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Fuzzy preferences in multiple participant decision making

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Abstract A fuzzy set theoretic approach for handling preference uncertainty within the paradigm of the Graph Model for Conflict Resolution is employed for systematically carrying out the strategic investigation of a conflict over the proposed export of water in bulk quantities. Following an overview of the literature regarding fuzzy preferences and their applications in decision making, the graph model is restructured to incorporate fuzzy preferences into calculations of stability. Nash and sequential stability definitions, which reflect human behavior in conflict, are modified to accommodate fuzzy preferences. The conflict over the potential large-scale export of water from Lake Gisborne, located in Canada's Newfoundland and Labrador province, is modeled, assuming that one of the four Decision Makers (DMs) in the dispute has fuzzy or uncertain preferences, while the preferences of the remaining DMs are crisp. The strategic insights gained by varying the satisficing behavior of the DM with fuzzy preferences are discussed.

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1. Introduction

Decision making is one of the most common activities in society. It ranges from simple decisions in one's daily life to strategic decisions in war. Depending on the number of DMs and objectives, decision making can be categorized as:

- (i) Single-participant single-objective,
- (ii) Single-participant multiple-objective,
- (iii) Multiple-participant single-objective,
- (iv) Multiple-participant multiple-objective [1].

The last case is the most complicated, and the objective of the present article is to suggest an appropriate solution methodology for it, especially in cases where the participants or DMs have uncertain preferences over the alternatives or states.

In multiple participant-multiple objective decision making problems, DMs usually have clashing interests and objectives. Hence, strategic conflict is a common feature in these decision problems. A number of formal methodologies have been proposed to facilitate the analysis of strategic conflicts, including Game Theory [2], Metagame Analysis [3], Conflict Analysis [4], Drama Theory [5] and the Graph Model for Conflict Resolution [6]. These methods share many general characteristics: they were designed to study conflict situations in which there are at least two DMs, each of whom has multiple options and multiple objectives that imply distinct preferences over the outcomes. They are either game theory or game theory variants that have been designed to yield better decision advice or more compelling structural insights [7]. Descriptions of a useful range of techniques for modeling and analyzing conflict are provided in some recently published handbooks [8–10].

Among these conflict methodologies, the Graph Model for Conflict Resolution is especially attractive because it is simple and flexible to use. Other advantages of the graph model include its ability to handle irreversible moves, model common moves, provide a flexible framework for defining, comparing and characterizing various stability definitions of human behavior under conflict, and be easily applied to real-world disputes. The graph model has been used for resolving conflicts arising in many different fields including water resources management, sustainable development and environmental engineering (see [11–16] for references).

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A graph model is a formal methodology for analyzing a multiple-participant dispute based on a number of stability definitions, describing different kinds of human behavior. Useful stability definitions include Nash stability (R) [17,18], general metarationality (GMR) [3], symmetric metarationality (SMR) [3] and sequential stability (SEQ) [4]. In general, a state is stable for a DM if that DM thinks that it is not advantageous for him or her to unilaterally move away from it [6]. A stability definition explores explicitly how moves and countermoves result in a given state being stable or not. The key input in calculating the stability is each DM's preferences over the states or alternatives. At the time of the development of the graph model, the preference input considered was crisp, that is preferences between any two states were expressed using the binary relations "is (strictly) preferred to" and "is indifferent to". However, while analyzing a real-world dispute, DMs may be unclear or uncertain about preferences between two states, because of their cultural and educational backgrounds, personal habits, lack of information about the decision parameters and the inherent vagueness of human judgment.

Until recently, only three attempts have been made to incorporate DMs' uncertain preferences into the Graph model for Conflict Resolution from different points of view. Li et al. [19] introduced a new preference structure for the graph model, which is neither fuzzy nor probabilistic. This preference structure includes uncertain or unknown preference in the comparison of two states, considering a situation in which a DM, for the time being, may be uncertain about the preference between two states, but knows that with full information, he or she would strictly or equally prefer one state to the other. Then the stability definitions for R , GMR , SMR and SEQ are modified in order to accommodate this incomplete binary preference into the graph model structure. Ben-Haim and Hipel [20] employed information-gap theory to ascertain how systematic changes in preferences of DMs affect individual stability results and equilibria. A fuzzy approach was proposed by Al-Mutairi et al. [21] to model uncertainty in the preferences of DMs involved in a conflict. The authors used five linguistic labels: much more preferred, more preferred, indifferent, less preferred, and much less preferred to split the fuzzy domain of preferences. Then, adapting the concepts of strong and weak stability proposed by Hamouda et al. [22,23], they introduced an analogous strong and weak stability and hence strong and weak equilibrium, to suggest possible resolutions of the dispute. But these approaches do not accommodate uncertainty about preferences between two states in a general sense.

A recent addition to the Graph Model for Conflict Resolution is a fuzzy preference framework developed by Bashar et al. [24,25]. This structure accommodates DMs' preference uncertainty in the form of fuzzy preference in stability calculations. A fuzzy preference relation is a generalized way of representing both certain and uncertain preferences between two states or alternatives. Within the fuzzy preference framework, four basic stability definitions are redefined as: *fuzzy Nash stability* or *fuzzy rationality* (FR), *fuzzy general metarationality* ($FGMR$), *fuzzy symmetric metarationality* ($FSMR$), and *fuzzy sequential stability* ($FSEQ$) [24,25].

As explained by the authors in the book edited by Jamshidi [26], a System of Systems (SoS) constitutes an informative paradigm in which reality can be visualized. In fact, many authors maintain that societal, technological and natural SoS are inhabited by multiple participants or agents who compete or cooperate with one another as they strive to achieve their goals within and among these interconnected SoS for which unexpected properties and behavior emerge over time [15,27–29].

Being able to model preference uncertainty via fuzzy preferences for DMs or agents participating in conflicts within these complex adaptive SoS makes the conflict models more realistic. Hence, one can argue that the original pioneering work of Professor Lotfi Zadeh, the "Father" of fuzzy sets, greatly influenced leading-edge research in the field of systems engineering and many other areas.

The organization of the remainder of the paper is now described. A brief literature review on preference uncertainty, fuzzy preferences and their applications in various decision making circumstances are presented in Section 2. Within Section 3, the definitions of a fuzzy set, fuzzy number, fuzzy relation and fuzzy preference are given. A general description of the Graph Model for Conflict Resolution is presented in Section 4, while in Section 5, the fuzzy preference framework of the Graph Model, together with the fuzzy stability definitions, are put forward. In Section 6, the new fuzzy graph model is employed for formally studying strategic decisions, regarding the dispute over the potential export of water in bulk from Lake Gisborne, located in the Canadian Province of Newfoundland and Labrador. Conclusions are presented in the final section of the paper.

2. Literature review of preference uncertainty and fuzzy preferences

Preference information is crucial for determining favored outcomes in decision making. However, the preference can be certain or uncertain. A certain preference is a situation in which a DM definitely prefers one state over another or is indifferent between them. On the other hand, an uncertain preference is the case where a DM is not quite sure whether he or she definitely prefers one state over another, even if he or she picks one. Preference uncertainty is very common in real world decision making, such as in engineering, the social sciences and economics.

2.1. Preference uncertainty

Preference uncertainty is modeled quantitatively or qualitatively. Quantitatively, it is represented by numbers indicating preference intensities [30–32] or degrees of preference [33,34]; while qualitatively, it is represented by linguistic labels, such as good, fair and poor [35,36]. Because of its importance in various decision making techniques, uncertain preference relations have been an active area of research, and many variants have been developed over the last few decades.

Frequently used uncertain preference relations include multiplicative preferences [31,37], incomplete multiplicative preferences [38,39], interval multiplicative preferences [40,41], incomplete interval multiplicative preferences [42], triangular fuzzy multiplicative preferences [43,44], incomplete triangular fuzzy multiplicative preferences [42], the fuzzy preference relation [32–34,45,46], the incomplete fuzzy preference relation [47,48], interval fuzzy preferences [49,50], incomplete interval fuzzy preferences [42], triangular fuzzy preferences [51], incomplete triangular fuzzy preferences [42], linguistic preferences [35,36] and incomplete linguistic preferences [52,53]. Among these preference relations, fuzzy preference relations are a convenient way of representing both certain and uncertain relative preferences between two states or alternatives.

A fuzzy preference between two states is represented by a preference degree, which is interpreted as the grade or strength of certainty of the preference of one state over another.

In the representation of a fuzzy preference, the maximum preference degree is 1.0 which implies definite preference, and the minimum preference degree is 0 which means definite reverse preference. The preference degree 0.5 indicates that the states, which are being compared, are likely to be indifferent.

2.2. Fuzzy preferences and their applications

Zadeh [54,55] developed fuzzy logic and fuzzy sets as effective tools for mathematically modeling uncertainty or vagueness. Based on Zadeh's notion of fuzzy logic, Orlovsky [33] proposed a fuzzy preference relation to generalize crisp preference in a decision making situation. He introduced and studied fuzzy preference and its properties, and the fuzzy set of non-dominated alternatives. He established that if the fuzzy preference relation in a fuzzy decision-making problem satisfies some topological properties, then the problem has "un-fuzzy" (crisp) non-dominated solutions.

Keeping in mind that fuzzy utilities could be a flexible way of representing utilities of states, Nakamura [56] proposed a method to construct a fuzzy preference, given a set of fuzzy utilities, to allow rational decision making. Tanino [32] discussed the use of fuzzy preference orderings in group decision making. He defined a fuzzy preference ordering as a fuzzy binary relation, satisficing reciprocity and max-min transitivity, and developed group fuzzy preference orderings, applicable when individual preferences are represented by utility functions, developing a method for group decision processes analogous to the extended contributive rule.

Chiclana et al. [57] introduced a general multipurpose decision model that is able to handle problems with a range of preference information: preference orderings, utility functions or fuzzy preference relations. First, the preference information is made uniform using fuzzy preference relations, and then selection processes are introduced based on the concept of fuzzy majority [58] and on ordered weighted averaging operators [59]. Chiclana et al. [46] also carried out research on how to integrate multiplicative preference relations into fuzzy multipurpose decision models using preference orderings, utility functions or fuzzy preference relations. Together with the work in [57], the authors provided a more flexible framework to manage different preference structures. This constituted a decision model that approximated real decision situations involving experts from different knowledge areas.

3. Fuzzy preferences

As mentioned above, a fuzzy preference relation models uncertain preference in a quantitative manner. Basically, a fuzzy preference relation is a fuzzy binary relation. This representation is based on Zadeh's fuzzy logic and fuzzy sets [54,55]. Within Zadeh's fuzzy logic concept, uncertain preferences can also be expressed as fuzzy utilities or as fuzzy weighted sums in a Fuzzy Multi-Criteria Decision Making (FMCDM) setting. Both fuzzy utilities and fuzzy weighted sums are essentially fuzzy numbers. There are existing techniques, such as in [56], to calculate fuzzy preference from a given set of fuzzy utilities or fuzzy numbers.

3.1. Fuzzy set, fuzzy number and fuzzy relation

Fuzzy Set: As is well known, the concept of a fuzzy set was originally introduced by Zadeh [54,55] as an extension of the classical (crisp) notion of set. Classical set theory determines the

membership of elements in a set in a binary manner: an element either belongs to, or does not belong to, the set. However, fuzzy set theory allows the membership of elements in a set to be described in terms of a membership function valued in the real unit interval $[0, 1]$, representing the degree or grade or strength of membership. The formal definition of a fuzzy set is given below.

Definition 1. Let X denote a nonempty collection of objects and $I = [0, 1]$, a unit interval of real numbers. A *fuzzy set* in X is characterized by a function δ ,

$$\delta : X \longrightarrow I, \quad x \longmapsto \delta(x),$$

where $\delta(x)$ is interpreted as the "degree or grade of membership" of $x \in X$ in the fuzzy set. The nearer the value of $\delta(x)$ is to 1, the higher is the grade of membership of x in the fuzzy set. The function δ is called the membership function of the fuzzy set. Note that a fuzzy set is often denoted by its membership function.

A crisp set is a special case of a fuzzy set, in the sense that the membership function of a crisp set can be considered as a characteristic function that assigns a 1 for each element belonging to the set, and a 0 for each element not belonging to the set; so, δ is a crisp subset (or, set) if and only if $\delta(x) = 0$ or 1, for all $x \in X$.

Fuzzy Number: A fuzzy number is a special type of fuzzy set defined on the set of real numbers. In fact, fuzzy numbers are regarded to be an extension of real numbers. Because of their close association with real numbers, fuzzy numbers are able to account for uncertainties in parameters, utilities, properties, geometry, initial conditions and other issues. Moreover, the basic arithmetic operations – addition, subtraction, multiplication and division – allow the application areas of fuzzy numbers to be extended enormously [60].

Definition 2 ([60,61]). Let \mathbb{R} denote the set of real numbers. A *fuzzy number* is a fuzzy set $\delta : \mathbb{R} \longrightarrow I$ with the following properties:

1. δ is upper semi-continuous;
2. $\delta(x) = 0$ outside some interval $[c, d]$;
3. There are real numbers $a, b : c \leq a \leq b \leq d$ for which:
 - (i) $\delta(x) = 1$ for all $x \in [a, b]$;
 - (ii) $\delta(x)$ is monotonic increasing on $[c, a]$;
 - (iii) $\delta(x)$ is monotonic decreasing on $[b, d]$.

Fuzzy relation: Traditionally, preference or crisp preference is formalized using a binary relation. In consequence, fuzzy preference is modeled using a fuzzy binary relation or simply a fuzzy relation. A classical or crisp relation indicates that an object, $x \in X$, is either related to, or not related to, another object, $y \in Y$. However, a fuzzy relation describes the uncertainty of the relationship between x and y as a degree or grade of relationship.

Definition 3 ([60]). Let X and Y denote nonempty collections of objects (not necessarily the same). A *fuzzy relation* from X to Y , denoted \mathcal{R} , is a fuzzy set in $X \times Y$ with membership function:

$$\mu_{\mathcal{R}} : X \times Y \longrightarrow [0, 1],$$

where $\mu_{\mathcal{R}}(x, y)$ represents the degree, grade or strength of the relationship of $x \in X$ with $y \in Y$.

A fuzzy relation from X to Y is usually represented by a matrix in which the rows and columns are labeled by the objects of X and Y , respectively. The entries of the matrix indicate the degree of relationship of the row object to the column object. Note that if $X = Y$, \mathcal{R} is said to be a fuzzy relation on X or on Y .

3.2. Fuzzy preferences

A fuzzy preference is an important type of fuzzy binary relation. It models both the certainty and uncertainty of preferences between two states or alternatives as a degree of preference. A formal definition of a fuzzy preference relation is presented below.

Definition 4 ([32,34,45,46]). Let S denote the set of m states or alternatives: s_1, s_2, \dots, s_m where $m > 1$. A fuzzy preference over S is a fuzzy relation on S , represented by a matrix $\mathcal{R} = (r_{ij})_{m \times m}$, with membership function:

$$\mu_{\mathcal{R}} : S \times S \longrightarrow [0, 1],$$

where $\mu_{\mathcal{R}}(s_i, s_j) = r_{ij}$ denotes the preference degree of state s_i over s_j , satisfying:

$$r_{ij} + r_{ji} = 1 \quad \text{and} \quad r_{ii} = 0.5, \quad \text{for all } i, j = 1, 2, \dots, m.$$

The condition $r_{ij} + r_{ji} = 1$ is referred to as the additive reciprocity.

One often writes $r_{ij} = r(s_i, s_j)$. Interpretations of the values of $r(s_i, s_j)$ are as follows:

1. $r(s_i, s_j) > 0.5$ indicates that state s_i is likely to be preferred to state s_j ; the larger $r(s_i, s_j)$, the more likely s_i is preferred to s_j ; $r(s_i, s_j) = 1$ means that s_i is definitely preferred to s_j .
2. $r(s_i, s_j) = 0.5$ means that state s_i is likely to be indifferent to state s_j .
3. $r(s_i, s_j) < 0.5$ indicates that state s_j is likely to be preferred to state s_i ; the smaller $r(s_i, s_j)$, the more likely s_j is preferred to s_i ; $r(s_i, s_j) = 0$ indicates that s_j is definitely preferred to s_i .

It is important to note that the amount of preference cannot be inferred from the degree of preference; rather, the degree of preference is the level of certainty that a DM will prefer one state to the other, but implies nothing about how great this preference is likely to be.

Thus, when s_i is definitely preferred to s_j by a DM, it means that it is certain that s_i is preferred to s_j , but there is no implication as to how much more preferred s_i is than s_j by the DM.

4. The graph model for conflict resolution

4.1. Structure of a graph model

A graph model of a conflict is a set of directed graphs, one for each DM, in which the states are nodes and the DMs' possible moves in one step are the directed arcs. The common vertices within all DMs' graphs are the feasible states.

Suppose that $N = \{1, 2, \dots, n\}$ is the set of DMs and $S = \{s_1, s_2, \dots, s_m\}$ is the set of feasible states. $D_k = (S, A_k)$ is DM k 's directed graph where A_k is DM k 's set of directed arcs. If \succsim_k represents DM k 's preferences over S , then a graph model can be given mathematically as:

$$\{(D_k, \succsim_k) : k \in N\}.$$

A directed arc is represented as an ordered pair of states, say (s_i, s_j) , where s_i is the tail and s_j is the head of the arc. DM, k , controls moves according to the directed arcs in A_k . Graphically, the direction of the move from one state to another is shown using directional arrows. Note that the moves may or may not be reversible, and that there may also be common moves [6].

4.2. Unilateral moves

To permit the analysis within a graph model, it is necessary to identify states to which a DM can unilaterally move from a given state. The reachable list, or the set of moveable states for a DM from a specified starting state, is a record of all the states that the DM can reach in one step. Formally, the *reachable list* from state $s \in S$ for DM k is defined as:

$$R_k(s) = \{s_i \in S : (s, s_i) \in A_k\}.$$

The reachable list defined above is the set of unilateral moves under the control of DM k . However, the graph model considers moves and countermoves in its stability calculations. When there are more than two DMs in the graph model, the countermoves are performed by more than one DM. Hence the definition of unilateral moves by a group or coalition of DMs is needed.

Let $s \in S$ and $H \subseteq N, H \neq \emptyset$. Let $R_H(s)$ denote the set of all states that can be reached from s through any legal sequence of unilateral moves by some or all of the DMs in H . Note that a sequence of moves is considered as legal if no DM moves twice consecutively. For any $s_1 \in R_H(s)$, let $\Omega_H(s, s_1)$ denote the set of all last DMs in legal sequences from s to s_1 .

A *unilateral move* from s by H is a member of $R_H(s) \subseteq S$, defined inductively as follows:

1. If $k \in H$ and $s_1 \in R_k(s)$, then $s_1 \in R_H(s)$ and $k \in \Omega_H(s, s_1)$.
2. If $s_1 \in R_H(s), k \in H$, and $s_2 \in R_k(s_1)$, then:
 - (i) if $|\Omega_H(s, s_1)| = 1$ and $k \notin \Omega_H(s, s_1)$, then $s_2 \in R_H(s)$ and $k \in \Omega_H(s, s_2)$.
 - (ii) if $|\Omega_H(s, s_1)| > 1$, then $s_2 \in R_H(s)$ and $k \in \Omega_H(s, s_2)$.

The induction stops if no new state (s_2) is added to $R_H(s)$, and if there is no change from $|\Omega_H(s, s_1)| = 1$ to $|\Omega_H(s, s_1)| > 1$ for any $s_1 \in R_H(s)$.

Below is an algorithm to implement this definition. Let $R_H(s, i)$ denote the set of unilateral moves from state s by coalition H in the i th iteration, and $\Omega_H(s, s_1, i)$ denote the set of all last DMs in legal sequences from s to s_1 at the i th iteration. Then:

1. For $i = 1$,

$$R_H(s, 1) = \{s_1 : s_1 \in R_k(s) \text{ for some } k \in H\}.$$

$$\text{If } s_1 \in R_H(s, 1), \text{ then } \Omega_H(s, s_1, 1) = \{k : s_1 \in R_k(s)\}.$$

2. For $i > 1$,

$$R_H(s, i) = R_H(s, i - 1)$$

$$\cup \{s_2 : s_2 \in R_k(s_1) \text{ for some } s_1 \in R_H(s, i - 1),$$

$$\text{provided } |\Omega_H(s, s_1, i - 1)| > 1$$

$$\text{or } k \notin \Omega_H(s, s_1, i - 1)\}.$$

If $s_2 \in R_H(s, i)$, then:

$$\Omega_H(s, s_2, i) = \Omega_H(s, s_2, i - 1)$$

$$\cup \{k : s_2 \in R_k(s_1) \text{ for some } s_1 \in R_H(s, i - 1)\}.$$

Induction stops when $R_H(s, i) = R_H(s, i - 1)$ and for all $s_1 \in R_H(s, i)$,

$$\Omega_H(s, s_1, i - 1) = \Omega_H(s, s_1, i).$$

5. Fuzzy preferences in the graph model for conflict resolution

In the next subsection, a fuzzy preference structure is presented for employment within the Graph Model for Conflict

Resolution. Subsequently, fuzzy stability definitions are given for Nash and sequential stability. Detailed descriptions of the foregoing concepts, as well as the definitions for fuzzy general metarationality and fuzzy symmetric metarationality, are furnished by Bashar et al. [24,25].

5.1. Fuzzy preference framework within a graph model

Fuzzy relative strength of preference: Fuzzy preference captures preference uncertainty using numbers between 0 and 1, indicating a pairwise preference degree to which one state is preferred over the other. Fuzzy preference can be considered as an increasing function of preference degrees for which a larger preference degree means more likely preferred. A preference degree of 1.0 means a definite preference. When a preference degree is less than 1.0, it indicates that there is something that persuades the DM that either state of the pair may be preferred to the other, even if the DM “leans” towards one of the states. Thus if $r(s_i, s_j) < 1$, then there must be some reason for the DM not to definitely prefer state s_i to state s_j . Due to the additive reciprocity, the number $r(s_j, s_i) = 1 - r(s_i, s_j)$ can be interpreted as the degree to which state s_i is not preferred over state s_j . Hence, the concept of an uninterrupted preference intensity of one state over another is defined below.

Definition 5. Let $k \in N$, and let $r^k(s_i, s_j)$ denote the preference degree of state s_i over s_j for DM k . Then the k th DM's fuzzy relative strength of preference of state s_i over s_j , denoted $\alpha^k(s_i, s_j)$, is defined to be:

$$\alpha^k(s_i, s_j) = r^k(s_i, s_j) - r^k(s_j, s_i).$$

Note that $-1 \leq \alpha^k(s_i, s_j) \leq 1$ for any $k \in N$ and $i, j = 1, 2, \dots, m$. In particular:

1. $\alpha^k(s_i, s_j) = 1$ indicates that DM k definitely prefers state s_i to state s_j .
2. $\alpha^k(s_i, s_j) = 0$ means that DM k tends to be indifferent between states s_i and s_j .
3. $\alpha^k(s_i, s_j) = -1$ indicates that DM k definitely prefers state s_j to state s_i .

The number $\alpha^k(s_i, s_j)$ measures how strongly DM k prefers state s_i over state s_j . Denoting $\alpha^k(s_i, s_j) = \alpha_{ij}^k$ for any $i, j = 1, 2, \dots, m$, the k th DM's fuzzy relative strength of preference over S can be represented by matrix $(\alpha_{ij}^k)_{m \times m}$. Keep in mind that for any $k \in N$ and any $i, j = 1, 2, \dots, m$:

$$\alpha^k(s_j, s_i) = -\alpha^k(s_i, s_j).$$

Fuzzy satisfying threshold: In a graph model, every DM has to decide whether to stay at the current state or to move to an advantageous state. Different DMs may have different criteria to identify their advantageous states from a given state. In other words, every DM in a conflict may wish to satisfy a certain level of fuzzy relative strength of preference to decide whether to move to a reachable state. This level of fuzzy relative strength of preference is referred to as the fuzzy satisfying threshold of the DM.

Definition 6. The level of fuzzy relative strength of preference that would motivate a particular DM to move from one state to another is called the *Fuzzy Satisficing Threshold* (FST) of that DM. DM k 's fuzzy satisfying threshold is denoted by γ_k , or simply by γ , if there is no confusion about which DM is being considered.

The fuzzy satisfying threshold of a DM reflects his or her satisfying behavior in a strategic conflict. From Definition 5, one can find that the fuzzy satisfying threshold of a DM is a positive number that cannot exceed 1. That is for any $k \in N$, $0 < \gamma_k \leq 1$.

Fuzzy unilateral improvements: Once the fuzzy satisfying threshold of a DM is selected, the DM can identify his or her advantageous states (if any) from the current state. A fuzzy unilateral improvement from a given state, $s \in S$, for a DM, is a reachable state from s , such that the DM's fuzzy relative strength of preference of the state over s is not less than his or her fuzzy satisfying threshold. A fuzzy unilateral improvement signals a DM's attractiveness to move.

Definition 7. Let $s \in S$ and $k \in N$, and let γ be the fuzzy satisfying threshold of DM k . A state $s_i \in R_k(s)$ is called a *fuzzy unilateral improvement* from s for DM k , if:

$$\alpha^k(s_i, s) \geq \gamma.$$

Recall that $R_k(s)$ is the set of reachable states from s for DM k .

Definition 8. The set of all fuzzy unilateral improvements from state s for DM k is called the *fuzzy unilateral improvement list* from s for DM k , and is denoted by $\tilde{R}_{k,\gamma}^+(s)$ or simply by $\tilde{R}_k^+(s)$. Mathematically:

$$\tilde{R}_{k,\gamma}^+(s) = \{s_i \in R_k(s) : \alpha^k(s_i, s) \geq \gamma\}.$$

The Graph Model (fuzzy) stabilities reflect a DM's unwillingness to move from the current state. A DM may be indisposed to move because the reachable states are disadvantageous, or moving to each advantageous state leads to an unfavorable outcome due to the responses (countermoves) by the opponents. Note that the responses could be a unilateral move or a fuzzy unilateral improvement. The fuzzy unilateral improvement and fuzzy unilateral improvement list defined above are for individual DMs. However, when the number of DMs in a conflict is more than two, the opponents of the focal DM is a coalition or group of two or more DMs. Therefore, it is also necessary to define fuzzy unilateral improvements from a given starting state for a coalition or group of DMs.

Definition 9. Let $s \in S$ and $H \subseteq N, H \neq \emptyset$. Let $\tilde{R}_H^+(s)$ denote the set of all fuzzy unilateral improvements from s by H , and for any $s_1 \in \tilde{R}_H^+(s)$, let $\tilde{\Omega}_H^+(s, s_1)$ denote the set of all last DMs in legal sequences from s to s_1 . A *fuzzy unilateral improvement* from s by H is a member of $\tilde{R}_H^+(s)$ defined inductively as follows:

1. If $k \in H$ and $s_1 \in \tilde{R}_k^+(s)$, then $s_1 \in \tilde{R}_H^+(s)$ and $k \in \tilde{\Omega}_H^+(s, s_1)$.
2. If $s_1 \in \tilde{R}_H^+(s)$, $k \in H$, and $s_2 \in \tilde{R}_k^+(s_1)$, then:
 - (i) if $|\tilde{\Omega}_H^+(s, s_1)| = 1$ and $k \notin \tilde{\Omega}_H^+(s, s_1)$, then $s_2 \in \tilde{R}_H^+(s)$ and $k \in \tilde{\Omega}_H^+(s, s_2)$.
 - (ii) if $|\tilde{\Omega}_H^+(s, s_1)| > 1$, then $s_2 \in \tilde{R}_H^+(s)$ and $k \in \tilde{\Omega}_H^+(s, s_2)$.

The induction stops if no new state (s_2) is added to $\tilde{R}_H^+(s)$, and if there is no change from $|\tilde{\Omega}_H^+(s, s_1)| = 1$ to $|\tilde{\Omega}_H^+(s, s_1)| > 1$ for any $s_1 \in \tilde{R}_H^+(s)$.

5.2. Fuzzy stabilities for a graph model

The concept of fuzzy stability is incorporated into the graph model to accommodate fuzzy preferences. Fuzzy stability implies a reluctance to move from a state reflecting a particular behavioral pattern. Keep in mind that different DMs may show different behavioral patterns in responding to a strategic conflict. Rational DMs usually move and countermove according to fuzzy unilateral improvements. Believing that most DMs are rational, fuzzy Nash stability and fuzzy sequential stability are considered to cover a majority of DMs' behavioral patterns in practical real world conflicts. Hence fuzzy Nash stability and fuzzy sequential stability are defined below.

Definition 10 (Fuzzy Nash Stability). Let $k \in N$ and $s \in S$. State s is said to be *fuzzy stable*, or *fuzzy Nash stable*, or *fuzzy rational* (FR) for DM k if and only if:

$$\tilde{R}_k^+(s) = \emptyset.$$

Under FR stability, DM k only cares about his or her own potential fuzzy unilateral improvements, without taking into account possible responses by the opponents. Thus state s is FR stable for DM k if and only if DM k has no fuzzy unilateral improvements from s .

Definition 11 (Fuzzy Sequential Stability). A state $s \in S$ is said to be *fuzzy sequentially stable* (FSEQ) for DM k if and only if for every $s_1 \in \tilde{R}_k^+(s)$ there exists an $s_2 \in \tilde{R}_{N-k}^+(s_1)$, such that $\alpha^k(s_2, s) < \gamma_k$.

For FSEQ, the focal DM contemplates whether each of his or her potential fuzzy unilateral improvements is sanctioned by the opponents using a fuzzy unilateral improvement. If the focal DM has no fuzzy unilateral improvements from the current state, the state is FR stable, which implies that it is also FSEQ stable.

Definition 12. *Fuzzy Equilibrium*: A state $s \in S$ that is fuzzy stable for all DMs under a specific fuzzy stability definition is called a *fuzzy equilibrium* (FE) under that definition.

A fuzzy equilibrium is regarded as a possible resolution of the conflict. Note that different DMs may have different fuzzy satisficing thresholds in identifying their own fuzzy stable states.

6. Application of the graph model fuzzy preference framework to strategic decisions in the lake Gisborne water export conflict

Lake Gisborne is located near the south coast of Newfoundland, Canada, approximately 10 kilometers upstream from the nearest community, Grand Le Pierre, a small town. The sources of water of the lake are mainly melted snow and rainfall. The water supply is plentiful and pure.

In 1995, Canada Wet Incorporated, a division of the McCurdy Group of Companies in Newfoundland, proposed a project to export bulk water from Lake Gisborne. The project included the construction of a water supply intake, a pipeline and an access road along the pipeline route, a marine loading facility for Ultra Large Crude Carrier vessels and a bottling plant in an accessible area close to the pipeline. Maximum water usage was estimated to be 300,000 cubic meters per week [62]. As a result of possible economic benefits, the Provincial Government of Newfoundland and Labrador registered the project.

Although at the time of the request there was no policy on bulk water exports, the provincial Department of the Environment and Labour, in conjunction with the Departments of Industry, Trade and Technology, and Justice, developed a new policy which was approved by the Cabinet in September, 1996. Under this policy, there was a strong possibility that the proposals for the exportation of water might be accepted.

The opposition and many environmental lobby groups criticized the province's water export policy. They argued that the policy would take the province closer to the export of water in bulk quantities from Lake Gisborne. The environmentalists claimed that permitting water from Lake Gisborne to be sold in bulk would make Canadian water a "commodity" which would allow any other company to bring

similar projects under the terms and conditions of World Trade Organization (WTO) and North American Free Trade Agreement (NAFTA) [12]. Removal of bulk water may cause unpredictable and harmful consequences to basin habitat, biodiversity, shorelines, jobs and culture, particularly to First Nations. Disregarding these environmental and other harmful consequences, several groups supported the project, with the hope of an economic development of job-poor Newfoundland, especially the small community of Grand Le Pierre.

To protect its natural resources and save the basin habitat from possible environmental disaster, the Federal Government of Canada announced its water export policy on February 10, 1999. This policy prohibited bulk water removal from major drainage basins in Canada. The Provincial Government of Newfoundland and Labrador endorsed and supported a permanent national ban on bulk water export projects, except for Lake Gisborne. (see Figure 1) But facing tremendous pressure from the Federal Government and lobby groups, the Provincial Government introduced new legislation prohibiting bulk water removal from Newfoundland and Labrador, and consequently the Gisborne Water Export project was stopped [63].

Early in 2001, Roger Grimes, the new Premier of Newfoundland and Labrador, revived the Lake Gisborne water export project, with a view to generate some badly needed cash for the province. He initiated a review of the project and thought that there was a good chance Newfoundland might go it alone. But the idea drew criticism across the province from politicians, environmentalists, students and others. In October 2001, Justice Minister, Kelvin Parsons, announced that the government would not introduce legislation to remove the ban on bulk water exports during the upcoming session of the legislature [64]. One reason was the low water price in international markets at that time.

By studying the background and consequences of the Lake Gisborne water export dispute, one can see that it is dynamic in nature. The water export was an issue at different points in time. Fang et al. [65] carried out a static analysis of the dispute for October, 1999, using the Graph Model for Conflict Resolution. Although the water export project is currently stopped, when the water price in the world trade market rises so that it is possible to make high profits from water exports, the Government of Newfoundland and Labrador may initiate a more thorough analysis to reach another conclusion in favor of exporting water in bulk from Lake Gisborne. Following a summary of the study of Fang et al. [65], Hipel et al. [66] investigated the strategic aspects of water exports from Lake Gisborne at a future time when the price of water is high.

Table 1 represents DMs and their options in the Gisborne conflict as of October, 1999, where the status quo situation is shown in the right column. Here, a "Y" indicates that the option against it is selected by the DM controlling it, and "N" means that the option is not taken. As found in [65], the evolution of the dispute from the status quo state (s_{12}) to the equilibrium state (s_6) is shown in Table 2. The arrows indicate the option changes that took place to cause the conflict to progress from one state to another. The final state in this sequence (the equilibrium state s_6) represents the situation where the Federal Government proposed a Canada wide accord on the prohibition of bulk water removal from lake basins, Newfoundland Provincial Government supported a full prohibition on bulk water removal, Canadian Support appealed to continue the Gisborne project and the Canadian opposition did not petition, as their major demand was satisfied. This is what actually happened when the Gisborne water export project was stopped.

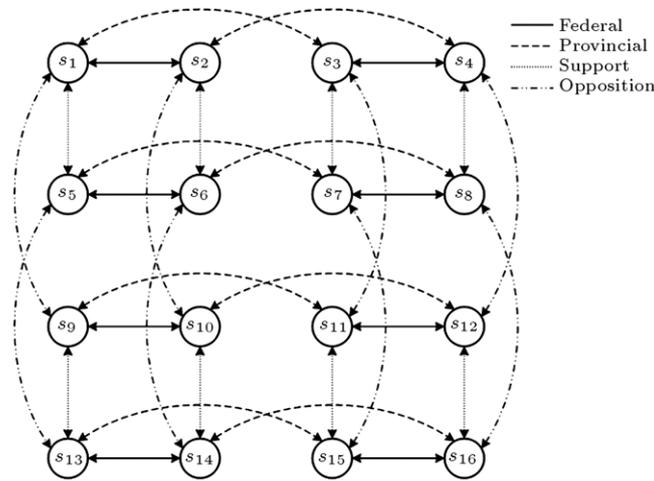


Figure 1: Unilateral moves in the future Lake Gisborne conflict.

Table 1: Decision makers and options in the Gisborne conflict as of October, 1999.

Decision makers	Options	Status quo
Federal Government (Federal)	1. Propose a Canada wide accord on prohibition of bulk water removals (Prohibition)	Y
Provincial Government of Newfoundland and Labrador (Provincial)	2. Full prohibition on bulk water removal from lake basins (Full) 3. Prohibition on bulk water removal from lake basins except the Gisborne project (Exception)	N Y
Canadian Support (Support)	4. Appeal to continue Gisborne project (Continue) 5. Appeal to seek compensation (Compensation)	N N
Canadian Opposition (Opposition)	6. Petition for introducing new legislation to prevent water export (Petition)	Y

Table 2: Evolution from the status quo state to equilibrium in the Gisborne conflict as of October, 1999.

Decision makers and options	Status quo state	Intermediate states	Equilibrium state
Federal			
1. Prohibition	Y	Y	Y
Provincial			
2. Full	N →	Y	Y
3. Exception	Y	N	N
Support			
4. Continue	N	N	N →
5. Compensation	N	N	N
Opposition			
6. Petition	Y	Y →	N
State	s_{12}	s_{10}	s_2

The background of the Gisborne water export conflict, as well as the evolution shown in Table 2, reveal that the Provincial Government of Newfoundland and Labrador is the key player in this conflict. During the evolution of the conflict, this DM changed its strategy on a number of occasions. Moreover, Table 2 reflects the fact that the historical result (or equilibrium) was controlled by the Provincial Government’s strategy to either fully prohibit the removal of bulk water from lake basins or not.

In this paper, a fuzzy preference framework of the graph model is employed to study the Lake Gisborne water export conflict for a future time. The DMs and their options are considered to be the same as those utilized by Hipel et al. [66]. In particular, the DMs in this future conflict are the Federal Government of Canada (Federal), Provincial Government of

Newfoundland and Labrador (Provincial), Canadian Support (Support) representing the group that believes the province would benefit from water exports, and the Canadian Opposition (Opposition) standing for the group consisting of the oppositions in the Parliament, environmental lobby groups and many others who oppose the project. The DMs of the Gisborne conflict in the present study, as well as in [66], are the same as in [65]; however, the DMs’ options are a little different, reflecting the status of the dispute after October, 1999. Table 3 lists the DMs the available movements between states by the DMs in the Gisborne water export conflict at some future date after October, 1999.

The study of the Gisborne water export conflict, based on a situation in the future when the water price is high, executed by Hipel et al. [66], identified an equilibrium or resolution different from the one in [65]. The final equilibrium (state s_{10} in Tables 4 and 5) represents the situation where the Newfoundland Provincial Government disregards the Federal Government’s proposition to prohibit bulk water removal from basins and the Opposition’s petition. The evolution from the status quo state to this equilibrium is displayed in Table 5. It shows that the Provincial Government, again, plays a key role in reaching this equilibrium by changes in its strategy. The high water price in the global market influences the Newfoundland Provincial Government in ignoring the Federal Government’s decision, which is reflected in its preferences over the feasible states. The Provincial Government most prefers states not maintaining the ban on bulk water removal over others (see [66] for details).

The Federal Government of Canada is the highest authority to protect Canada’s natural resources. Therefore, at any time, the Federal Government can compel the Provincial Government

Table 3: Decision makers and options in the Gisborne conflict at a future date.

Decision makers	Options	Status quo
Federal Government of Canada (Federal)	1. Continue a Canada wide accord on prohibition of bulk water removals (Prohibition)	Y
Provincial Government of Newfoundland and Labrador (Provincial)	2. Maintain the ban on bulk water removal from lake basins (Maintain)	Y
Canadian Support (Support)	3. Appeal for continuing the Gisborne project based on the NAFTA (Appeal)	N
Canadian Opposition (Opposition)	4. Petition for prohibition on water export (Petition)	N

Table 4: Feasible states in the Gisborne conflict at a future date.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
Federal																
1. Prohibition	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
Provincial																
2. Maintain	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
Support																
3. Appeal	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
Opposition																
4. Petition	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
States	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}

Table 5: Evolution from the status quo state to equilibrium in the future Gisborne conflict as studied in [66].

Decision makers and options	Status quo state	Intermediate states	Equilibrium state
Federal			
1. Prohibition	Y	Y	Y
Provincial			
2. Maintain	Y	Y → N	N
Support			
3. Appeal	N → Y	Y → N	N
Opposition			
4. Petition	N	N	N → Y
State	s_4	s_8	s_{10}

to comply with its accord on the prohibition of bulk water export. Hence for the Newfoundland Provincial Government, the states where the Federal Government continues a Canada wide accord, and the Provincial Government does not maintain

a ban on bulk water removal, represent a direct conflict with the Federal Government. Although the Provincial Government does not want to maintain the ban on bulk water removal, it prefers not to be in direct conflict with the Federal Government. Therefore, for the analysis of the conflict at any point in time after October, 1999, the authors of this paper recognize a preference uncertainty between some states for the Newfoundland Provincial Government. For example, the Provincial Government may not definitely prefer state s_2 (which represents the Federal Government's wish to continue a Canada wide accord, while the Provincial Government does not maintain the ban on bulk water removal) over state s_4 (which represents that the Federal Government is continuing with a Canada wide accord and the Provincial Government maintains the ban). As discussed earlier, a fuzzy preference relation may well represent this type of preference.

The matrix $\mathcal{R}^{\text{Provincial}}$ under Table 6 represents the fuzzy preferences for the Newfoundland Provincial Government. An

Table 6: Matrix $\mathcal{R}^{\text{Provincial}}$: fuzzy preferences of the Newfoundland Provincial Government.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
s_1	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
s_2	0	0.5	0.8	0.7	0	0.1	0.8	0.7	0	1.0	0.9	0.8	0	0.2	0.8	0.9
s_3	0	0.2	0.5	0.8	0	0	0.7	0.9	0	0.3	0.7	0.8	0	0.3	0.8	0.9
s_4	0	0.3	0.2	0.5	0	0.1	0.3	0.2	0	0.2	0.3	1.0	0	0.1	0.2	0.4
s_5	0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
s_6	0	0.9	1.0	0.9	0	0.5	0.9	0.8	0	0.8	0.9	0.7	0	0.9	0.8	0.9
s_7	0	0.2	0.3	0.7	0	0.1	0.5	0.8	0	0.4	0.8	0.9	0	0.2	0.8	0.7
s_8	0	0.3	0.1	0.8	0	0.2	0.2	0.5	0	0.3	0.3	0.8	0	0.1	0.3	1.0
s_9	0	1.0	1.0	1.0	0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0
s_{10}	0	0	0.7	0.8	0	0.2	0.6	0.7	0	0.5	0.8	0.8	0	0.3	0.8	0.8
s_{11}	0	0.1	0.3	0.7	0	0.1	0.2	0.7	0	0.2	0.5	0.8	0	0.1	0.4	0.8
s_{12}	0	0.2	0.2	0	0	0.3	0.1	0.2	0	0.2	0.2	0.5	0	0.1	0.3	0.1
s_{13}	0	1.0	1.0	1.0	0	1.0	1.0	1.0	0	1.0	1.0	1.0	0.5	1.0	1.0	1.0
s_{14}	0	0.8	0.7	0.9	0	0.1	0.8	0.9	0	0.7	0.9	0.9	0	0.5	0.8	0.9
s_{15}	0	0.2	0.2	0.8	0	0.2	0.2	0.7	0	0.2	0.6	0.7	0	0.2	0.5	0.7
s_{16}	0	0.1	0.1	0.6	0	0	0.3	0	0	0.2	0.2	0.9	0	0.1	0.3	0.5

Table 7: Preferences of Federal, Support and Opposition in the Gisborne conflict at a future date.

DMs	Ranking of states from most preferred on the left to least preferred on the right															
Federal	s_4	s_{12}	s_8	s_{16}	s_{10}	s_2	s_{14}	s_6	s_3	s_{11}	s_7	s_{15}	s_9	s_1	s_{13}	s_5
Support	s_1	s_9	s_2	s_{10}	s_5	s_{13}	s_6	s_{14}	s_7	s_{15}	s_8	s_{16}	s_3	s_{11}	s_4	s_{12}
Opposition	s_{12}	s_4	s_8	s_{16}	s_{10}	s_2	s_{14}	s_6	s_3	s_{11}	s_{15}	s_7	s_9	s_1	s_{13}	s_5

Table 8: Fuzzy stability results for the future Lake Gisborne conflict.

FST	States	FR				FE	FSEQ				FE						
		Federal	Provincial	Support	Opposition		Federal	Provincial	Support	Opposition							
$\gamma_{\text{Federal}} = 1.0$ $\gamma_{\text{Provincial}} = 0.6$ $\gamma_{\text{Support}} = 1.0$ $\gamma_{\text{Opposition}} = 1.0$	s_1		✓	✓				✓									
	s_2	✓	✓	✓			✓	✓	✓								
	s_3		✓					✓						✓			
	s_4	✓	✓				✓	✓						✓			
	s_5		✓					✓						✓			
	s_6	✓	✓					✓	✓					✓			
	s_7				✓				✓				✓	✓			
	s_8	✓			✓	✓		✓	✓	✓	✓	✓	✓	✓	✓		✓
	s_9		✓		✓	✓			✓	✓	✓	✓	✓	✓	✓		
	s_{10}	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	s_{11}														✓		
	s_{12}	✓						✓							✓		
	s_{13}		✓							✓					✓		
	s_{14}	✓	✓					✓	✓						✓		
	s_{15}					✓							✓	✓	✓		
	s_{16}	✓				✓			✓				✓	✓	✓		
$\gamma_{\text{Federal}} = 1.0$ $\gamma_{\text{Provincial}} = 0.8$ $\gamma_{\text{Support}} = 1.0$ $\gamma_{\text{Opposition}} = 1.0$	s_1		✓	✓					✓		✓						
	s_2	✓	✓	✓				✓	✓	✓							
	s_3										✓				✓		
	s_4	✓	✓					✓	✓	✓					✓		
	s_5		✓								✓				✓		
	s_6	✓	✓						✓	✓					✓		
	s_7				✓					✓			✓	✓	✓		
	s_8	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	s_9		✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓		
	s_{10}	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	s_{11}														✓		
	s_{12}	✓	✓					✓	✓	✓					✓		
	s_{13}		✓							✓					✓		
	s_{14}	✓	✓					✓	✓						✓		
	s_{15}					✓						✓	✓	✓	✓		
	s_{16}	✓				✓			✓			✓	✓	✓	✓		

FST: fuzzy satisfying threshold.
 FE: fuzzy equilibrium.

entry in $\mathcal{R}^{\text{Provincial}}$, for instance 0.7, in the second row and fourth column, stands for the Provincial Government's preference degree of state s_2 over state s_4 . The preferences of the Federal Government, Canadian Support and Canadian Opposition are considered to be the same as in [66], which are given in Table 7 where the states are ranked from most to least preferred for each of the DMs.

A fuzzy stability analysis is carried out for fuzzy Nash stability and fuzzy sequential stability, defined in Definitions 10 and 11, respectively, the results of which are shown in Table 8. For this analysis, two sets of fuzzy satisfying thresholds are considered:

- (i) $\{\gamma_{\text{Federal}} = 1.0, \gamma_{\text{Provincial}} = 0.6, \gamma_{\text{Support}} = 1.0, \gamma_{\text{Opposition}} = 1.0\}$,
- (ii) $\{\gamma_{\text{Federal}} = 1.0, \gamma_{\text{Provincial}} = 0.8, \gamma_{\text{Support}} = 1.0, \gamma_{\text{Opposition}} = 1.0\}$.

It can be seen that the fuzzy satisfying thresholds of the DMs are the same, except for the Provincial Government. A (✓) in a cell in the table indicates that the state is fuzzy stable for the indicated DM or a fuzzy equilibrium under the indicated fuzzy stability definition, for a particular set of fuzzy satisfying thresholds of the DMs.

Table 9: Evolution from the status quo state to fuzzy equilibrium s_8 in the future Gisborne conflict (when $\gamma_{\text{Provincial}} = 0.8$).

Decision makers and options	Status quo state	Fuzzy equilibrium state
Federal		
1. Prohibition	Y	Y
Provincial		
2. Maintain	Y	Y
Support		
3. Appeal	N	→ Y
Opposition		
4. Petition	N	N
State	s_4	s_8

When $\gamma_{\text{Provincial}} = 0.6$, state s_{10} is fuzzy equilibrium for both FR and FSEQ. It represents the same final equilibrium as in [66]. State s_{10} can be reached from the status quo, state s_4 , by state transitions in the same way as shown in Table 5. In this case, while the Federal Government continues a Canada wide accord, the Provincial Government's fuzzy satisfying threshold allows some states, representing the Provincial Government's not maintaining the ban, to be a fuzzy unilateral improvement

Table 10: Evolution from the status quo state to fuzzy equilibrium s_{10} in the future Gisborne conflict (when $\gamma_{\text{Provincial}} = 0.8$).

Decision makers and options	Status quo state		Intermediate states		Fuzzy equilibrium state
Federal					
1. Prohibition	Y	Y	Y	Y	Y
Provincial					
2. Maintain	Y	Y	Y	→ N	N
Support					
3. Appeal	N	N	→ Y	Y	→ N
Opposition					
4. Petition	N	→ Y	Y	Y	Y
State	s_4	s_{12}	s_{16}	s_{14}	s_{10}

for the Provincial Government over states maintaining the ban. For example, state s_6 is a fuzzy unilateral improvement from state s_8 for the Provincial Government. This case represents the Newfoundland Provincial Government's aggressiveness to complete the Lake Gisborne water export project.

When $\gamma_{\text{Provincial}} = 0.8$, states s_8 and s_{10} are found to be fuzzy equilibria under both *FR* and *FSEQ*. The two evolutions from the status quo state to fuzzy equilibria, s_8 and s_{10} , are shown in Tables 9 and 10, respectively. As can be seen in Table 10, the fuzzy equilibrium, s_{10} , is reached from the status quo state through an evolution path different from the one in Table 5. This is because state s_6 is no longer a (fuzzy) unilateral improvement from state s_8 for the Provincial Government. Starting at the status quo state, s_4 , the Canadian Opposition can invoke a unilateral move to state s_{12} , by launching a petition for prohibition on water exports. Then, the Canadian Support can cause a unilateral move to state s_{16} by appealing for continuing the Gisborne water export project. The Provincial Government then makes a unilateral move to state s_{14} , by removing the ban on water exports. Finally, the Canadian Support stops its appeal. The state transitions in Table 10 make it clear that the Newfoundland Provincial Government may disregard the Federal Government's proposition and the Canadian Opposition's petition to implement the Gisborne water export project.

In Table 9, starting at state s_4 , the Canadian Support makes a unilateral move to fuzzy equilibrium s_8 by appealing for continuing the Gisborne water export project. State s_8 represents the situation in which the Federal Government continues a Canada wide accord on the prohibition of bulk water removal from lake basins, the Newfoundland Provincial Government maintains the ban on bulk water removal, and the Canadian support appeals for continuing the Gisborne water export project. This can happen at any point in time in the future. Moreover, the fuzzy equilibrium, s_8 (in Table 9), is equivalent to the equilibrium state, s_6 (in Table 2), as found by Fang et al. [65], based on the conflict, as of October, 1999. It can also be seen from Table 8 that s_8 is a *FSEQ* equilibrium when $\gamma_{\text{Provincial}} = 0.6$. This indicates that the fuzzy stability analysis predicts a realistic resolution of the dispute for any future time, which the crisp analysis fails to determine.

7. Conclusions

The fuzzy preference framework of the graph model opens the opportunity to analyze multiple-participant multiple-objective decision models when the DMs' preferences are not certain. It offers flexibility for analysts and researchers to study conflicts in two steps:

(i) Identify uncertain preferences (if any) for a DM and model them as a fuzzy preference relation.

(ii) Choose a suitable fuzzy satisficing threshold for the DM.

The methodology also allows each DM to supply his or her very own fuzzy satisficing threshold. This flexibility makes the strategic investigation of a multiple-participant conflict, using the fuzzy preference framework of the graph model, more realistic. Moreover, an analyst may extract valuable information by studying the change in stabilities for various fuzzy satisficing thresholds, which may help guide the DMs toward a win-win resolution. A fuzzy preference framework is applicable for conflict decision making, not only for a case in which DMs have fuzzy preferences over the states, but also when DMs have crisp preferences over the states. In fact, crisp stabilities are special cases of fuzzy stabilities. This significantly broadens the applicability of the Graph Model for Conflict Resolution.

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