Physics Letters B 700 (2011) 254–260

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The unified first law in "cosmic triad" vector field scenario

Yi Zhang [a,b,*], Yungui Gong [a], Zong-Hong Zhu [b]

[a] College of Mathematics and Physics, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[b] Department of Astronomy, Beijing Normal University, Beijing 100875, China

ARTICLE INFO

Article history:
Received 17 February 2011
Received in revised form 2 May 2011
Accepted 3 May 2011
Available online 11 May 2011
Editor: S. Dodelson

Keywords:
The unified first law
Vector field
Apparent horizon

ABSTRACT

In this Letter, we try to apply the unified first law to the "cosmic triad" vector field scenario both in the minimal coupling case and in the non-minimal coupling case. After transferring the non-minimally coupling action in the Jordan frame to the Einstein frame, the correct dynamical equation (Friedmann equation) is gotten in a thermal equilibrium process by using the already existing entropy while the entropy in the non-minimal coupled "cosmic triad" scenario has not been derived. And after transferring the variables back to the Jordan frame, the corresponding Friedmann equation is demonstrated to be correct. For complete arguments, we also calculate the related Misner–Sharp energy in the Jordan and Einstein frames.

1. Introduction

The profound connections between gravity and thermodynamics are suggested by many phenomena, such as the discovery of Hawking radiation and the four laws of classical black hole mechanics [1–4]. Based on the geometric feature of thermodynamic quantities of black holes [5], a remarkable connection for cosmology is found by Jacobson who pointed out it is possible to derive the Einstein equations of gravitational fields from a view of thermodynamics. The keys to derive the Einstein equation are the fundamental relation (Clausius relation) $\delta Q = T\,dS$ and the form of the entropy which is proportional to the horizon area. Further studies between thermodynamics and gravity have been extended to various cosmological settings [6–11], such as the Lovelock gravity [12–16], the brane-world scenario [17–19], the scalar–tensor theory [20–23], the loop quantum gravity [24–28], the Horava–Lifshitz gravity [29], the logarithm correction scenario [30–35], the trace anomaly correction scenario [36], and the $f(R)$ gravity [37–48,51,52].

In cosmology, the scalar field could be assumed to be isotropic and homogeneous to correspond with the FRW (Friedmann–Robertson–Walker) background. It is the most popular candidate of dynamical sources for the accelerations in our universe [53–56]. However, the fundamental scalar field [57,58] has not to be probed yet. On the other hand, the vector field is common in our realistic world. The inflationary scenario with vector fields was proposed by Refs. [59,60]. Despite the later discovered instability problems [61] in perturbations [62–64], this vector field scenario was even extended to higher spin fields in cosmology [65–67]. The "cosmic triad" scenario is one of those models that coincide with the observable isotropic and homogeneous FRW background [68–73] (see also "N-inflation" vector field scenario proposed by Refs. [74–76] which is similar to "N-flation" in scalar field [77], the time-like vector filed scenario proposed by Refs. [78–86], and the exact isotropic solutions of the Einstein–Yang–Mills system proposed by Refs. [87–90]). The "cosmic triad" scenario of vector field has three spatial components equal and orthogonal to each other where $A_1 = (0, A, 0, 0)$, $A_2 = (0, 0, A, 0)$ and $A_3 = (0, 0, 0, A)$. In this Letter, we will use view to study the relation between gravity and thermodynamics.

In a special kind of spherically symmetric black hole spacetimes, Padmanabhan showed that the Einstein equations on the black hole could be written into the first law of thermodynamics [6]. Cai and Kim [10] derived the Friedmann equation by assuming that the apparent horizon has temperature and entropy and applying the fundamental relation $\delta Q = T\,\delta S$ to the apparent horizon of FRW universe. The Clausius relation requires the equilibrium thermodynamics. In the Einstein gravity, the Clausius relation for the equilibrium thermodynamics could always hold. However, there are some arguments on the existence of thermal equilibrium process for the non-Einstein gravity, such as the scalar–tensor theory (the $f(R)$ theory as well). The field equation for scalar–tensor gravity needs the non-equilibrium thermodynamics arguments in Refs. [91,92]. To get the Friedmann equation, the related thermal dynamical discussion has used the bulk viscosity entropy production

* Corresponding author at: College of Mathematics and Physics, Chongqing University of Posts and Telecommunications, Chongqing 400065, China.
E-mail addresses: zhangyia@cqupt.edu.cn (Y. Zhang), gongyg@cqupt.edu.cn (Y. Gong), zhuzh@bnu.edu.cn (Z.-H. Zhu).

0370-2693
doi:10.1016/j.physletb.2011.05.005

term [37–40]. Therefore, it is proposed to add the entropy production term to get the Friedmann equation in Ref. [38]. Meanwhile, it was noticed that the entropy of static horizon is well defined by Wald's definition in Refs. [41,49,50], which is a Noether charge associated with the horizon killing vector and the correct Friedmann equation could be gotten in the non-minimally coupled gravity with equilibrium thermodynamics.

The non-minimally coupled vector fields bring us a new physical background [74–76]. The non-minimally coupled "cosmic triad" vector field scenario is quite similar to the scalar–tensor theories of gravity. Therefore, it is rather natural to ask whether the corresponding physical process is thermal equilibrium or not. Even worse, we have no idea of the entropy definition in the non-minimally coupled "cosmic triad" vector field scenario. Fortunately, the Einstein frame could be used as a bridge. The exact form of the entropy of "cosmic triad" scenario in the Jordan frame is not prerequisite. Based on the conformal transformations and the entropy form of the Einstein gravity, we can still derive the Friedmann equation.

Our derivations of the Friedmann equation will also be affected by the definition of energy. To make our arguments complete and consistent, we try to discuss general Misner–Sharp energies [93,94] in a spherically symmetric spacetime by integral method. The generalized Misner–Sharp energy is argued to be related to the Einstein equation whose definition is clear in the Einstein gravity, but not in the non-Einstein gravity [93,94]. The thermal equilibrium process in scalar–tensor gravity has been presented in Refs. [37–40] with the effective geometric part included in the total energy density. However, our results will not include the obvious effective geometric part. We use units of $k_B = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by $\kappa^2 = 8\pi m_{Pl}^{-2}$ where $m_{Pl} = G^{-1/2}$ is the Planck mass.

We arrange our Letter as follows. In Section 2, we introduce basic notions in thermodynamics, the temperature, the apparent horizon, the unified first law and the Clausius relation. After that, we present the minimally coupled "cosmic triad" vector field model and deduce its dynamical equation in Section 3. Then, in the non-minimal coupling case, considering the similarity between scalar–tensor theories and the discussed vector fields theory, we manage to get the Friedmann equation with the help of Einstein frame in Section 4. For consistency, the results of the general Misner–Sharp energy are presented by integral method in Section 5. The Letter is concluded in Section 6.

2. The unified first law

The FRW metric is one kind of spherically symmetric spacetime. If the closure of a hypersurface was foliated by future or past, outer or inner marginal sphere, it is the so-called trapping horizon. However, in the FRW universe, the "outer trapping horizon" does not exist, instead there are a kind of cosmological horizons called "inner trapping horizon" which is the apparent horizon in the context of the FRW cosmology. In this Letter, we will not distinguish the two horizons. And, the associated thermodynamics will be discussed. The $(3+1)$-dimensional FRW universe has the metric

$$ds^2 = -dt^2 + a^2\gamma_{ij}\,dx^i\,dx^j, \tag{1}$$

where a is the scale factor, the metric γ_{ij} is given by $\gamma_{ij} = d\rho^2/(1-k\rho^2) + \rho^2 d\Omega_{n-1}^2$ and the three-dimensional spatial curvature of the hypersurface is parameterized as negative, zero or positive, respectively. The FRW metric could be rewritten in the double null form as well

$$ds^2 = h_{ab}\,dx^a\,dx^b + r^2\,d\Omega_{n-1}^2, \tag{2}$$

where $r = a(t)\rho$, $x^0 = t$, $x^1 = \rho$ and the two-dimensional metric is $h_{ab} = \mathrm{diag}(-1, 1/(1-k\rho^2))$.

The thermodynamics will be established on the apparent horizon where the future inner trapping horizon is the boundary of a system. The dynamical apparent horizon is defined as

$$r_A = h_{ab}\partial_a r\partial_b r = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter. And, the surface gravity of the trapping horizon κ_s is defined as

$$\kappa_s = \frac{1}{2}\nabla^a\nabla_a r = r_A\left(1 - \frac{\dot{r}_A}{2Hr_A}\right), \tag{4}$$

where the subscript "s" is used to note the variables for the thermodynamics specially. Then, the corresponding temperature is

$$T = \frac{\kappa_s}{2\pi} = -\frac{r_A}{2}\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right). \tag{5}$$

For dynamical black holes, Hayward [96,97] has proposed a relation called "unified first law" to deal with the gravity and the thermodynamics associated with trapping horizon of a dynamical black hole in four-dimensional Einstein theory. For spherically symmetrical spacetimes, the time–time component of the Einstein equations could be rewritten in the "unified first law" form

$$dE = A_s\Psi + W\,dV, \tag{6}$$

where A_s and V are the area and volume of the three-dimensional space. The first term in the unified first law could be interpreted as an energy-supply term, analogous to the heat-supply term in the classical first law of thermodynamics. One has

$$\begin{aligned}\Psi &= \Psi_t\,dt + \Psi_\rho d\rho \\ &= -\frac{1}{2}\big(\rho^{(m)} + p^{(m)}\big)Hr\,dt + \frac{1}{2}\big(\rho^{(m)} + p^{(m)}\big)a\,d\rho,\end{aligned} \tag{7}$$

where the superscript "(m)" notes the variables for the total matter which includes not only the pressure matter, but also the matter field part. In this Letter, we just neglect the radiation part which could be added conveniently by rewriting the Lagrangian and it would not affect our results. And, the second term in the unified first law could be interpreted as a work term. Following Refs. [10,94,96,97], the work density at the apparent horizon is

$$W = -\frac{1}{2}T^{ab}h_{ab} = \frac{1}{2}\big(\rho^{(m)} - p^{(m)}\big), \tag{8}$$

which should be regarded as the work done by a change of the apparent horizon. Finally, on the left-hand side of the unified first law, the energy on the apparent horizon is the generalized Misner–Sharp energy

$$\begin{aligned}dE &= A_s\Psi + A_sW\,dr_A \\ &= -\big(\rho^{(m)} + p^{(m)}\big)A_sHr_A\,dt + A_s\rho^{(m)}\,dr_A = d\big(\rho^{(m)}V\big).\end{aligned} \tag{9}$$

On the other side, during the time interval dt, the Clausius relation gives out an energy flux

$$\delta Q = T\,dS, \tag{10}$$

where δQ and T are the variation of heat flow and the Unruh temperature seen by an accelerated observer just inside the horizon. Then, by matching the heat flux of energy and the amount of energy crossing the apparent horizon, one has

$$\delta Q = T\,dS = A\Psi. \tag{11}$$

In the Einstein gravity, the unified first law also implies the Clausius relation $\delta Q = T\,dS$ [95]. The Clausius relation holds for all local Rindler causal horizons through each spacetime point in the equilibrium thermodynamics. Therefore, in equilibrium thermodynamics, by matching Eqs. (6) and (10), it obtains

$$T\,dS = dE - W\,dV. \tag{12}$$

Combined with the temperature (5), the above equation could be rewritten as

$$\frac{1}{2\pi r_A}\left(1-\frac{\dot r_A}{2Hr_A}\right)dS = 4\pi r_A^3 H\left(\rho^{(m)}+p^{(m)}\right)dt - 2\pi r_A^2\left(\rho^{(m)}+p^{(m)}\right)dr_A \tag{13}$$

where the equilibrium thermodynamics must hold. It has been shown that the above equation is held in the Einstein gravity with the pressureless matter. However, if the vector fields were added, it is a question whether this situation will be changed or not. However, given the exact form of entropy, Eq. (13) will give out the Raychaudhuri equation which connects the geometry and the matter. Furthermore, by considering the conserved equation of the energy density, the Friedmann equation will be easily derived.

3. "Cosmic triad" vector field scenario

The "cosmic triad" vector field scenario [69] is composed of three vector fields minimally coupled with gravity, which are a set of three identical self-interacting vectors. This kind of vector fields naturally arise from a gauge theory with $SU(2)$ or $SO(3)$ gauge group. In this Letter, Latin indices are used to label the different fields ($a, b, \ldots = 1\ldots3$), and Greek indices are used to label the different spacetime components ($\mu, \nu, \ldots = 0\ldots3$). In minimal coupling case, the action of "cosmic triad" scenario is

$$\mathcal{I} = \int d^4x\sqrt{-g}\times\left[\frac{R}{16\pi G} - \sum_{a=1}^{3}\left(\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + V\left(A^{a2}\right)\right) + \mathcal{L}_m\right], \tag{14}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$, $A^{a2} = g^{\mu\nu}A^a_\mu A^a_\nu$, A^a_ν is the vector field and $\mathcal{L}_m$ is the Lagrangian of pressureless matter. The term $F^a_{\mu\nu}F^{a\mu\nu}/4$ could be considered as the Maxwell type kinetic energy term, and the term $V(A^2)$ as the potential of the vector field. We assume the energy density of pressureless matter conservation

$$\dot\rho_m + 3H(\rho_m + p_m) = 0, \tag{15}$$

where the dot means a derivative with respect to time t.

In "cosmic triad" vector field scenario, the ansatz that the three vectors are equal and orthogonal to each other could be expressed as

$$A^b_\mu = \delta^b_\mu B(t)\cdot a. \tag{16}$$

Following Ref. [69], we could define a new variable called "physical" vector field which is $B_i = A_i/a$, where A_i is called "comoving" vector field. The related equation $B^2 = B_iB_i = A_\mu A^\mu = A^2$ could be conveniently obtained in the FRW background. Then we could express most of our equations in term of B_i and B^2 in the following discussions. The corresponding energy density and pressure are given by

$$\rho_v = \frac{3}{2}(\dot B + HB)^2 + 3V\left(B^2\right), \tag{17}$$

$$p_v = \frac{1}{2}(\dot B + HB)^2 - 3V\left(B^2\right) + 2B^2V', \tag{18}$$

where the subscription "v" means the variable corresponding to the vector fields, and the prime denotes a derivative with respect to the square of vector field B, for example $V' = dV/dB^2 = dV/dA^2$. In the minimal coupling case, the energy density of the vector field is conserved as well

$$\dot\rho_v + 3H(\rho_v + p_v) = 0. \tag{19}$$

And the equation of motion of vector field is

$$\ddot B + 3H\dot B + V' + \left(2H^2 + \dot H\right)B = 0. \tag{20}$$

Obviously, compared with scalar fields, the term $(2H^2 + \dot H)B$ is an additional term and therefore the dynamics of vector field is different.

In thermodynamics, there are different definitions of entropy. Hayward [96,97] has studied black hole's entropy in generalized theories of gravity and proposed that the correct dynamical entropy of stationary black hole's solution with bifurcate Killing horizon is the Noether charge entropy. In the Einstein gravity, the two definitions seem to be consistent, the entropy has such a form

$$S = \frac{A_s}{4\pi G}. \tag{21}$$

Putting the variables (17), (18) and (21) into Eq. (13), we could get the Raychaudhuri equation in the "cosmic triad" vector field scenario

$$\dot H - \frac{k}{a^2} = -4\pi G\left(2(\dot B + HB)^2 + 2B^2V' + \rho_m + p_m\right). \tag{22}$$

By using the conserved equations (15) and (19), the Friedmann equation is obtained

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\frac{3}{2}(\dot B + HB)^2 + 3V\left(B^2\right) + \rho_m\right). \tag{23}$$

During the process, an integral constant has emerged which could be regarded as a cosmological constant and could be incorporated into the energy density as a special component. Here, the two energy components are conserved separately, but in the non-minimal coupling case, the situation is more complicated.

4. Non-minimally coupled "cosmic triad" vector field case

In the vector field scenario, the non-minimal coupling term is used to satisfy the slow-roll conditions [71]. Without the non-minimal coupling term, the vector field could only be used as curvaton [67,62–64]. Let us start with the action of non-minimal coupled "cosmic triad" vector field

$$\mathcal{I}_n = \int d^4x\sqrt{-g}\times\left[\frac{f(A^2)R}{16\pi G} + 3\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V\left(A^2\right)\right) + \mathcal{L}_m(g_{\mu\nu})\right], \tag{24}$$

where the subscript "n" denotes the non-minimal coupling case, the function $f(A^2)$ shows the non-minimal coupling effect and it will go back to the minimal coupling case when $f(A^2) = 1$. Some gauge-dependent second order derivatives of the vector field A_μ come from the $f(A^2)R$ term which breaks the gauge invariance of the vector's kinetic term.

Conformal (or Weyl) transformations are widely used in scalar–tensor theories of gravity, the theory of scalar fields coupled non-minimally to the Ricci curvature R. Due to the similarities between the scalar–tensor theory and the non-minimal coupling "vector filed" scenario, we could perform the conformal transformation from the Jordan frame to the Einstein frame. One could introduce auxiliary fields, or even simply redefine fields for one's convenience. There is no unique prescription of redefining the fields of a theory. Acting on the metric by a suitable conformal transformation, the action (24) could be recast into the one in the Einstein frame with the new metric,

$$\bar{g}_{\mu\nu} = f\left(A^2\right) g_{\mu\nu}, \tag{25}$$

where the bar represents variables in the Einstein frame. And this frame is expected to excite the generic helicity-0 ghost of the non-invariant vector theories. The corresponding action in the Einstein frame is changed to [98]

$$\bar{I}_n = \int d^4x \sqrt{\bar{g}} \left[\frac{\bar{R}}{2\kappa} - \frac{3}{4\kappa} Z^2 \left(\partial_\mu \bar{A}^2\right)^2 - \frac{1}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - W\left(\bar{A}^2\right) \right] + \int d^4x \, L_m(\bar{g}_{\mu\nu}), \tag{26}$$

where the kinetic terms of the vector A_μ and the tensor $g_{\mu\nu}$ are now diagonalized in a covariant way, and

$$\bar{F}_{\mu\nu} = F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \bar{F}^{\mu\nu} = \bar{g}^{\mu\rho} \bar{g}^{\nu\sigma} F_{\rho\sigma} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{27}$$

$$U = \frac{V}{f^2}, \qquad \bar{A}^2 = \frac{A^2}{f}, \qquad Z = -\frac{d\ln(1/f)}{d\bar{A}^2} = \frac{ff'}{f - A^2 f'}. \tag{28}$$

The energy densities of pressureless matter and vector fields are being rescaled as

$$\bar{\rho}_m = \frac{\rho_m}{f^2}, \tag{29}$$

$$\bar{\rho}_v = \frac{3}{2f^2}(\dot{B} + HB)^2 + 3\frac{V}{f^2} + \frac{\dot{f}^2}{f^2}. \tag{30}$$

The energy densities of two components are not conserved separately any more. However, the total energy of matter is still conserved which includes the rescaled pressureless matter and the rescaled vector fields

$$\dot{\bar{\rho}}^{(m)} + 3H\left(\bar{\rho}^{(m)} + \bar{p}^{(m)}\right) = 0, \tag{31}$$

where $\bar{\rho}^{(m)} = \bar{\rho}_m + \bar{\rho}_v$ and $\bar{p}^{(m)} = \bar{p}_m + \bar{p}_v$.

In the Einstein frame, the entropy could be written as

$$\bar{S} = \frac{\bar{A}_s}{4\pi G}. \tag{32}$$

In order to get the heat δQ in the Clausius relation, we have to consider the contribution from matter fields. In the Einstein frame, by using the unified first law, one could get the Raychaudhuri equation

$$\dot{\bar{H}} - \frac{k}{\bar{a}^2} = -4\pi G(\bar{\rho}_v + \bar{p}_v + \bar{\rho}_m + \bar{p}_m). \tag{33}$$

Combining the above equation with the conserved equations (31), the Friedmann equation is obtained

$$\bar{H}^2 + \frac{k}{\bar{a}^2} = \frac{8\pi G}{3}(\bar{\rho}_v + \bar{\rho}_m). \tag{34}$$

In the Einstein frame, the energy density has been rescaled and even the energy density of matter is no more conserved.

It should be noted that the energy measured by an observer is the one in the Jordan frame. Based on the rescaled metric, the relations of the scalar factor and the Hubble parameter between the two frames hold as

$$\bar{a} = \sqrt{f} a, \qquad \bar{H} = \frac{d\bar{a}}{\bar{a}\,dt} = H + \frac{\dot{f}}{2f}. \tag{35}$$

Then the Friedmann equation (34) in the Einstein frame could be transferred to the one in the Jordan frame

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\frac{3}{2}\left(\dot{B}^2 + HB\right)^2 + 3V + 6H(\dot{f} + Hf) + \rho_m\right). \tag{36}$$

It is just the correct Friedmann equation in the non-minimally coupled "cosmic triad" vector field scenario.

In a short summary, the form of the entropy in the non-minimally coupled "cosmic triad" scenario is needed to directly get the Friedmann equation. Unfortunately, such entropy is unknown. Therefore, we transfer the Jordan frame to the Einstein frame where the definition of the entropy is clear. In the Einstein frame, we have obtained the dynamical equation with the rescaled variables. At last, by transferring these variables back to the Jordan frame, one has the correct Friedmann equation.

The equilibrium thermodynamics could be held in the Einstein frame. As the exact physics in the Jordan frame is unknown, there are clearly at least two possibilities for this theory. If the thermodynamics in the Jordan frame is in equilibrium, the thermal process is equilibrious both before and after the conformal transformation. But, if it is a non-thermal equilibrium process in the Jordan frame which is contrast to the Einstein frame case, the derived Friedmann equation is just a coincidence. This problem could be left to quantum gravity.

5. The generalized Misner–Sharp energy

Due to the strong equivalence principle, the energy–momentum pseudo-tensor of gravitational field will vanish at any point of spacetime in a locally flat coordinate. Therefore, a local energy density of gravitational field does not make any sense [99]. However, there exist two well-known definitions of total energy: the Bondi–Sachs (BS) energy [100] and the Arnowitt–Deser–Misner (ADM) energy [101]. And, considering a boundary of a given region in spacetime, it is possible to define quasi-local energy, for instance, Brown–York energy [102], Misner–Sharp energy [93], etc. In particularly, at null and spatial infinity, the Misner–Sharp mass reduces to the BS and ADM energies [96,97]. When the notion of generalized Misner–Sharp energy (or mass) is introduced, it seems clear to write and interpret the unified first law [96,97].

Based on the method developed in Ref. [95] where the Einstein equations are used, we will calculate the corresponding Misner–Sharp energy E_M which is defined in a spherically symmetric spacetime of the "cosmic triad" vector field model. The integral method which is introduced in Ref. [95] shows that the definition of the generalized Misner–Sharp energy depends on a constraint condition. For convenience, another form of double-null metric is considered in Ref. [95],

$$ds^2 = -dt^2 + e^{2\psi(t,\rho)}\,d\rho^2 + r^2(t,\rho)\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \tag{37}$$

where $r(t,\rho) \equiv a(t)\rho$ and $e^{\psi(t,\rho)} = a(t)/\sqrt{1 - k\rho^2}$. Following the integral method, we try to list the generalized Misner–Sharp masses.

5.1. Minimal coupling case

Under the double-null metric (37), the generalized Misner–Sharp energy acts as the boundary of a finite region under consideration in the Einstein gravity. Here, we choose the method developed in Ref. [95] to calculate the generalized Misner–Sharp mass which could be used both for the minimal and for the non-minimal coupling cases. Based on the definition, the general Misner–Sharp mass is

$$E_M = \frac{r}{2G}\left(1 - h^{ab}\partial_a r \partial_b r\right) = \frac{r^3}{2G}\left(H^2 + \frac{k}{a^2}\right). \tag{38}$$

In the small-sphere limit, the leading term of E_M is the production of the volume and the energy density of matter [96],

$$E_M = \rho^{(m)} V = \frac{4\pi r^3}{3}\left(\frac{3}{2}(\dot{B} + HB)^2 + 3V\left(B^2\right) + \rho_m\right). \tag{39}$$

Matching the above Eqs. (38) and (39), the Friedman equation could be gotten. However, the Einstein equation is used in the derivation of Eq. (39). Therefore, it is not a surprise to get the Friedmann equation. This calculation demonstrates that the Misner–Sharp is a consistent variable in the Einstein equation. Therefore, for the unified first law, the Misner–Sharp energy is also a consistent quantity.

5.2. Non-minimal coupling case in the Jordan frame

The generalized Misner–Sharp mass is related to the Einstein equation closely. And, the integral method could be used both in the Jordan and in Einstein frames. Therefore, even in the non-minimal coupling case, we could get the generalized Misner–Sharp mass. For metric (37), by using the action (24), the component of the matter part of the stress–energy tensor is

$$8\pi G T^{(m)}_{tt} = 3f\left(\frac{k}{a^2 + H^2}\right) + 3H\dot{f}, \qquad 8\pi G T^{(m)}_{t\rho} = 0, \tag{40}$$

$$8\pi G T^{(m)}_{\rho\rho} = \frac{a^2}{1 - k\rho^2}\left(-f\left(\frac{k}{a^2} + H^2 + \frac{2\ddot{a}}{a}\right) - \ddot{f} - 2H\dot{f}\right) \tag{41}$$

and based on the unified first law, the generalized Misner–Sharp mass is

$$dE_{nM} = A_s \Psi + W\,dV = C(t,\rho)\,dt + D(t,\rho)\,d\rho, \tag{42}$$

where

$$C(t,\rho) = 4\pi r^2 e^{-2\psi}\left(T^{(m)}_{t\rho} r_{,\rho} - T^{(m)}_{\rho\rho} r_{,t}\right) = \frac{1}{2}Hr^3\left[f\left(\frac{k}{a^2} + H^2 + \frac{2\ddot{a}}{a}\right) + \ddot{f} + 2H\dot{f}\right], \tag{43}$$

$$D(t,\rho) = 4\pi r^2\left(T^{(m)}_{tt} r_\rho - T^{(m)}_{t\rho} r_{,t}\right) = \frac{1}{2}\rho^2 a^3\left[3f\left(\frac{k}{a^2}\right) + 3H\dot{f}\right]. \tag{44}$$

Then, the energy could be calculated as:

$$E_{nM} = \int D(t,\rho)\,d\rho + \int\left[C(t,\rho) - \frac{\partial}{\partial t}\int D(t,\rho)\,d\rho\right]dt. \tag{45}$$

If the parameters C and D satisfy the constraint condition

$$\frac{\partial C(t,\rho)}{\partial \rho} - \frac{\partial D(t,\rho)}{\partial t} = 0, \tag{46}$$

the generalized Misner–Sharp mass will be gotten

$$E_{nM} = \frac{r^3}{2G}\left(f\left(B^2\right)\left(\frac{k}{a^2} + H^2\right) + H\dot{f}\left(B^2\right)\right). \tag{47}$$

And in the small-sphere limit of the non-minimal coupling case, the leading term in E_{nM} is the production of volume and the energy density of the matter

$$E_{nM} = \frac{4\pi r^3}{3}\rho^{(m)} = \frac{4\pi r^3}{3}\left(\rho_m + \frac{3}{2}(\dot{B} + HB)^2 + 3V\right). \tag{48}$$

It is a consistent result that the Friedmann equation is given out by combining Eqs. (47) and (48).

5.3. Non-minimal coupling case in the Einstein frame

In the Einstein frame, the definition of the Misner–Sharp energy gives out the geometric representation

$$\bar{E}_{nM} = \frac{r}{2G}\left(1 - \bar{h}^{ab}\partial_a r \partial_b r\right) = \frac{r^3}{2G}\left(\frac{k}{a^2} + \bar{H}^2\right). \tag{49}$$

And, in the non-minimal coupling case, the total matter contains the redefined vector field and the pressureless matter. In small-sphere limit, by using the Einstein equation, the leading term of $\bar{E}_{nM}$ is the production of the volume and the energy density of total matter

$$\bar{E}_{nM} = \frac{4\pi r^3}{3}\bar{\rho}^{(m)} = \frac{4\pi r^3}{3}(\bar{\rho}_m + \bar{\rho}_v). \tag{50}$$

Then, combined with Eqs. (49) and (50), the Friedmann equation (36) is gotten in the Einstein frame. After another conformal transformation, we could get the correct Friedmann equation (36) in the Jordan frame. The correctness of Friedmann equation makes sure that our arguments on the unified first law are consistent.

Compared with Eqs. (48) and (50), the generalized Misner–Sharp energy is being rescaled. However, the Misner–Sharp energy is corresponding to the production of the volume and the energy density of the matter ($\rho^{(m)}V$ in the Jordan frame and $\bar{\rho}^{(m)}V$ in the Einstein frame). The conformal transformation extracts the freedom in the non-minimally coupled "cosmic triad" vector field theory, and the energy density and the Misner–Sharp mass are both rescaled.

6. Conclusion

Compared with scalar fields, the dynamics of vector fields are more complicated. In this Letter, we try to find out the relations between thermodynamics and "cosmic triad" vector field scenario.

In the minimal coupling case of "cosmic triad" scenario, considering the entropy is proportional to the area of horizon in the Einstein gravity, $dS = dA/4\pi G$ is used for the Clausius relation. Additionally, with the unified first law, we get the correct Friedmann equation as expected. However, in the non-minimally coupled "cosmic triad" system, there is no corresponding entropy. Because of the similarity between "cosmic triad" scenario and scalar–tensor theory, we transformed the non-minimally coupled vector field in the Jordan frame to the Einstein frame. In the Einstein frame, the form of entropy $d\bar{S} = d\bar{A}/4\pi G$ could be used. The Friedmann equation was gotten successfully by using the unified first law of thermodynamics. By matching the variables in the two frames, the Friedmann equation is demonstrated to be correct even in the Jordan frame. Furthermore, we calculated the generalized Misner–Sharp energy as well which is a key variable for the derivations of dynamical equations. The generalized Misner–Sharp energy is the production of the volume and the energy density of the matter and is demonstrated to be consistent with the unified first law.

In conclusion, the unified first law which connects gravity and thermodynamics is a useful way to get the Friedmann equation in the "cosmic triad" vector field scenario. The correct Friedmann equation is obtained by means of the Einstein frame and the generalized Misner–Sharp energy.

Acknowledgements

Y.Z. thanks Prof. Rong-gen Cai and Dr. Ya-peng Hu for useful discussions. We thank the valuable comments from the anonymous referee. This work was supported by the Ministry of Science and Technology of China national basic science Program (973 Project) under grant Nos. 2007CB815401 and 2010CB833004, the National Natural Science Foundation of China key project under grant No. 10935013, the National Natural Science Foundation of China under grant Nos. 11005164 and 11073005, the Distinguished Young Scholar Grant 10825313, CQ CSTC under grant Nos. 2009BA4050 and 2010BB0408, and CQ MEC under grant No. KJTD201016. Y.Z. was partially supported by China Postdoc Grant No. 20100470237.

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