaching at the Ecole Polytechnique, a more philosophical approach was adopted and no diagrams were furnished; yet there was little trace of the principles favored by Lagrange (recently retired as his colleague professor of analysis at that school, incidentally). De Prony's philosophy was *encyclopédiste* (on which see also Subsection 8.3), especially the classification of theories; some emphasis laid on empiricism ("force envisaged as by its effects," and so on); and distinctions stressed between notations, definitions, theorems, and problems, which were laid out in separate columns on the right-hand pages.

5.3. Laplace

The *Mécanique céleste* of Laplace, the first four volumes of which appeared in 1799, 1799, 1802, and 1805 in a total of around 1400 pages, lies between the other two works in some ways. On the one hand, the influence of Lagrange was quite considerable (although it was not stated by its author, one of the less scrupulous of attributers), starting no doubt with his title. He covered in very great detail all aspects of celestial mechanics, covering all the known heavenly bodies; in planetary mechanics, he dealt at length with the shape of the earth, precession and nutation, sea-flow and tides, and lunar theory (or, to be more precise, lunar theories). Among the other branches of mechanics, he began the study of molecular mechanics which was to grow substantially over the next decade and (as was mentioned in Subsection 1.4) significantly helped the rise of physics; he also considered several parts of the corporeal branch. Like Lagrange, he furnished no diagrams (apart from those in a supplement of 1806 on capillary theory), but rather than explicating an ideology he merely rendered his text even more difficult to follow than it was already. His treatise is cited as Laplace [Mechanics].

In terms of the approaches described in Section 2, Laplace deployed variational methods from time to time (and proposed a proof of the principle of virtual velocities); but he relied mostly on derivatives and variants of Newton's laws, using Euler's innovation of expansions in trigonometric series of the key variables of celestial mechanics (as did Lagrange) and drawing on (and even inventing) certain methods of solving partial differential equations to yield (where necessary) iterative solutions. He continued and indeed greatly encouraged the French love of long equations in mathematical astronomy, in contrast to the German penchant for compact and feasible methods (on which see [Grattan-Guinness 1990, Interlude 641.1]). He is notoriously hard to read, and not only from the point of view of nonattributions: the only recommendable edition of his treatise is Bowditch's English translation *Celestial mechanics* (1829–1839), which also contains extensive footnotes and appendices on his procedures and on various other contemporaneous developments.

5.4. *Montucla*

The parts of Montucla's *Histoire des mathématiques* which are relevant to mechanics come in the third and fourth volumes of 1802. The first 336 pages of the third volume were written and proofread by Montucla himself, but the rest was edited and largely compiled by Lalande after Montucla's death. Following convention, I cite the volumes as [Montucla 1802], although in some compensation to their main author I mention that his bibliography and history of astronomy, published in the year following [Lalande 1803], is a valuable source for the last twenty years of the century.

In one respect all the four volumes of Montucla's work are disappointing: the text is always prosodic, so that very many of the mathematical details are glossed over or at best shown in plastic-replica prose. The third volume began with an extensive survey of the calculus and related subjects, continued with optics (which, as was mentioned in Subsection 1.3, was not very mathematical outside the geometrical aspects), and came to mechanics in the last quarter of its 800 pages. The various approaches of Section 2 in the present paper were noted, and then, interestingly, a chapter on machines was added (by Lalande: his own contributions seem largely to start here). The fourth volume, in its main text of nearly 600 pages, ran through celestial mechanics in much detail (following the specialty of its real author), with planetary mechanics rather patchily treated; there then followed navigation. Six short supplements followed; two of them dealt respectively with geography and music, the latter often regarded as a classical science in the 18th century and leading to mathematical (though not mechanical) problems such as temperament. While the prosodic character severely limits the measure of the knowledge conveyed, overall the coverage gives a better impression of the range of mechanics than do most of the later histories.

5.5. *Delambre*

As the *secrétaire perpétuel* for "mathematical" sciences of the scientific class of the *Institut de France*, in 1808 Delambre read to Napoleon a report [Delambre 1810] on "the progress in mathematical sciences since 1789, and their current state." The points just made about Montucla apply here also: while popular in level, the coverage was good (including note of some work outside France), and it is interesting to see how the topics of our branches were divided among the headings "Analytical Mechanics," "Mechanics," "Geography and Voyages," "Mathematical Physics," and "Manufactures and Arts."

6. SOME OTHER IMPRESSIONS AROUND 1800

One obvious feature of these works is that they were all written by French scientists; but, as was mentioned at the beginning of the last section, France was the dominant country of the time. I make here a few remarks on developments elsewhere by taking two moderately significant countries, the German states and Britain. Among other countries, Italy is in particular need of detailed study, and I cannot fill the gap here, offering only my study [Grattan-Guinness 1986b] of the mathematical achievements of one scientific society.

6.1. German Sources

The renown of the French works just described was recognized in the German-speaking lands, and German translations were soon prepared: of Lagrange in 1797, of de Prony's two volumes in 1798 and 1801, and of the first two volumes (only) of Laplace in 1800 and 1802. At the time certain German authors were writing general surveys of mathematics, and I shall briefly note the results here.

The works give a poor impression of the understanding in Germany of the main concerns in mechanics of the time (although, as was noted in Subsection 5.3, research-level astronomy was developing well). The state of professionalization of science in Germany was not then strong, and in any case the mathematicians showed some preference for pure mathematics, with a huge concern for combinatorics [Mehrtens 1980]. Kaestner, one of the leaders of this movement, also worked in areas of mechanics, and began publishing a multivolume history of mathematics in 1796 which treated both pure and applied areas [Kaestner 1796–1800]; but unfortunately he reached only the mid-17th century at the time of his death in 1800. However, he also wrote a preface for Rosenthal [1794–1803], another large project in mathematics in production from 1794 which provided 12 volumes over the next decade. It began with 4 volumes of the "first part," on pure mathematics, which only reached "F"; it was followed by 8 more of the fifth part on the unusual subject of "War Sciences" ("Kriegswissenschaften"), which stopped when in the letter "K." Surprisingly little engineering mathematics appeared in these pages, though some aspects of friction theory and embankments were presented in articles such as "Bollwerk," "Futtermauer," and "Hauptwall," while a few features of ballistics appeared in "Kaliber" (of a bullet) and "Kanonen." On this concern with war sciences, see [Jähns 1891].

When Rosenthal's project (grandiose but in fact rather untypical for Germans) petered out in 1803, Klügel started putting out his "dictionary of mathematics"; but true to form, it covered only pure mathematics during the 33 years of its duration [Klügel 1803–1836]. However, in the third volume of 1808 there is an interesting entry of 22 pages on "mathematics," in which he distinguished mechanics from astronomy, and both from "technical mathematics," which was divided into eight parts (including "war sciences").

6.2. British Sources

Another major scientific country was Britain; but once again, in mechanics, and in mathematics in general, it did not create a strong tradition. Long self-bound to

Newtonian methods in both mechanics and the calculus, it was only just beginning to take seriously the massively superior achievements of the Continentals—being much stimulated, in fact, by the appearance from 1799 of Laplace's great work (see [Guicciardini 1989, Chaps. 7–9]). In mechanics and the calculus it had kept a respectable second place to Continental work up to midcentury, with MacLaurin justly well regarded; but when Continental work expanded greatly with the introduction of multivariate and variational techniques, the British were left pathetically behind, and made no attempt either to imitate the methods within their own Newtonian fluxional calculus or to extend the range of (their) mechanics itself. (See especially [Wallis 1986] on the British community of mathematicians.) A dreary catalog of textbooks or manuals was produced, almost always restricted to the first three sections of Book 1 of the *Principia*, some simple machines, and the law of free fall. It is not clear why this apathy occurred; there was no technical barrier within the framework of Newton's theories, either in mechanics or in the calculus, to prevent such extensions.

These remarks are particularly pertinent to England. They are fair also for Ireland, where the state of affairs was not encouraging; reforms there did not start until the mid-1810s, although a rapid rise to importance then ensued [Grattan-Guinness 1988]. Wales produced nothing. But in Scotland somewhat more awareness of Continental developments is evident. Even there, though, parochialism is evident. Take the third edition of the *Encyclopaedia britannica*, which was produced at the end of the 18th century. In 1797 a substantial piece of 60 pages on mechanics was published. It is distinguished by its attention to engineering mechanics, which took up the first 45 pages and went into much more detail than the simple cases just mentioned: then followed some basic remarks on corporeal mechanics, but based only upon Newton's laws (in the form presented by Atwood), and throughout the piece almost all citations were of British work. I list the article as Robison [1797], as he seems to have been responsible for the articles on "natural philosophy."

I can fairly reflect the better side in Britain by noting the Englishman Hutton, one of the very few Britons who was aware of Continental methods before Laplace's book began to appear. He produced the elaborate *Mathematical and philosophical dictionary* in two volumes in 1796 and 1795 (in that order) ([Hutton 1795–1796; see [Grattan-Guinness 1986a]). He still showed his British preferences in covering "Calculus" in half a column and sending the reader for the details to the 23 columns of "Fluent" and "Fluxion"; but his biographical articles on Continental mathematicians were fair, those for Euler and d'Alembert being very warm. His coverage of applications was much better than that of the Germans; however, most entries were curiously lacking in technical details, unlike the main articles on pure mathematics, some of which were lengthy. In addition, he partitioned mechanics in rather surprising ways. "Dynamics" and "Statics" took only preliminary entries, with the information given under titles such as "Newtonian Philosophy," "Force," "Descent," "Motion," "Planet," "Tides," "Precession of Equinoxes" ("Astronomy," while very long, was only historical), and "Nutation" (presented in the supplement of the book). Some instruments were well handled, especially "Barometer" and "Pendulum"; and he also gave good space to "Steam" (again in the supplement), in which he described a version of de Prony's speculation on gases noted in Subsection 5.2. By contrast, entries such as "Elasticity," "Action," "Geodesy," and "Hydraulics" are extremely disappointing, and overall the grasp of the subject shown even by this most capable author was pretty patchy. The French lead was to last for three more decades.

7. CONCLUDING COMMENTS

The standard first approximation to our knowledge of the history of mechanics in the 18th century, that the subject was dominated by Newtonian methods and laws, was rejected in the opening section, and a more varied interpretation suggested in the following sections. However, this picture now presented is far from a full appraisal, and for reasons other than that of the necessary brevity of this paper. Roughly speaking, standard interpretations of the period used to take Lagrange's own historical paragraphs in his *Méchanique analitique* of 1788 as a definitive sketch; but work in recent decades has rejected such a view, somewhat downgrading his own contributions and increasing the significance of Euler's (see especially Truesdell's writings, and the general volumes [Euler 1983, 1985, 1988]). My own impression is that Euler may now have *too* good a press, and that some other figures need upgrading of their own. I suspect that proper appraisals are lacking of at least d'Alembert, Daniel Bernoulli, Boscovich [Whyte 1961], Hermann, Lambert [1979], MacLaurin, and Varignon.

As regards subject matter, the branch of engineering mechanics must be taken far more seriously as an integral component of the whole scheme. In addition, the history of mechanics education is poorly known, relative to all its branches (although there is some information *passim* in [Taton 1964; Gillispie 1980] for the important case of France). The specific volumes discussed or mentioned in Section 4 were treatises rather than textbooks (de Prony had educational ambitions for his *Nouvelle architecture hydraulique*, but its content, and even size and price, would have made them pious). We still have a long way to go; the varieties are by no means yet fully exposed.

8. APPENDIX: COMMENTS ON THE LITERATURE

8.1. The Primary Literature

One reason for the reception of Euler and Lagrange and the neglect of the six other figures just mentioned is the existence or not of modern(ish) editions of their works. Lagrange's works were completed by the early 1890s: as an edition, it leaves much to be desired (for example, the *Méchanique analitique* is not in it). Laplace's edition was finished just before the First World War, but it is another pillar to nonscholarship and in any case he remains impenetrably hard to comprehend; thus, for the reasons mentioned in Subsection 5.3, he has never gained his proper due. At that time the Euler edition began, and most of his published works have now been dealt with (although in fact the series for mechanics is not yet quite complete, a matter currently in the hands of E. J. Aiton).

Regarding the other figures, the *Opera* produced by Boscovich in five volumes in 1785 is not an edition in the sense intended here, though it does contain pertinent material. The so-called *Opera mathematica* of the 1940s for Lambert contains only his works on pure mathematics, while the edition for the Bernoulli family has only started in this decade; and the other figures have no relevant edition at all.

Even in the Euler edition, publication of the correspondence has only begun in this decade, and the notebooks are still awaited (in the meantime Knobloch [1989] provides a valuable index). Thus one still has to rely on older partial editions and selections, such as [Fuss 1843], for this type of source.

It is worth mentioning the series of *Ostwald's Klassiker der exakten Wissenschaften*, which began in 1889. It included good editions of some primary items pertinent to our story, with good notes and commentary (and, where necessary, German translations).

For references to the primary literature, in addition to the citations in the secondary sources, the usual bibliographies are handy; the *Royal Society catalogue of scientific papers* even picks up material for some authors from the late 18th century (for example, it is the easiest place to track the publication of Euler's posthumous papers). Less well known than it deserves is Reuss's "Repertorium" [1801–1821], a Poggendorff on the publication of papers in society journals which was produced in the early 19th century, before Poggendorff. It is very valuable for all mathematical topics: Volumes 4, 5, and 7 are the most pertinent ones in its series.

8.2. *The Secondary Literature: Histories and Textbooks*

As was mentioned in Subsection 1.2, the bibliography below consists only of those (primary or secondary) works which have been cited in the text. Some additional explanation is provided here and in the next Subsection.

The literature has not received a good overall appraisal above, and I would add that the general histories of mechanics, while containing valuable commentaries on some aspects of the story, are pretty disappointing in the overall impression that they give. (The most substantial works are those of Mach [Mechanics] (a very overrated volume, though itself influential), Duhem [1903], Dugas [1955], Szabo [1977], and Bogolyubov [1976, 1978].) For example, the importance of the differences between the three approaches is often not properly emphasized; and the five branches are not delineated as such, with the engineering branch frequently played right down. As an example of the omissions, frequently not a line is devoted to Poinsot's theory of the couple mentioned in Subsection 5.2. Even among more specialized histories, a very variable quality of scholarship is evident.

The general histories of mathematics are necessarily still patchier: but in addition, many of them have been written in the last 100 years, during which time pure mathematics has come to dominate over applied mathematics to the extent that in these books applications of all kinds are lightly treated (see even [Cantor 1908; Kline 1972], the best of the more substantial histories). Naturally for the history of all mathematics before that time, a fundamental travesty is thereby committed.

A better impression comes from Jouguet [1909], who presents "mechanics taught by the original authors." His commentary is interspersed by passages (in translation where necessary) from the primary literature, some of them extensive. The coverage was mostly restricted to corporeal and molecular mechanics, but the variety was quite clearly indicated. The book deserves to be better known. In our own time, Truesdell's writings deserve an honorable mention for highlighting elasticity theory, hydrodynamics, and acoustics, and thereby widening the range of topics which historians of mechanics must address.

As a rule, textbooks on mechanics are disappointing as historical sources, although they are a valuable indication of how the subject was being presented and taught at a given time. Even the more historically minded writers of the 19th century, such as E. J. Routh, rarely cast their references back beyond their own century.

8.3. The Secondary Literature: Encyclopaedias and Dictionaries

I turn now to other types of sources, starting with large-scale encyclopaedias involving many authors. From the 1780s the French began to put out the *Encyclopédie méthodique*, as a successor to the Diderot/d'Alembert *Encyclopédie*: the majority of the volumes came out fairly soon, though others continued to appear until the 1820s. Mechanics turns up in quite a large number of parts of the encyclopaedia, doubtless following a taxonomy which however I have not been able to unravel. The series on "Mathematics" has several pertinent articles, of course, but others are to be found in the runs for "Military Arts," "Marine," "Manufacturers Arts and Trades," and "Mechanical Arts and Trades" and in the *Dictionary of Physics* (whose four volumes were published between 1793 and 1822, late in the project and thereby indicative of the low status of physics noted in Subsection 1.4). De Prony planned a supplementary volume on "the art and the science of the engineer," but the project never reached print beyond a short preface [1788].

Some of the British encyclopaedias produced in the early 19th century contained excellent surveys of (parts or branches) of mechanics (an example of 1797 was described in Subsection 6.2); and even when not explicitly historical, they are both impressive in range and detail and valuable to us in providing a perception of the view of 18th-century mechanics held not long afterward. I have furnished a table of these encyclopaedias at the end of [Grattan-Guinness 1981]: I mention here various items in the editions of the *Encyclopaedia britannica* of the period 1800–1830, especially the supplementary volumes from 1815 on; the remarkable ensemble of long pieces published in the 1820s by Barlow in the *Encyclopaedia metropolitana*; and some of the longer articles on technology in *Rees's cyclopaedia*, which was completed by 1819.

Two other sources started around the turn of the 19th century and this century, largely under German leadership. First, the *Encyklopädie der mathematischen Wissenschaften* was primarily intended to report on the most recent developments in the various branches of mathematics; but several articles in its fourth and sixth parts (which are devoted respectively to corporeal and to planetary and celestial mechanics) go back beyond the immediate developments and have some information on our period. A few articles in the second and fifth parts, on the calculus and related subjects and on physics, are also to be scanned. Second, (and as much Swiss as German), the Euler edition contains valuable editorial commentaries in several of the volumes of the second series: only those pieces which contain a substantial amount of information on other figures have been cited here.

To conclude, some of the biographical dictionaries and encyclopaedias produced over the decades have valuable articles on the 18th-century figures. The *Dictionary of scientific biography* (1970–1980) is of course outstanding in this regard. In addition, the biography of the French mathematician and instrument maker Borda by Mascart [1919] contains information (unfortunately not always accurate) on many other figures, mostly French compatriots of his subject.

8.4. Tertiary Literature

Four particular surveys can be mentioned. First, Rousseau & Porter [1980] present a general study of 18th-century science and the historiographical difficulty that it presents (of seeming to be more boring than the centuries on either side); the articles by E. G. Forbes and H. J. M. Bos are pertinent. Second, Grigoryan & Filatova [1983] survey Soviet writings on the history of mechanics. Third, Whitrow's [1976] volume of cumulated secondary literature for the period 1913–1965 contains exhaustive listings for mechanics, under various headings. Finally, the bibliography for the history of mathematics by Dauben [1985] has parts on our topic and on all neighboring topics, while May [1973] provides good references to individuals and to some topics.

ACKNOWLEDGMENTS

Versions of this paper were delivered as lectures at the conferences on "Newton and the world of science" held in Moscow in October 1987 under the auspices of the Soviet Academy of Sciences, and on "Hegel and Newtonianism" which took place at Trinity College Cambridge in September 1989. For providing funds to let me attend these meetings I am grateful respectively to the Royal Society of London and to the Istituto Italiano per gli Studi Filosofici. For comments on this written version, I am grateful to H. Pulte and to an anonymous referee.

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