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On the weak N-dependence of $SO(N)$ and $SU(N)$ gauge theories in $2+1$ dimensions

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ABSTRACT

We consider (continuum) mass ratios of the lightest 'glueballs' as a function of N for $SO(N)$ and $SU(N)$ lattice gauge theories in $D=2+1$. We observe that the leading large N correction is usually sufficient to describe the N-dependence of $SO(N\geq 3)$ and $SU(N\geq 2)$, within the errors of the numerical calculation. Just as interesting is the fact that the coefficient of this correction almost invariably turns out to be anomalously small, for both $SO(N)$ and $SU(N)$. We point out that this can follow naturally from the strong constraints that one naively expects from the Lie algebra equivalence between certain $SO(N)$ and $SU(N')$ theories and the equivalence of $SO(\infty)$ and $SU(\infty)$. The same argument for a weak N-dependence can in principle apply to $SU(N)$ and $SO(N)$ gauge theories in $D=3+1$.

1. Introduction

Gauge theories at $N\to\infty$ are often more tractable than those at finite N [1]. Since physically relevant theories tend to be at small N, it is interesting to determine whether the latter are sufficiently 'close' to $N=\infty$ for this limit to be physically useful.

Lattice studies of $SU(N)$ gauge theories in $2+1$ and in $3+1$ dimensions do indeed suggest a weak N-dependence for the few observables that have been calculated with adequate precision (see e.g. [2,3] and [4,5] respectively), as do exploratory studies of $SO(N)$ gauge theories in $D=2+1$ [6]. In this paper we shall use the (preliminary) results of current calculations of glueball mass ratios in $SU(N)$ [7] and $SO(N)$ [8] gauge theories in $D=2+1$ to analyse the N-dependence with greater reliability and accuracy than hitherto.

We shall find that the N-dependence is remarkably weak. (Something that was already apparent for $SU(N)$ from earlier calculations.) Not only can the variation of many mass ratios be accurately described with just a leading $O(1/N^2)$ correction for $SU(N)$, and $O(1/N)$ for $SO(N)$, but the coefficient of the correction term turns out to be $\ll 1$. We point out that this can follow naturally from the strong constraints imposed by the fact that $SO(N)$ and $SU(N)$ gauge theories share identical $N\to\infty$ planar limits plus the equivalence between certain $SO(N)$ and $SU(N')$ theories at smaller N.

In Section 2 we summarise some expectations for $SO(N)$ and $SU(N)$ gauge theories. We then present in Section 3 some results from [7,8] for the N-dependence of continuum mass ratios in $SO(N)$ and $SU(N)$ gauge theories. The range of N extends up to $N=16$ in both cases, so it is plausible that it makes sense to apply large-N expansions. In Section 4 we give examples of some constraints on the N-dependence which plausibly arise from the (Lie algebra) equivalence between some of the theories. We then briefly comment on $3+1$ dimensions, and finish with some conclusions.

2. Preliminaries

2.1. Large N

In $SU(N)$ gauge theories all-order diagrammatic arguments [1], supported by non-perturbative lattice calculations (see [9] for recent reviews), suggest that a mass ratio will approach its $N=\infty$ value as

$$\frac{M_i}{M_j} \overset{N\to\infty}{=} \tilde{r}_{ij} + \frac{\tilde{c}_{1,ij}}{N^2} + \frac{\tilde{c}_{2,ij}}{N^4} + \dots \quad : \quad SU(N). \tag{1}$$

In $SO(N)$ gauge theories a similar diagrammatic analysis [10] suggests

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$$\frac{M_i}{M_j} \overset{N\to\infty}{=} r_{ij} + \frac{c_{1,ij}}{N} + \frac{c_{2,ij}}{N^2} + \ldots \qquad : \quad SO(N) \tag{2}$$

One can show that the leading planar diagrams are the same in both cases up to a factor of 2 in g^2 [10]. Moreover, $SU(N)$ and $SO(2N)$ gauge theories are related by an orbifold projection [11], and it can be shown that this implies an identical common particle spectrum at $N=\infty$ [12]. So we can expect identical mass spectra at $N=\infty$, i.e.

$$\tilde{r}_{ij} = r_{ij} \tag{3}$$

in the common $C=+$ sector of the two theories.

2.2. Small N

Certain low N pairs of $SO(N)$ and $SU(N')$ theories are known to possess the same Lie algebras. These are: $SU(2)$ and $SO(3)$, $SU(2)\times SU(2)$ and $SO(4)$, $SU(4)$ and $SO(6)$. The Lie algebra equivalence suggests that ratios of glueball masses may well be identical within each pair of such theories, in which case

$$\left.\frac{M_i}{M_j}\right|_{SO(3)} = \left.\frac{M_i}{M_j}\right|_{SU(2)} = \left.\frac{M_i}{M_j}\right|_{SU(2)\times SU(2)} = \left.\frac{M_i}{M_j}\right|_{SO(4)}. \tag{4}$$

(The single particle spectrum of $SU(2)\times SU(2)$ should be the same as that of $SU(2)$, although the former will have extra multiparticle glueball states consisting of glueballs from the two groups.) We also may expect

$$\left.\frac{M_i}{M_j}\right|_{SO(6)} = \left.\frac{M_i}{M_j}\right|_{SU(4)}. \tag{5}$$

All this assumes that the differing global structure of the groups does not affect the particle spectrum. Whether this plausible assumption is indeed the case, or if not whether it is true for some states, is one of the interesting questions motivating the $SO(N)$ study in [8]. It would also be interesting to understand the place of the 'Pfaffian' particles of $SO(2N)$ in this context (see [13] for a discussion).

Note that, to include the $SO(N)$ fundamental string tension in these relations, one must take care to match with the correct $SU(N)$ representation. For example, an $SO(3)$ confining flux tube carrying fundamental flux corresponds to the $SU(2)$ flux tube carrying adjoint flux [6,8]. In a finite volume some 'glueball' states are composed of a pair of (conjugate) flux tubes closed around a spatial torus. Of course, as the volume increases these states become heavier and, eventually, unimportant.

3. Lattice results

The $SO(N)$ results we use are taken from [8] and those for $SU(N)$ from [7]. We refer to these papers for all the details of the calculations. The methods are entirely standard. The lattice action is the simple plaquette action. The Monte Carlo is a heat bath for $SO(N)$ [6] and a mixed heat bath plus over-relaxation for $SU(N)$. We use a moderately large basis of operators with various spin (J), parity (P) and charge conjugation (C) quantum numbers and calculate their correlators. Their exponential decay as the time separation increases provides an estimate of the ground state mass for the specified J^{PC} quantum numbers. The large basis of operators allows us to perform a systematic variational calculation which provides estimates of excited states as well. (Note that we label states with the lowest continuum spin J that contributes to the particular representation for a square spatial lattice. This will not always be correct [14].)

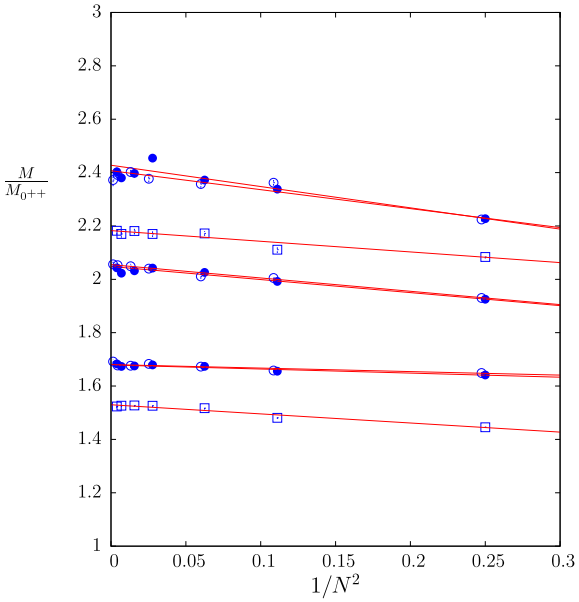

Fig. 1. Some masses versus $1/N^2$ in $SU(N)$, in units of the mass gap. In ascending order: the first excited $J^{PC}=0^{++}$, □, the 2^{++}, •, and 2^{-+}, ○, ground states, the 2^{++}, •, and 2^{-+}, ○, first excited states, the 0^{-+} ground state, □, and the 1^{++}, •, and 1^{-+}, ○, ground states. The $P=-$ partners have been shifted horizontally to be more visible. Lines are corresponding best fits of the form $c_0 + c_1/N^2$.

The calculations in [8] and in [7] attempt to calculate the masses of a large number of excited states. There are important systematic errors in such calculations, as discussed in [8,7]. Here we wish to focus on the typical N-dependence of mass ratios and so we restrict ourselves to a few of the best determined masses. We therefore include only the ground states of each (square) representation and the first excitations of the lightest of these, i.e. the $0^{\pm+}, 2^{\pm+}, 1^{\pm+}$ ground states and the $0^{++\star}$ and the $2^{\pm+\star}$ first excited states. The lattice volumes used are such that there should be no significant contamination from winding flux tube states, and the masses are not so large that we need to be concerned with contamination by multi-glueball states (even at smaller N). However the heavier a state the more rapidly the exponentially decreasing correlator disappears into the statistical fluctuations and (potentially) the larger the systematic error in extracting the mass. This is a caveat to consider in the case of the heavier glueballs i.e. the 0^{-+}, the $1^{\pm+}$ and the $2^{\pm+\star}$, and particularly so at coarser lattice spacings. This problem is enhanced if the overlap of the desired state onto the basis of operators is smaller – and this tends to be the case for $SO(N)$ at small N. Nonetheless the systematic error induced by these factors in the qualitative behaviour of the N-dependence of mass ratios – our main interest here – should not be substantial.

3.1. SU(N)

We calculate [7] our mass ratios in units of the mass gap, the 0^{++} ground state, since this is our most accurately calculated mass. For each $SU(N)$ we extrapolate the lattice values of each mass ratio to the continuum limit using an $O(a^2)$ correction. We do so for each of $N=2,3,4,6,8,12,16$. The resulting mass ratios are plotted in Fig. 1. We also plot best fits of the form

$$\frac{M}{M_{0^{++}}} = c_0 + \frac{c_1}{N^2}. \tag{6}$$

The values of c_0 and c_1 are listed in Table 1.

We observe in Fig. 1 the parity doubling of $J\neq 0$ states which is expected in the continuum limit of $D=2+1$. (For $J=2$ the

Table 1
Coefficient of leading large-N fits to mass ratios $M/M_{0^{++}}$ in $SO(N)$ and $SU(N)$, using fits $c_0 - c_1/N^2$ for $SU(N)$ and $c_0 - c_1/N$ for $SO(N)$. Errors are statistical.

State	c_1/c_0		c_0	
	$SO(N)$	$SU(N)$	$SO(N)$	$SU(N)$
$0^{++\star}$	0.278(46)	0.223(13)	1.593(12)	1.530(3)
0^{-+}	0.030(70)	0.183(21)	2.193(27)	2.183(6)
2^{++}	0.139(41)	0.091(14)	1.744(13)	1.679(3)
2^{-+}	0.110(42)	0.078(14)	1.720(14)	1.681(4)
$2^{++\star}$	0.098(61)	0.239(17)	2.132(23)	2.048(5)
$2^{-+\star}$	0.050(58)	0.243(18)	2.111(22)	2.055(4)
1^{++}	0.118(75)	0.329(21)	2.548(36)	2.427(7)
1^{-+}	0.164(86)	0.295(22)	2.560(40)	2.407(7)

Table 2
Normalised coefficient of leading (and sub-leading) large-N correction to mass ratios $M/M_{0^{++}}$ in $SO(N)$ and $SU(N)$ from fits $c_0 + c_1/N^2 + c_2/N^4$ for $SU(N)$ and $c_0 + c_1/N + c_2/N^2$ for $SO(N)$. Errors are statistical.

State	$SO(N)$	$SU(N)$	
	$\lvert c_1\rvert/c_0$	$\lvert c_1\rvert/c_0$	$\lvert c_2\rvert/c_0$
$0^{++\star}$	0.47(21)	0.266(48)	0.16(18)
0^{-+}	0.25(30)	0.198(77)	0.05(27)
2^{++}	0.17(18)	0.117(36)	0.10(13)
2^{-+}	0.24(19)	0.145(52)	0.25(19)
$2^{++\star}$	0.38(27)	0.190(72)	0.18(26)
$2^{-+\star}$	0.04(25)	0.249(75)	0.03(28)
1^{++}	0.95(38)	0.191(77)	0.51(27)
1^{-+}	0.59(50)	0.010(90)	1.13(32)

parity doubling may be broken by finite volume effects, but these should be small here.)

We also observe that the mass ratios can be described with just a leading $O(1/N^2)$ correction all the way from $SU(16)$ down to $SU(2)$. (We assume $N = 16$ is large enough that there will be no surprises at larger N.) The χ^2 of these fits is reasonable in most cases. Only for the 1^{++} is it very large ($\sim$6 per degree of freedom), and for the 1^{-+} and $0^{++\star}$ it is moderately large ($\sim$2.5 per degree of freedom). In the case of $1^{\pm+}$ the problem is a large (and presumably unphysical) mass splitting for $SU(6)$, rather than the expected degeneracy, and for the $0^{++\star}$ the problem is a large downward fluctuation in $SU(3)$. None of this is helped by including an extra $O(1/N^4)$ term in the fit.

The fact that an $O(1/N^2)$ correction suffices for all N already tells us that the deviations from $N = \infty$ cannot be large at any N. However a glance at Fig. 1 tells us that the deviations are even smaller than this would suggest if we had natural coefficients $c_1 \sim c_0$. Indeed one finds $c_1/c_0 \ll 1$ for the best fits, as shown in Table 1. One might wonder if this result is stable under the inclusion of an additional $O(1/N^4)$ correction term (even if the statistical analysis does not demand such an extra term). As we see from Table 2 this result is indeed stable. At least in $SU(N)$ glueball mass ratios show remarkably little variation as N varies from $N = 2$ to $N = \infty$.

3.2. $SO(N)$

We calculate [8] the continuum mass ratios in $SO(N)$ just as for $SU(N)$. We do so for each of $N = 3, 4, 5, 6, 7, 8, 12, 16$. The resulting mass ratios are plotted in Fig. 2. We also plot there best fits of the form

$$\frac{M}{M_{0^{++}}} = c_0 + \frac{c_1}{N} \tag{7}$$

and list the values of c_0 and c_1 in Table 1.

We observe in Fig. 2 the parity doubling of $J \neq 0$ states just as we saw for $SU(N)$. However it is clear from the scatter of points

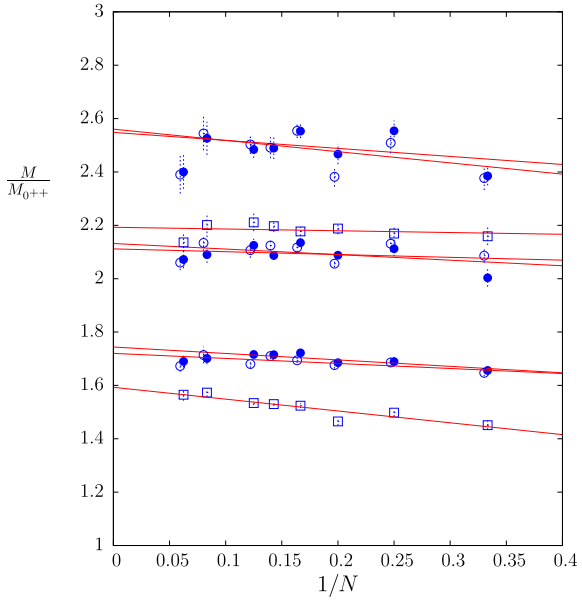

Fig. 2. Some masses versus $1/N$ in $SO(N)$, in units of the mass gap. In ascending order: the first excited $J^P = 0^+$, □, the 2^+, •, and 2^-, ○, ground states, the 2^+, •, and 2^-, ○, first excited states, the 0^- ground state, □, and the 1^+, •, and 1^-, ○, ground states. The $P = -$ partners have been shifted horizontally to be more visible. Lines are corresponding best fits of the form $c_0 + c_1/N$.

that our $SO(N)$ results are considerably less accurate than those for $SU(N)$. This must be partly due to the fact that the overlap of the states on our basis is significantly smaller in $SO(N)$ than in $SU(N)$, particularly at small N, and it may also be that the lack of over-relaxation in the Monte Carlo update means that it explores the phase space more slowly. Nonetheless, the weakness of the N-dependence is evident and we observe that the mass ratios can be described with just a leading $O(1/N)$ correction all the way from $SO(16)$ down to $SO(3)$. The χ^2 of these fits is reasonable in most cases, only being somewhat large, with $\chi^2/n_{dof} \sim 2$–3, for the $1^{\pm}$ and the $2^{\pm\star}$. Fits including an extra $O(1/N^2)$ correction term improve the χ^2/n_{dof} for the $2^{+\star}$ and the 1^+ but not for the $2^{-\star}$ and the 1^-. Since the $P = \pm$ pairs of states should be degenerate (at each N) it is not clear if the need for such an extra term in these states is being indicated or not.

In any case, what we clearly see is a very weak N-dependence that in most cases can be described with a leading $O(1/N)$ correction. Moreover, just as for $SU(N)$, the coefficient of this correction is small, $c_1/c_0 \ll 1$, as we see in Table 1. One might again wonder if this result is stable under the inclusion of an additional $O(1/N^2)$ correction term and the indications from Table 2 are that this is indeed the case, albeit with large uncertainties. So, just as for $SU(N)$, the glueball mass ratios in $SO(N)$ show remarkably little variation with N over the whole range of N.

3.3. $SO(N)$ and $SU(N)$: a comparison

We see from the best fits listed in Table 1 that the $N \to \infty$ limits of the $SU(N)$ and $SO(N)$ mass ratios are very similar and, given that the errors listed are purely statistical, that they are broadly compatible. The apparent differences are $\sim$1–5% with the $SO(N)$ values being always higher. This is in the direction one would expect from the smaller $SO(N)$ overlaps leading to a slightly too-early identification of the effective mass plateaux that then leads to a small systematic over-estimate of the masses.

There is no obvious best way to compare the $SU(N)$ and $SO(N)$ mass ratios at finite N given the different powers of the leading

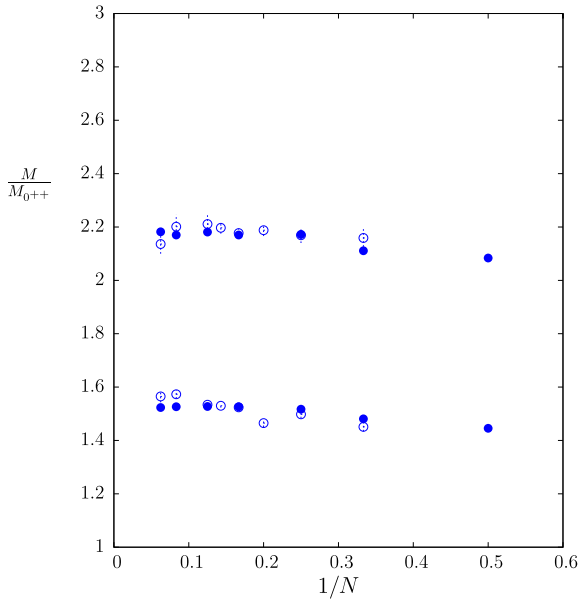

Fig. 3. Some masses versus $1/N$ in $SO(N)$, ∘, and $SU(N)$, •, in units of the mass gap. In ascending order: the first excited $J^{PC} = 0^{++}$ and the 0^{-+} ground state.

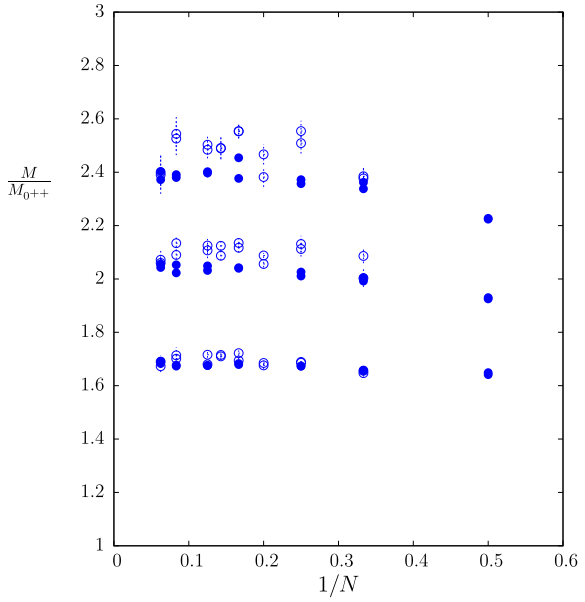

Fig. 4. Some masses versus $1/N$ in $SO(N)$, ∘, and $SU(N)$, •, in units of the mass gap. In ascending order: the $J^{PC} = 2^{\pm+}$ ground states, the $2^{\pm+}$ first excited states, and the $1^{\pm+}$ ground states.

corrections. Here we shall simply overlay in Figs. 3, 4 the mass ratios for $SU(N)$ and $SO(N)$. (The reader can use the fits in Table 1 to construct alternative comparisons.) Without going into fine details (see [8] for a careful comparison) we see that the mass ratios of the two theories are broadly similar, taking into account the larger systematic errors on the most massive states. A similar comment applies to the comparison between $SU(2)$ and $SO(3)$, $SU(2)$ and $SO(4)$, and $SU(4)$ and $SO(6)$, with similar caveats concerning the most massive states.

The reader will have noticed that so far we have not considered the fundamental string tension, σ_f, in our mass ratios. This is usually the physical quantity that is most accurately obtained in lattice calculations of energies and so one often sees continuum glueball masses presented as a ratio $M/\sqrt{\sigma}$. The reason we

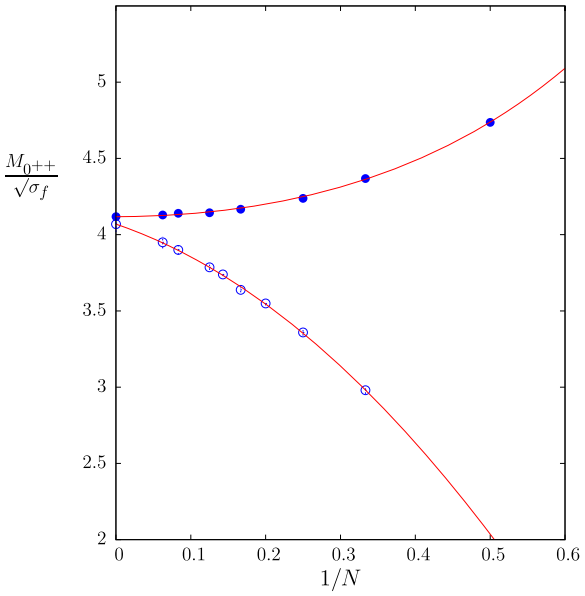

Fig. 5. The mass of the lightest scalar glueball in units of the fundamental string tension versus $1/N$: for $SO(N)$, ∘, and for $SU(N)$, •. Lines are best fits as described in text. Extreme left points are values extrapolated to $N = \infty$.

have not done so is that, for example, the fundamental $f = \underline{3}$ of $SO(3)$ corresponds to the adjoint $A = \underline{3}$ of $SU(2)$, and the $f = \underline{6}$ of $SO(6)$ corresponds to the $k = 2$ antisymmetric of $SU(4)$. That is to say, the fundamental string tensions for e.g. $SO(3)$ and $SU(2)$ are not the same physical quantities. This is in contrast to colour singlet glueball masses that do not care about the representation of the fundamental fields. Since $\sigma_A[SU2] \sim 2.5\sigma_f[SU2]$ [7,8] and $\sigma_{2a}[SU4] \sim 1.35\sigma_f[SU4]$ (see e.g. [15]) we know in advance that a mass ratio $M/\sqrt{\sigma_f}$ will have a strong N-dependence for either $SU(N)$ or for $SO(N)$ or for both. To illustrate this we show in Fig. 5 the ratio of the mass of the ground state $J^{PC} = 0^{++}$ glueball (the mass gap) to the fundamental string tension for both $SO(N)$ and $SU(N)$. The corrections to the $N = \infty$ limit are clearly much greater than in the mass ratios shown in Figs. 1, 2. It is equally clear from Fig. 5 that a straight line fit to the $SO(N \geq 3)$ ratios will not work: one needs to include an $O(1/N^2)$ term in addition to the leading $O(1/N)$ term. This is also the case for the $SU(N \geq 2)$ ratios: one needs to include an $O(1/N^4)$ term in addition to the leading $O(1/N^2)$ term.

4. Constraints on the N-dependence

We assume in this section that the glueball spectrum of both $SO(3)$ and $SO(4)$ is the same as that of $SU(2)$ and that the spectrum of $SO(6)$ is the same as that of $SU(4)$. (In the case of $SO(4)$ there will be extra multi-glueball states from the $SU(2) \times SU(2)$ structure, but that does not affect our argument here.) We also assume the $N = \infty$ glueball spectra of $SO(N)$ and $SU(N)$ are identical in their common $C = +$ sector. These constraints become quite powerful when we assume in addition that we only need a few of the terms in the expansions in eqs. (1), (2) to describe the glueball spectra $\forall N$. This last assumption is quite strongly supported by the lattice calculations which, as we have seen, typically require only a leading order correction to reproduce the spectra for all N, within the errors.

How strongly these constraints determine the mass spectra will depend on how many terms we need to retain in the expansions in eqs. (1), (2) to accurately reproduce the mass spectra for all N. We illustrate the possibilities with the following sample of scenarios.

1. Assume we know the $SU(N)$ spectrum. Then, given our above assumption, we also know the spectra of $SO(3)$, $SO(4)$, $SO(6)$ and $SO(\infty)$. This is enough to predict the spectrum of $SO(N)$ for all N, if the large N expansion, when truncated to 4 terms,

$$\frac{M}{M_{0^+}} \overset{N\geq 3}{\simeq} = c_0 + \frac{c_1}{N} + \frac{c_2}{N^2} + \frac{c_3}{N^3} \quad : \quad SO(N), \tag{8}$$

is sufficiently accurate, as is strongly supported by our calculations which show that just the leading term is mostly good enough within our statistical errors. That is to say: the $SU(N)$ spectrum predicts that of $SO(N)$ $\forall N$ within the accuracy of eq. (8). An example of such a prediction was displayed in Fig. 1 of [6].

2. Suppose that the spectrum is accurately reproduced by eq. (8) with only the first $O(1/N)$ correction non-zero. Then the equality of the $SO(3)$ and $SO(4)$ spectra, immediately tells us that $c_1 = 0$, i.e. there is no N-dependence at all in $SO(N)$. This then demands that the $SU(2)$, $SU(4)$ and $SU(\infty)$ spectra should also be equal. So if the expansion for $SU(N)$

$$\frac{M}{M_{0^+}} \overset{N\geq 2}{\simeq} = \tilde{c}_0 + \frac{\tilde{c}_1}{N^2} + \frac{\tilde{c}_2}{N^4} + \dots \quad : \quad SU(N), \tag{9}$$

is sufficiently accurate with just the first two correction terms (not implausible given what the $SU(N)$ lattice calculations indicate) we have no N-dependence for $SU(N)$ either.

3. Suppose that two correction terms, i.e. $c_1/N + c_2/N^2$, suffice for $SO(N \geq 3)$. Subtracting the expansions for $SO(3)$ and $SO(4)$, and using the equality of the mass ratios, we see that $c_2 = -12c_1/7$. This can then be used to reduce the number of fitted parameters from two to one. If we additionally assume that a single correction term, i.e. $\tilde{c}_1/N^2$, suffices for $SU(N \geq 2)$ (as often appears to be the case) then we have three relations between these coefficients (using also $c_0 = \tilde{c}_0$) whose only solution is $\tilde{c}_1 = c_1 = c_2 = 0$, i.e. no N-dependence at all.

4. Suppose we ignore the constraints from $SU(2)$ because, for example, we do not trust the large-N expansion for such low N, and similarly for $SO(3)$. In such a case, if we make the relatively weak assumption that a single correction, $\tilde{c}_1/N^2$, suffices for $SU(N \geq 4)$, and similarly a single correction, c_1/N, suffices for $SO(N \geq 6)$ then we immediately obtain $\tilde{c}_1 = 16c_1/6$, i.e. the N-dependence of $SU(N \geq 4)$ is completely constrained by that of $SO(N \geq 6)$ (or vice versa).

An important feature of these arguments is that if the number of significant terms in expansions around $N = \infty$ is small enough, then the expansion coefficients will be zero. Since the lattice calculations are indeed consistent with such a small number of terms, this perhaps provides an explanation why the coefficients of these terms turn out to be unexpectedly small and why there is so little dependence on N for both the $SO(N)$ and $SU(N)$ glueball spectra.

Given the importance of our lattice results in supporting such arguments, and given that our errors are finite, we need to ask how large a higher order term might be concealed within these errors. To address this question we show in Table 2 the subleading coefficient, c_2, in fits that include such a term. We do so only for $SU(N)$ since the errors in $SO(N)$ are too large to provide any kind of tight constraint. What we see is that apart from $J = 1$, where the fits are very poor and the systematic errors are largest, the coefficients of the $O(1/N^4)$ term are small and indeed consistent with zero (as one would expect given that the leading term by itself gives acceptable fits). So for $SU(N)$ at least, arguments based on a low order expansion in $1/N^2$ do appear to have a significant motivation.

Of course, future calculations with much smaller errors will inevitably expose the presence of higher order terms in the $1/N$ expansions. Our above analysis suggests that the coefficients of these will be small enough for the arguments of this section to retain an approximate validity. However this will certainly make the comparison between $SO(N)$ and $SU(N)$ more delicate, in much the same way as in the comparison between $SU(N)$ adjoint and bifundamental chiral condensates [16] where the quantities are very accurately determined and the corrections are respectively in powers of $1/N^2$ and $1/N$, just as in our case.

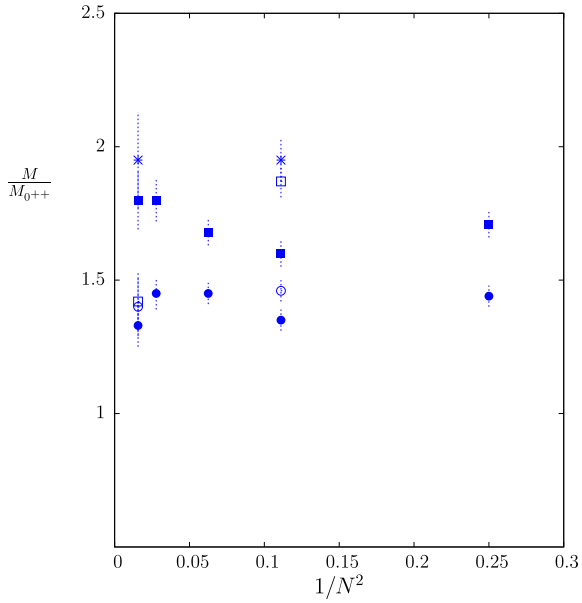

Fig. 6. Some masses versus $1/N^2$ in $SU(N)$ in $D = 3 + 1$, in units of the mass gap. The first excited $J^{PC} = 0^{++}$, □, the ground state 2^{++}, ∘, and the first excited 2^{++}, ⋆: all from [5]. Also the first excited $J^{PC} = 0^{++}$, ■, and the 2^{++}, •, ground state from [4].

5. $D = 3 + 1$

The discussion in Sections 2 and 4 carries over unchanged to $D = 3+1$. So the question is whether lattice calculations encourage us to assume that low-order large-N expansions are accurate all the way down to $SU(2)$ and $SO(3)$, or not.

For $SU(N)$ some calculations exist. In Fig. 6 we show some results from [5] and [4]. We see that the 2^{++} ground and first excited states show little variation with N. The first excited 0^{++} is, however, not consistent between the two calculations: one indicates a large variation, the other a modest one! The main conclusion here must be that much more accurate lattice calculations are needed if we wish to pursue this question.

In the case of $SO(N)$ a few calculations exist, [6], but at no N has a continuum extrapolation been performed. The problem here is that there is a first-order phase transition in the lattice (bare) coupling, separating the weak and strong coupling phases, which, for small N, occurs for a very small lattice spacing (measured on the weak coupling side) [6]. Thus for small N extremely large lattice volumes are needed if one is to be on the weak coupling side, from where one can take a continuum limit. It may be that improved lattice actions will help to overcome this obstacle, but at present no calculations useful for our purposes exist for $SO(N)$ in $D = 3 + 1$.

6. Conclusions

In this paper we have considered the N-dependence of glueball masses, in units of the mass gap, for $SU(N)$ and $SO(N)$ gauge

theories in $2+1$ dimensions. Out of the glueball spectra, calculated in [7,8], we have selected a few of the most reliably and precisely calculated states. We have seen that the N dependence is very weak and in most cases can be described with just the leading large-N correction for all values of N. Moreover, we saw that the coefficient of the correction term is unexpectedly small. We noted that the Lie algebra equivalences between certain $SO(N)$ and $SU(N')$ groups can become very constraining if the N-dependence of the spectra can be described by sufficiently few terms in the large-N expansions for both $SU(N)$ and $SO(N)$, and that this provides a possible explanation for the very weak N-dependence. In principle such arguments carry over to $D=3+1$ but at present we do not have usefully precise indications that one or two correction terms suffice to describe the N-dependence of these theories.

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