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N-spheres in general relativity: Regular black holes without apparent horizons, static wormholes with event horizons and gravastars with a tube-like core

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Abstract

We consider a way to avoid black hole singularities by gluing a black hole exterior to an interior with a tube-like geometry consisting of a direct product of two-dimensional AdS, dS, or Rindler spacetime with a two-sphere of constant radius. As a result we obtain a spacetime with either "cosmological" or "acceleration" (event) horizons but without an apparent horizon. The inner region is everywhere regular and supported by matter with the vacuum-like equation of state $p_r + \rho = 0$ where $p_r = T_r^r$ is the longitudinal pressure, $\rho = -T_0^0$ is the energy density, T_μ^ν is the stress–energy tensor. When the surface of gluing approaches the horizon, surface stresses vanish, while p_r may acquire a finite jump on the boundary. Such composite spacetimes accumulate an infinitely large amount of matter inside the horizon but reveal themselves for an external observer as a sphere of a finite ADM mass and size. If the throat of the inner region is glued to two black hole exteriors, one obtains a wormhole of an arbitrarily large length. Wormholes under discussion are static but not traversable, so the null energy condition is not violated. In particular, they include the case with an infinite proper distance to the throat. We construct also gravastars with an infinite tube as a core and traversable wormholes connected by a finite tube-like region.

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The nature of inner structure of black holes and the problem of their singularity is one of central issues in black hole physics [1]. Different attempts were undertaken to remove a singularity by making composite spacetimes that reveal themselves as a black hole for an external observer but contain a regular inner region. In doing so, the special role is played by the de Sitter (dS) metric which is supposed to mimic vacuum-like media [2]. Here, different possibilities arise: one can (1) replace the part of a black hole metric by the dS one inside the horizon [3], (2a) consider some conceivable distribution of matter that interpolates smoothly between the Schwarzschild and dS metrics [4] or (2b) sew two exact solutions smoothly due to special fine-tuning of parameters, the composite spacetime having a horizon [5], (3) sew the black hole and dS metrics (or its generalization) outside the horizon in such a way that the horizon does not form at all (so-called gravastars [6]).

All the aforementioned approaches assume that the central singularity is replaced by some regular interior in which this singularity is smoothed out in the centre. In the present work we suggest a quite different way—to get rid off the singularity in the centre by simply getting rid of the centre by itself. This idea is realized by sewing an outer black hole region with spacetimes having no centre of symmetry such as Bertotti–Robinson (BR) [7] or Nariai metric [8] or the direct geometrical product of two-dimensional Rindler spacetime and a fixed two-sphere ($\mathrm{Rindler}_2 \times \mathrm{S}_2$). For all such spacetime the algebraic structure of the stress energy tensor $T_0^0 = T_r^r$ is invariant under radial boosts similarly to properties of metrics considered in [4,5]. However, spacetime structure is qualitatively different. In particular, the event horizon is not accompanied by the apparent

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horizon. Apart from this, in some particular examples the role of black hole horizons is played by the acceleration ones that usually represent a pure kinematical effect and disappear after passing to the proper chosen frame. We will see that such composite spacetimes automatically possess one more important features: although on the boundary stresses persist, they asymptotically vanish in the limit as the shell approaches the horizon.

As far as the spacetime structure of the inner region is concerned, the aforementioned options (1) and (2) correspond to T-regions in the sense that $(\nabla r)^2 < 0$ where r is the areal radius (we use the terminology of Ref. [9]). In the case (3) the interior spacetime represents R-region for which $(\nabla r)^2 > 0$. In this sense, our case occupies the intermediate position since $(\nabla r)^2 = 0$ inside just because of constancy of r. For brevity, we will call it N-region. Thus, the whole spacetime consists of gluing one R- and one N-region.

Consider the static metric

$$ds^2 = -dt^2 f + dl^2 + r^2(l)\left(d\theta^2 + d\phi^2 \sin^2\theta\right), \qquad f = b^2. \quad (1)$$

If r can be chosen as a variable (that is not always the case, see below), it can be rewritten in the equivalent form

$$ds^2 = -dt^2 f + \frac{dr^2}{V} + r^2\left(d\theta^2 + d\phi^2 \sin^2\theta\right), \qquad V = \left(\frac{dr}{dl}\right)^2. \quad (2)$$

We would like to glue to different spacetimes along the timelike surface (shell) $r = r_0$. Following the general formalism [10], one can write

$$8\pi S_\mu^\nu = \left[K_\mu^\nu\right] - \delta_\mu^\nu [K], \quad (3)$$

where $S_\mu^\nu \equiv \int_{r_0-0}^{r_0+0} dl\, \tilde{T}_\mu^\nu$, $\tilde{T}_\mu^\nu$ is the stress–energy tensor of the shell, K_μ^ν is the tensor of the extrinsic curvature calculated on the surface $r = r_0$, $K = K_i^i$ $(i = 0, 2, 3)$ and $[\cdots] = (\cdots)_+ - (\cdots)_-$, signs "+" and "−" correspond to the outer and inner regions, respectively. If $[K_\mu^\nu] = 0$, the quantity S_μ^ν vanishes and both regions match smoothly. Calculating K_μ^ν from (3) one can easily obtain

$$K_0^0 = -\frac{b'}{b}, \qquad K_2^2 = -\frac{r'}{r} = K_3^3, \qquad K = -\frac{2r'}{r} - \frac{b'}{b}, \quad (4)$$

where prime denotes differentiation with respect to the proper length l. We have ($\sigma \equiv -S_0^0$, $\Theta \equiv -S_2^2$)

$$8\pi\Theta = \left[K_0^0\right] + \left[K_2^2\right] = -\frac{(br)'_+ - (br)'_-}{br}, \quad (5)$$

$$8\pi\sigma = 2\left[K_2^2\right] = -\frac{2(r'_+ - r'_-)}{r}. \quad (6)$$

Let the stress–energy tensor be represented in the form $T_\mu^\nu = \mathrm{diag}(-\rho, p_r, p_\perp, p_\perp)$. If b' has different signs from both sides of the boundary (like it happens for gravastars [6] or their simplified version [11]), the tensor S_μ^ν does not vanish and, moreover, as the boundary approaches the horizon, the stresses grow unbound. If b'_+ and b'_- have the same sign, one can combine known exact solutions to obtain smooth gluing [5]. We consider now a quite different situation. We choose the metric of interior "−" to obey the Einstein equations with $r = r_0 = \mathrm{const}$. Then it follows from 00 and 11 equations that $\rho^- = -p_r^- = \frac{1}{8\pi r_0^2}$ and 22 equation gives us $\frac{b''}{b} = 8\pi p_\perp^-$, where $(\cdots)_\pm \equiv \lim(\cdots)_{r\to r_0 \pm 0}$. Thus, the interior should be vacuum-like in the sense that $\rho + p_r = 0$, and there are three different cases depending on the sign of $p_\perp$. If (1) $p_\perp > 0$, then (a) $b = a \sinh \kappa l$, where a is a constant, $\kappa^2 = 8\pi p_\perp$, (b) $b = a\exp(\kappa l)$ or (c) $b = a\cosh \kappa l$. If (2) $p_\perp < 0$, by a suitable linear transformation of l we can achieve $b = a \sin\kappa l$ with $\kappa^2 = -8\pi p_\perp$, if (3) $p_\perp = 0$, we have (a) $b = al$ or (b) $b = a$. Particular examples of corresponding physical sources are electromagnetic field (case 1 with $p_\perp = \rho$—BR solution), cosmological constant (case 2 with $p_\perp = -\rho$—Nariai solution), string dust [12] (case 3).

In all these cases formulas (5) and (6) can be rewritten as

$$4\pi\sigma = -\frac{\sqrt{V_+}}{r_0}, \quad (7)$$

$$8\pi\Theta = -\frac{\sqrt{V_+}}{r_0} - \left[\frac{\partial \ln b}{\partial l}\right]. \quad (8)$$

For any gluing outside the horizon one cannot glue smoothly the N-region with the R-one (in agreement with the remark about Nariai solution in Section IVa of [5]) but, nonetheless, we will see now that in the horizon limit both σ and Θ asymptotically vanish. Let us discuss separately the cases when the exterior represents (i) a non-extremal black hole, (ii) an extremal one. Let $r_0 \to r_h$, where r_h corresponds to the horizon. Consider first the case (i). Then $b'_h \neq 0$ by definition and we have in the "+" region for small l the asymptotic expansion $b = b'_h l[1 + O(l^2)]$. The quantity $\sqrt{V(r)}$ behaves like $\sqrt{r - r_h} \sim l$. We glue the "+" region with versions 1a, 2 or 3a of the "−" region. Then $\frac{\partial \ln b}{\partial l}$ has the same asymptotic form $\frac{\partial \ln b}{\partial l} = \frac{1}{l} + O(l)$ on both sides of the shell, in the "+" region $\frac{\partial r}{\partial l} \to 0$ and in the "−" region $\frac{\partial r}{\partial l} = 0$ exactly. As a result, we obtain that $\sigma, \Theta \sim l \to 0$. It is worth stressing that one can glue any two spacetimes of the kind under discussion.

Consider case 3a as an example. Inside the shell, one can introduce the new variables according to $X = l\cosh(a\tau)$, $T = l\sinh a\tau$, perform the transformation and obtain in the interior the new metric of the same form but with the new $b = \mathrm{const}$, in other words the Minkowski two-dimensional spacetime in agreement with well-known relation between the Rindler and Minkowski spacetimes, the total metric being $ds^2 = -dT^2 + dX^2 + r_0^2(d\theta^2 + d\phi^2\sin^2\theta)$. Thus, the two-dimensional part mimics the empty space but because of the angular part the four-dimensional spacetime is curved. As is well known, in the two-dimensional flat spacetime the family of Rindler observers following the trajectories $l = \mathrm{const}$ covers not the whole spacetime but only one quadrant bounded by past and future acceleration horizons. If an observer passes to X, T frame, the acceleration horizon in accordance with its pure kinematic nature disappears and this new frame covers all X–T manifold, so that all signals can escape to corresponding infinity. (In cases 1a and 1b we are faced with the AdS two-dimensional geometry that also possesses acceleration horizons, in case 2 the geometry of two-dimensional part is of the dS type and the horizon is "cosmological".)

However, now the four-dimensional nature of spacetimes comes into play. Usually, the Carter–Penrose diagrams representing the structure of spacetime are pure two-dimensional, with the reservation that each point represents a two sphere of the areal radius r. In doing so, the coordinate r plays the double role: it enters spacetime diagrams and it measures the surface area. In particular, for the case of asymptotically flat spacetimes (that embraces Schwarzschild and Reissner–Nordström black holes) $r \to \infty$ at spatial and null infinity. Meanwhile, for the case under discussion coordinates X, T (or similar coordinates for the BR metric) have nothing in common with coordinates r, t of asymptotically flat spacetime since $r = r_0 = \text{const}$ inside the N-region. Therefore, although inside the shell only an acceleration horizon is present, signals from the interior cannot reach an observer at infinity (and even an observer with a finite l between the horizon and the shell). As a result, we have a black hole in the sense that there is a spacetime region from which light cannot escape to infinity. As the quantity r is constant inside, there are no trapped surfaces at all. Thus, we obtain a black hole with an event horizon but without apparent horizons.

Up to now, we considered the non-extremal horizons. In the extremal case (ii) we must select the only suitable candidate for smooth gluing, case 1b with $b_- = a\exp(\kappa l)$, so that $\frac{\partial \ln b_-}{\partial l} = \kappa$. Let in the outer region the metric have the asymptotics typical of extremal black holes: $b_+ = B(r - r_h) + O(r - r_h)^2$, $V = A^{-2}(r - r_h)^2 + O(r - r_h)^3$, where A and B are constants. Then $b_+ \sim (r - r_h) \sim \exp(\frac{l}{A})[1 + O(\exp(\frac{l}{A}))]$, $l \to -\infty$ and $\frac{\partial \ln b_+}{\partial l} = \frac{1}{A} + O[\exp(\frac{l}{A})]$. In the same manner, one can easily calculate stresses and obtain that they are proportional to $\exp(\frac{l}{A})$ and vanish in the limit under consideration, provided $\kappa = \frac{1}{A}$, whence $p_\perp^- = \frac{1}{8\pi A^2}$. Consider, for simplicity, the BR spacetime. Then $A = r_h$, $b = \sinh\frac{l}{r_h}$. By boosts in the radial direction satisfying

$$\sinh y = \frac{1}{2\xi}\left(t^2 - \xi^2 + 1\right),$$

$$\cos T \cosh y = \frac{1}{2\xi}\left(t^2 - \xi^2 - 1\right), \quad \xi \equiv e^{-l}, \tag{9}$$

where for a moment we put for simplicity $r_h = 1$, we may achieve b to have the form 1c according to known properties of BR spacetime, so that $ds^2 = -dT^2\cosh^2 y + dy^2 + d\theta^2 + \sin^2\theta\, d\phi^2$. It follows from (2) and 00 Einstein equation that the Hawking temperature $T_H = \frac{1}{4\pi r_+}(1 - \frac{\rho^+}{\rho^-})\exp(\psi_+)$, where $\psi_+ = 4\pi\int_\infty^{r_+} dr \frac{r(T_r^r - T_0^0)}{V}$. If the horizon is extremal, $T_H = 0$ and $\rho^+ = \rho^- = \frac{1}{8\pi r_h^2}$. With the regularity conditions on the horizon $T_r^r - T_0^0 = 0$, one obtains that $p_r^+ = -\rho^+ = -\rho^- = p_r^-$. Thus, the radial pressure is continuous. (In the particular case when both inside and outside $p_\perp = \rho = -p_r = \frac{e^2}{8\pi r^4}$ where e is an electric charge, the results of [13] for sewing the BR spacetime with the extremal Reissner–Nordström metric are reproduced.) However, it does not necessarily hold for non-extremal horizons in which case radial pressure can acquire a jump. For example, this happens in the case of the outer Schwarzschild metric: in the "+" region $p_r^+ = 0$ but in the "−" region $p_r^- = -\rho^- \neq 0$. Thus, the tangential stresses asymptotically vanish but the jump in p_r does not. Such a seemingly paradoxical combination is easily explained if one invokes the conservation law $T_{\mu;\eta}^{\nu} = 0$ with $\mu = l$, whence $(\sqrt{-g}T_1^1)' = r^2 b' T_0^0 + 2r'brT_2^2$. Then it is clear that it is combination $\sqrt{-g}T_1^1$ which enters the expression for the jump due to jumps in b' and r'. Usually, $\sqrt{-g} \neq 0$ and, if other components are continuous across the shell, continuity of $\sqrt{-g}T_1^1$ is equivalent to the continuity of T_1^1. However, as the shell approaches the horizon, $\sqrt{-g} \sim b \sim l \to 0$. Therefore, the jump in p_r is compatible with the continuity of $\sqrt{-g}T_1^1$.

The composite spacetimes under discussion have one more interesting property connected with the gravitational mass defect. The gravitational mass measured in the outer region is equal to $m(r) = m(r_0) + 4\pi\int_{r_0}^{r} dr\,\rho r^2$, the ADM mass $m(\infty)$ being finite since outside the shell matter is supposed to be bounded within some compact region or the density ρ decreases rapidly enough. In the limit $r_0 \to r_h$ the mass $m(r_0)$ tends to $m(r_h)$ and is finite. However, the total proper mass $m_p = 4\pi\int dl\,\rho^2$ measured on the hypersurface $T = \text{const}$ in the tube under the shell at r_0, obviously, diverges. It is not surprising that m_p is infinite for an extremal horizon in the outer region since l diverges ($dl \sim \frac{dr}{r - r_+}$). Meanwhile, now m_p diverges also for the *non-extremal* horizons due to an infinite tube inside the shell (for instance, as a result of integration over X in the two-dimensional Minkowski case). To some extent, it resembles the so-called T-spheres that can reveal themselves as a body of a finite mass and size for an external observer whereas they bind an infinite amount of matter inside the horizon [14] (see also [15]). By analogy, we call such objects N-spheres. As there is no singular centre here, N-spheres can be considered as realization of Wheeler's idea of "mass without mass", alternative to T-spheres [14]. However, we would like to stress that, while in the case of T-region matter collapses or starts from the singular state, in our case the interior is perfectly regular. As the media with the equation of state $p + \rho = 0$ can be thought of as gravitational vacuum condensate [2,4,6], the fact that an object with infinite "bare" energy reveals itself in physical observations as a body with a finite energy, can be viewed as a classical analogy of known properties of vacuum in quantum field theory.

Up to now we discussed gluing between two regions only. One can proceed further and glue in the same manner another Schwarzschild (or extremal black hole) region from the left, but again with the shell in the R-region. Actually, we have some generalization of notion of wormholes [16,17]—with a throat of an arbitrary length lying in the N-region and connecting two R-regions. Inside the throat the equation of state is exactly vacuum-like $p_r + \rho = 0$, the proper mass bounded inside the throat can be made as large as one wishes (the configuration considered in [18] tends to such a throat in some particular limit). The possibility of extended throats ("hyperspatial tubes") for generic static wormholes was briefly mentioned in [19,20]. We would like to stress that in our case such objects are combined with the event horizons. It looks natural to call wormholes with tube-like geometries inside "N-wormholes".

As a matter of fact, we have an object that interpolates between "ordinary" black holes and wormholes. In particular, this reveals itself in the following: the typical feature of black holes is the trapped surfaces while the typical feature of wormholes is the "antitrapped" surfaces (see remarks due to D. Page on p. 405 of Ref. [16] and [21]). But now we have neither first nor second case since $r=\text{const}$ in both directions inside the horizon. N-wormholes under discussion are simultaneously static, not traversable but safe for one-way travel (no tidal forces or spacetime singularities occurs inside the tube). In particular, this includes the extremal case, when the proper distance to throat is infinite. In the absence of horizons, the spacetime would be geodesically complete, time of travel would be infinite and there would be no wormhole at all. Now, thanks to the horizon, the time is finite, so that an observer is able to go through the tube but is unable to return.

The constructions under discussion contain horizons as a result of the limiting procedure. In doing so, we used cases 1a, 1b, 2, 3a for gluing. Meanwhile, there exists alternative to it. Let us take, as an exterior, a region from some traversable wormhole instead of a black hole and glue it to the N-region. Then the horizon is present neither in the original spacetime nor in the composite spacetime. To accomplish this, we should use cases 1c or 3b complementary to our previous choice since for them $b \neq 0$ on the throat and there are no horizons, as requested. Thus, in sum we exhaust all possible cases 1–3. The composite spacetime in the case under current discussion realizes literally the gravastar construction since there is no horizon. It is natural to call it "N-gravastar" since it contains a core with a tube inside. In contrast to original constructions [6], where surface stresses grow unbound as one approaches a would-be horizon, now these stresses are not only finite but vanish at all. To see this, it is worth noting that in the outer region $b' = \frac{\partial b}{\partial r} r' = 0$ on the throat due to the factor r. It follows from the explicit form of b inside that in cases 1c and 3b $b' = 0$ also in the N-region. In a similar way, $r' = 0$ both outside on the throat and everywhere inside. As a result, stresses (5), (6) vanish. One can take the position of the would-be horizon as close to the throat as one likes (for instance, one may take the metric similar to that in Eq. (7) of [20] with $b^2 \sim (r - r_0)^2 + \varepsilon^2$, $\varepsilon \to 0$) but this does not affect this circumstance. Proceeding further in the same manner as before, one can accomplish gluing from both sides of the N-region to obtain N-wormhole without horizons. By construction, this kind of N-wormholes is traversable. Actually, it is obtained with the help of cut and paste technique like in Chapter 15 of Ref. [17]. The difference consists in that now, instead of a thin shell, we work with tubes of a finite (but arbitrarily large) length.

As is well known, the existence of static traversable wormholes entails, as the necessary condition, the violation of NEC (null energy conditions) [16,17]. Meanwhile, in our case, this condition is satisfied (although, on the verge) inside the N-region, $p_r^- + \rho^- = 0$. If we consider traversable N-wormholes obtained by the surgery based on cases 1c or 3b, NEC is inevitably violated on the throat, $p_r^+ + \rho^+ < 0$ [16,17]. In the absence of horizons, smooth gluing entails $p_r^- = p_r^+$, so that we obtain, as by-product, that $\rho^+ < \rho^-$. However, if we glue according to prescriptions 1a, 1b, 2, 3a (when the horizon is present), NEC is marginally satisfied not only in the "$-$" region but also from the "$+$" side of the throat. The difference can be understood as follows. One can easily obtain from 00 and 11 Einstein equations that $G_1^1 - G_0^0 = \frac{2r'b'}{rb} - \frac{2r''}{r}$. If there is no horizon, $b \neq 0$ and the first term vanishes on the throat due to the factor r' (moreover, $b' = \frac{\partial b}{\partial r} r'$, so that b' also vanishes). As the throat is supposed to be a minimum of r, the second derivative $r'' > 0$, so that $G_1^1 - G_0^0 = 8\pi(p_r + \rho) < 0$, and NEC is violated. However, if $b \sim l$, $r - r_h \sim l^2$ with $l \to 0$ (non-extremal case) or $r - r_h \sim \exp(\frac{l}{r_h})$, $b \sim \exp(\frac{l}{r_h})$ with $l \to -\infty$ (extremal case), it follows from the above expression that $p_r^+ + \rho^+ \to 0$. Thus, NEC in the "$+$" region is satisfied just due to the properties of the horizon. As now a wormhole is not traversable, there is nothing wrong in that NEC is not violated.

To summarize, we constructed composite objects that interpolate between black holes and gravastars in that there is no horizon in the particular solution obtained by gluing different regions of spacetime but the horizon appears as a result of the limiting procedure when the object turns into what we called a N-sphere. In doing so, we obtained event horizons without apparent ones. Alternatively, we also obtained a gravastar with an infinite tube as a core (N-gravastar). Generalization of the procedure under consideration gave rise to objects interpolating between black holes and wormholes (not traversable N-wormholes) or connecting two external regions without horizons (traversable N-wormholes).

The type of geometry inside the N-region can be written as $N_2 \times S_2$ where N_2 is two-dimensional subspace—Rindler (if $p_\perp = 0$), AdS ($p_\perp > 0$) or dS ($p_\perp < 0$) one. Correspondingly, there are three possible types of N-spheres. As the geometry of the kind $N_2 \times S_2$ does not change its type when influenced by quantum backreaction (see [22] and references [6,8–15] therein), the whole construction survives at the semi-classical level.

The constructions considered in the present Letter can be also relevant for higher-dimensional generalization of BR-like solutions [23]. In particular, it concerns the issue of compactification of extra-dimensions in Kaluza–Klein theories where flux tubes of constant cross-sections can arise as compactified BR-like phase inside an uncompactified one [24,25].

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