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On the continuum limit of gauge-fixed compact U(1) lattice gauge theory

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Abstract

We investigate the continuum limit of a compact formulation of the lattice U(1) gauge theory in 4 dimensions using a nonperturbative gauge-fixed regularization. We find clear evidence of a continuous phase transition in the pure gauge theory for all values of the gauge coupling (with gauge symmetry restored). When probed with quenched staggered fermions with U(1) charge, the theory clearly has a chiral transition for large gauge couplings. We identify the only possible region in the parameter space where a continuum limit with nonperturbative physics may appear.

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1. Introduction

Although quantum field theories were first formulated on the lattice regulator to investigate the nonperturbative properties of quantum chromodynamics, the lattice regulator can be useful in general to study nonperturbative behavior of any field theory, in particular the theories involving nonasymptotically-free couplings. In this Letter, we have looked at quantum electrodynamics (QED) which on the lattice can be formulated in terms of compact group-valued gauge fields in the usual Wilson approach, or in terms of noncompact gauge fields as in the continuum. The noncompact formulation does not allow any nonperturbative behavior in the pure gauge sector (it does show nonperturbative behavior only in the presence of fermions). However, the compact formulation allows for self-interaction of gauge fields on the lattice regulator and hence it is interesting to study its phase structure and possible continuum limits. Obviously, for it to be a viable regularization, the lattice theory must have a weak coupling continuum limit that would produce free photons in the pure gauge sector. Once it has the expected weak coupling continuum limit, one is interested in finding a continuum limit with possible nonperturbative properties.

It is well known that compact formulation of U(1) gauge theory (with or without fermions) on the lattice has at least two different phases: a weak coupling phase (with usual QED-like properties on the regulator) called the Coulomb phase and a strong coupling confining phase which resembles QCD (again on the lattice regulator) in many ways—existence of gauge balls, confinement, nonzero chiral condensate, appear-

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ance of Goldstone bosons, etc. To remove the regulator, there ought to be a continuous phase transition in the intermediate region. Numerical studies have found the existence of such a phase transition [1,2], but the order of the phase transition is generally accepted as first order.

A continuum limit from the confinement phase sustaining the properties of the confinement phase would imply a hitherto unknown continuum abelian gauge theory which is likely to correspond to a nontrivial fixed point.

As far as continuum limits from the Coulomb phase is concerned, one would have to imagine that the critical manifold (obtained by appropriately expanding the parameter space) is not too far away from the weak coupling region since perturbative results on lattice match excellently with our familiar continuum QED. However, the lack of a continuum limit for compact lattice QED in the weak coupling region means that the issue of triviality (existence of Landau poles, etc.) cannot really be answered in a genuine (nonperturbative) way.

With the addition of a nonminimal plaquette term in the gauge action [3], there is new evidence that the Coulomb-confinement transition in the pure gauge theory is first order [4]. Only with inclusion of fermions (with a four fermion interaction [5,6]), there is indication of a continuous phase transition.

In this Letter we make an exploratory study of possible continuum limits of a compact lattice formulation of pure U(1) gauge theory using a different regularization in 4 space-time dimensions. We also probe the pure gauge theory by quenched staggered fermions which have U(1)-charge. This regularization of lattice U(1) theory was originally devised to 'tame' the 'rough gauge' problem of lattice chiral gauge theories [7]. Because of the gauge-invariant measure and the lack of gauge-invariance of the lattice chiral gauge theories, the longitudinal gauge degrees of freedom (lgdof) couple nonperturbatively to the physical degrees of freedom. To decouple the lgdof which are radially frozen scalar fields, a nonperturbative gauge fixing scheme (corresponding to a local renormalizable covariant gauge fixing in the naive continuum limit) for the compact U(1) gauge fields was proposed [8]. A key feature of this gauge fixing scheme is that the gauge fixing term is not the exact square of the expression used in the gauge fixing condition and as a

result not BRST-invariant (as required by Neuberger's theorem [9] for compact gauge fixing). It has, in addition, appropriate irrelevant terms to make the perturbative vacuum $U_{\mu x} = 1$ unique. Because the gauge fixing term obviously breaks gauge invariance, one needs to add counterterms to restore manifest gauge symmetry.

The parameter space of this regularization of compact U(1) theory now includes the gauge coupling, the coefficient of the gauge fixing term and the coefficients of the counterterms. In this extended parameter space it has been shown [10] that for a weak gauge coupling (g = 0.6) there exists a continuous phase transition at which the U(1) gauge symmetry is restored and the continuum theory of free photons emerge. In our study, determination of the phase diagram and the critical region as extensively as possible is very important because we are dealing with a gauge-noninvariant theory in general and it is necessary to have freedom along the critical manifold so that irrelevant parameters can be appropriately tuned to restore gauge invariance (protecting the theory from a violation of unitaritv).

Scanning a wide range of all the three parameters, in the pure gauge theory we have found phase transitions between a phase with broken rotational symmetry (FMD phase) and one with rotational symmetry (FM phase). The FM–FMD transition is the place where the gauge symmetry gets restored. To recover Lorentz invariance in the continuum, the FM–FMD transition needs to be approached from the FM side.

Probing the pure gauge theory with quenched U(1)charged fermions, we have also found evidence for a chiral transition for large gauge couplings although in this exploratory study we could assess its approximate location in a limited region of the parameter space. We have looked for chiral condensates only near the FM–FMD transition, because this is where the gauge symmetry is restored.

From our numerical evidence we expect that the chiral phase transition intersects the FM–FMD phase transition at the tricritical line where the order of the FM–FMD transition changes. The region of the FM–FMD transition where the chiral condensate is nonzero seems to be first order. The intersection region of the chiral and the FM–FMD transitions thus seems to be the only candidate where a continuum limit with nonperturbative properties may be achieved.

A preliminary account of our work can be found in [11].

2. The regularization

The action for the compact gauge-fixed U(1) theory [8], where the ghosts are free and decoupled, is

$$S[U] = S_g[U] + S_{gf}[U] + S_{ct}[U].$$
 (1)

 S_g is the usual Wilson plaquette action,

$$S_g = \frac{1}{g^2} \sum_{x\mu < \nu} (1 - \text{Re} \, U_{\mu\nu x})$$
(2)

where g is the gauge coupling and $U_{\mu x}$ is the groupvalued U(1) gauge field.

 S_{gf} is the BRST-noninvariant compact gauge fixing term

$$S_{gf} = \tilde{\kappa} \left(\sum_{xyz} \Box(U)_{xy} \Box(U)_{yz} - \sum_{x} B_x^2 \right), \tag{3}$$

where $\tilde{\kappa}$ is the coefficient of the gauge fixing term, $\Box(U)$ is the covariant lattice Laplacian, and

$$B_x = \sum_{\mu} \left(\frac{\mathcal{A}_{\mu,x-\mu} + \mathcal{A}_{\mu x}}{2}\right)^2,\tag{4}$$

where $\mathcal{A}_{\mu x} = \text{Im } U_{\mu x}$. As mentioned in the introduction, S_{gf} is not just a naive transcription of the continuum covariant gauge fixing term, it has in addition appropriate irrelevant terms. This makes the action have an unique absolute minimum at $U_{\mu x} = 1$, validating weak coupling perturbation theory around g = 0 or $\tilde{\kappa} = \infty$ and in the naive continuum limit S_{gf} reduces to $(1/2\xi) \int d^4 x (\partial_{\mu} A_{\mu})^2$ with $\xi = 1/(2\tilde{\kappa}g^2)$.

Validity of weak coupling perturbation theory together with perturbative renormalizability helps to determine the form of the counter terms to be present in S_{ct} . It turns out that the most important gauge counterterm is the dimension-two counterterm, namely, the gauge field mass counterterm given by

$$S_{ct} = -\kappa \sum_{\mu x} \left(U_{\mu x} + U_{\mu x}^{\dagger} \right).$$
⁽⁵⁾

In the pure bosonic theory there are possible marginal counterterms including derivatives. However, in the investigation of the gauge-fixed theory as given, the dimension-two counterterm has been mostly considered, because it alone could lead to a continuous phase transition that recovers the gauge symmetry. It was argued that the marginal counterterms would not possibly create new universality classes for the continuum theory corresponding to large $\tilde{\kappa}$ (for a discussion on other counterterms, please see [8,10]).

Our philosophy here has been an usual one, i.e., to take a lattice theory given by (1) having the expected weak coupling results, and then try and find out the strong coupling properties of the same theory.

Information about the phase diagram of the model can be obtained in the constant field approximation, first by expanding the link field $U_{\mu x} = \exp(igA_{\mu x})$ around $U_{\mu x} = 1$ and then requiring the gauge potential A_{μ} to be constant (thus all the terms containing derivatives of A_{μ} vanish). Since the WCPT is defined around $U_{\mu x} = 1$, the classical potential V_{cl} is the leading order approximation of the effective potential and is given by

$$V_{cl} = \kappa \left[g^2 \sum_{\mu} A_{\mu}^2 + \cdots \right] + \frac{g^4}{2\xi} \left[\left(\sum_{\mu} A_{\mu}^2 \right) \left(\sum_{\mu} A_{\mu}^4 \right) + \cdots \right], \tag{6}$$

where the dots represent terms of higher order in g^2 . Since the perturbation theory is defined around g = 0 or $\tilde{\kappa} = \infty$, the classical potential is expected to be reliable at least for the region of large $\tilde{\kappa}$.

From (6) it follows that for $\kappa > 0$, the gauge boson mass is nonzero and V_{cl} has a minimum at $A_{\mu} = 0$. The region $\kappa > 0$ therefore is a phase with broken gauge symmetry—the FM phase.

For $\kappa < 0$, the minimum of V_{cl} shifts to a nonzero value:

$$A_{\mu} = \pm \left(\frac{\xi |\kappa|}{3g^2}\right)^{1/4}$$
 for all μ

implying an unusual phase with broken rotational symmetry in addition to the broken gauge symmetry we call it the directional ferromagnetic phase (FMD) [8].

For $\kappa = 0 \equiv \kappa_c$ the minimum of V_{cl} is at $A_{\mu} = 0$ and at the same time the gauge boson mass vanishes, thus gauge symmetry is restored which signals a continuous phase transition or criticality. For large enough $\tilde{\kappa}$ the presence of the $\sim A_{\mu}^{6}$ term in the gauge fixing action (3) produces a continuous phase transition FM \ FMD at which the gauge boson mass scales to zero and hence the gauge symmetry is restored. The continuum limit now can be taken by approaching FM–FMD transition from within FM phase.

3. Numerical simulation and results

To obtain the phase diagram of the gauge-fixed pure U(1) theory, given by the action (1), in $(\kappa, \tilde{\kappa})$ -plane for fixed values of the gauge coupling *g*, we defined the following observables (for a L^4 -lattice):

$$E_P = \frac{1}{6L^4} \left(\sum_{x,\mu < \nu} \operatorname{Re} U_{\mu\nu x} \right), \tag{7}$$

$$E_{\kappa} = \frac{1}{4L^4} \left\langle \sum_{x,\mu} \operatorname{Re} U_{\mu x} \right\rangle,\tag{8}$$

$$V = \left\langle \sqrt{\frac{1}{4} \sum_{\mu} \left(\frac{1}{L^4} \sum_{x} \operatorname{Im} U_{\mu x} \right)^2} \right\rangle.$$
(9)

 E_P and E_κ are not order parameters but they signal phase transitions by sharp changes. We expect $E_\kappa \neq 0$ in the broken symmetric phases FM and FMD and $E_\kappa \sim 0$ in the symmetric (PM) phase. Besides, E_κ is expected to be continuous at a continuous phase transition (infinite slope in the infinite volume limit) and show a discrete jump at a first order transition [10, 12]. The true order parameter is V which allows us to distinguish the FMD phase (where $V \neq 0$) from the other phases where $V \sim 0$.

The Monte Carlo simulations were done with a 4-hit Metropolis algorithm on a variety of lattice sizes from 4⁴ to 16⁴, although investigations were mostly done on 10⁴ lattices. The phase diagram was explored in $(\kappa, \tilde{\kappa})$ -plane at gauge couplings g = 0.6, 0.8, 1.0, 1.1, 1.2, 1.3 and 1.4 over a range of 0.30 to -2.30 for κ and 0.00 to 1.00 for $\tilde{\kappa}$. The autocorrelation length for all observables was less than 10 for 10⁴ lattices and each expectation value was calculated from about a thousand independent configurations.

Fig. 1 collectively shows the phase diagram in $(\kappa, \tilde{\kappa})$ -plane for the different gauge couplings. The



Fig. 1. Phase diagram of pure U(1) gauge fixed theory on 10^4 lattice. FM–FMD transition for different gauge couplings: (a) g = 0.8 (\bigcirc), (b) g = 1.0 (\Box), (c) g = 1.1 (\triangle), (d) g = 1.2 (\diamond), (e) g = 1.3 (\bigtriangledown). The line joining the *-symbols separates the PM phase from the FM and the FMD phases. The PM transition line does not depend too much on the gauge coupling and an average line is drawn here for clarity.

diagram looks qualitatively the same for all gauge couplings. For zero or small values of $\tilde{\kappa}$, there is a FM– PM transition. The FM–FMD transition which ensures recovery of the gauge symmetry is obtained at larger finite values of the coupling $\tilde{\kappa}$. As seen from the figure, the FM–FMD transition can be approached by tuning just one parameter, namely, κ for given values of gand $\tilde{\kappa}$.

The phase diagram and behavior of the theory at g = 0.6 was investigated before [10]. We have reconfirmed the results of [10] and for reasons of clarity Fig. 1 does not include the data for g = 0.6. At weaker gauge couplings g = 0.6, 0.8, 1.0 the FM-FMD transition (the dotted lines roughly parallel to the $\tilde{\kappa}$ -axis) appears to be continuous. On the other hand, for stronger gauge couplings g = 1.1, 1.2, 1.3and also g = 1.4 (although not shown in Fig. 1) this transition is first order for smaller values of $\tilde{\kappa}$ and is continuous for larger values of $\tilde{\kappa}$. The critical value of $\tilde{\kappa}$ at which the order changes shifts to larger values with increasing gauge coupling. In this exploratory study we have not determined the precise value of the above-mentioned critical $\tilde{\kappa}$ for the whole range of gauge couplings investigated.

The order of the FM–FMD phase transition is inferred from the change of E_{κ} with κ . A E_{κ} versus κ



Fig. 2. Discontinuity in E_{κ} (for large gauge coupling) across FM–FMD transition (the dotted line) for (a) 10⁴ and (b) 16⁴ lattices.



Fig. 3. Continuous change in E_{κ} (for large gauge coupling) across FM–FMD transition (the dotted line) at larger $\tilde{\kappa}$, for (a) 10⁴ and (b) 16⁴ lattices.

plot is shown here only for a stronger gauge coupling, discussed next.

Fig. 2 which depicts the nature of change of E_{κ} versus κ across the FM–FMD transition at g = 1.3, $\tilde{\kappa} = 0.4$ for (a) 10^4 and (b) 16^4 lattices, shows a discrete jump, implying a first order transition. Although the figure is presented only for g = 1.3, our observation is that at $\tilde{\kappa} \sim 0.4$ and $g \gtrsim 1.1$ the FM–FMD transition seems to be first order, whereas for g < 1.1 and any $\tilde{\kappa}$ (large enough to accommodate a FM–FMD transition) this transition is continuous.

At large gauge couplings $(g \gtrsim 1.1)$, when we increase the value of $\tilde{\kappa}$, however, the discrete jump of E_{κ} , as κ changes across the FM–FMD transition, disappears. This is clearly shown in Fig. 3 for g = 1.3

and $\tilde{\kappa} = 0.8$ for (a) 10^4 and (b) 16^4 lattices. This actually would indicate a revival of a continuous FM–FMD transition at these couplings. From our experience it is reasonable to expect that for a larger gauge coupling, the transition would still remain continuous if $\tilde{\kappa}$ is made large enough.

Clearly there is a huge qualitative difference between Figs. 2 and 3 in the nature of change of E_{κ} versus κ across the FM–FMD transition. Please note that the scales of the Figs. 2(a) and 3(a), which show data for only the 10⁴ lattices, are exactly the same. In addition, although the Figs. 2 and 3 are shown only for 10⁴ and 16⁴ lattices, we have actually looked at all the observables on lattices from 4⁴ all the way upto 16⁴, and we have seen that the qualitative difference discussed



Fig. 4. Quenched chiral condensate on 10^4 lattice as a function of m_0 for different g at $\tilde{\kappa} = 0.4$.

above gets only more pronounced at larger lattices. At $\tilde{\kappa} = 0.4$, as shown in Fig. 2(a) and (b), the discrete jump of E_{κ} gets distinctly sharper on the 16⁴ lattice. On the other hand, at $\tilde{\kappa} = 0.8$ as shown in Fig. 3(a), E_{κ} is quite continuous across the FM–FMD transition on 10^4 lattices. Even a fine resolution of data points separated by $\Delta \kappa = 0.001$ does not show any discontinuity. As we go to 16^4 lattices as shown in Fig. 3(b), we critically investigate only the region around the transition (which obviously shifts a little with the change of lattice size) and absolutely no discontinuity is found with our resolution. In addition, a hint of a S-shape around the transition is visible which promises to evolve into an inflexion with infinite slope at the transition in the thermodynamic limit.

We have probed the gauge-fixed pure gauge system by quenched staggered fermions with U(1) charge by measuring the chiral condensate

$$\langle \bar{\chi} \chi \rangle_{m_0} = \frac{1}{L^4} \sum_x \langle M_{xx}^{-1} \rangle \tag{10}$$

as a function of vanishing fermionic bare mass m_0 . M is the fermion matrix. The chiral condensates are computed with the Gaussian noise estimator method [14]. Antiperiodic boundary condition in one direction is employed.

Fig. 4 shows quenched chiral condensates near the FM–FMD transition (remaining in the FM phase) obtained on 10^4 lattice for different gauge couplings as a function of staggered fermion bare mass m_0 at



Fig. 5. Quenched chiral condensate on 10^4 lattice as a function of m_0 for different $\tilde{\kappa}$.

 $\tilde{\kappa} = 0.4$. Fig. 4 clearly indicates that for weaker gauge couplings (g < 1.1) the chiral condensates vanish in the chiral limit. For stronger gauge couplings (g > 1.1) the chiral condensates are clearly not zero in the chiral limit. The dotted lines in Fig. 4 (and in Fig. 5 to follow) are only to guide the eye. It is to be noted that at $\tilde{\kappa} \sim 0.4$ as g changes from below 1.1 to above, the FM–FMD transition changes from continuous to first order (see Fig. 2).

Fig. 5 also shows chiral condensates near the FM– FMD transition again as function of the bare fermion mass but this time for different values of $\tilde{\kappa}$. Here we observe, interestingly, that for $\tilde{\kappa} \gtrsim 0.8$ and large gauge coupling (g = 1.3) the chiral condensates tend to vanish in the chiral limit. It is to be noted again that at g = 1.3 as $\tilde{\kappa}$ changes from roughly below 0.8 to above, the FM–FMD transition changes from first order to continuous.

The above discussion strongly suggests that inclusion of fermions here leads to a chiral phase transition that intersects the FM–FMD phase transition. For a fixed $\tilde{\kappa}$ there is chiral transition as g is changed, and for a fixed g the chiral transition shows up as $\tilde{\kappa}$ is changed (the third parameter κ is used to stay on the FM–FMD transition). A similar phenomenon occurs for the *order* of the FM–FMD transition with respect to changes in g and $\tilde{\kappa}$.

There is no chiral condensate in regions where we can take a continuum limit in the pure gauge theory (which is the expected perturbative result). On the other hand, chiral condensates appear where there is no continuum limit for the pure gauge theory.

The tricritical point (or line in the 3-dimensional parameter space) where the FM–FMD transition changes its order seems to be the likely place where the chiral transition intersects the FM–FMD transition. The tricritical line is therefore the only candidate where there is a possibility of a continuum limit with nonperturbative properties like the chiral condensate.

4. Conclusion

With the particular regularization of compact pure U(1) gauge theory with an extended parameter space, we have shown that there is clearly a continuum limit for the whole range of the bare gauge coupling g. Evidence of a continuum limit in other regularizations of a compact lattice U(1) gauge theory is either absent, inconclusive [1,2,4] or dependent on inclusion of fermionic interactions [5,6,13].

Given the long history of speculation about a confining strong coupling U(1) gauge theory and related issues of non-triviality, we have probed the pure gauge system by quenched staggered fermions and found a clear evidence for a chiral phase transition. However, the region with a nonzero chiral condensate does not allow a continuum limit. The continuum limit in the pure gauge theory is only attained with no chiral condensate. This is consistent with perturbative expectations.

We have found reasonable evidence to expect that the tricritical line at which the order of the FM–FMD phase transition changes in the pure gauge theory, coincides with the line where the chiral phase transition intersects the FM–FMD transition. This line is the only candidate for a possible continuum limit with nonperturbative properties like chiral condensates.

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