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Why do we live in $3+1$ dimensions?

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Abstract

In the context of string theory we argue that higher-dimensional Dp-branes unwind and evaporate so that we are left with D3-branes embedded in a $(9+1)$-dimensional bulk. One of these D3-branes plays the rôle of our Universe. Within this picture, the evaporation of the higher-dimensional Dp-branes provides the entropy of our Universe.

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1. Introduction

One of the open questions in modern cosmology is the dimensionality of spacetime. Why do we live in a Universe of $(3+1)$ dimensions? The question why the spatial dimension is not lower than 3 can be answered quite satisfactorily with a weak form of the anthropic principle: if it were, then there would be no intelligent life around to ask the question. But why is it not higher, e.g. 4, 5 or 42?

In this treatise we do not want to address the question in its full generality, but we restrict ourselves to string theory. We assume that a 10-dimensional superstring theory (type I, IIA or IIB; not a heterotic type theory since it does not have D-branes) be the correct description of the physical world. We also assume that the theory lives on a 9-dimensional spatial torus, the tenth dimension being time.

So far two possibilities to reduce the 10 dimensions from string theory to the 4 dimensions of the observed spacetime have been under discussion.

(i) The Kaluza–Klein approach, where the 6 extra-dimensions are rolled up in a small torus (or more generically a Calabi–Yau manifold) with a size

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given by the string scale $\sqrt{\alpha'}$ which is much smaller than all scales probed in the laboratory.

(ii) The braneworld approach, where our observed Universe represents a 3-brane on which open strings can end (Dirichlet-brane or D-brane, see Ref. [1]). Since gauge charges are attached to the ends of strings, gauge particles and fermions can propagate only along the 3-brane while gravitons (and dilatons, ...) which are closed string modes can move in the bulk. Since gravity has been probed only down to scales of about 0.1 mm, the dimensions of the bulk can much larger than the string scale.

In the braneworld context, the extra-dimensions can even be infinite, if the geometry is non-trivial and they are warped [2]. Large extra-dimensions can be employed to address the hierarchy problem [3]. This and other attractive features have led to a growing body of literature on braneworld models and their astrophysical and cosmological consequences [4,5].

In both approaches, the number of large spatial dimensions is set equal to 3 just in order to agree with observations, but without physical motivation. The first argument why the number of large spatial dimensions should be three has been made within the Kaluza–Klein approach by Brandenberger and Vafa [6]. They have argued that, when allowing strings to wind around a 9-torus, they intersect and unwind only inside a 3-dimensional sub-manifold, so that only three dimensions can grow large while the other 6 are held back by strings wrapping around them. This hypothesis has been verified numerically by Sakellariadou [7].

Here we argue within the braneworld approach. We claim that due to intersections leading to reconnection and unwinding, all Dp-branes of dimension $p > 3$ disappear; they are unstable. One of the stable 3-branes plays the rôle of our Universe. We focus our discussion on type IIB string theory, but our results could also hold for any other version of string theory which allows for 3-branes.

The picture we have in mind is that at very early times space was potentially large and filled with Dp-branes and $\bar{\mathrm{D}}p$-anti-branes of all possible dimensions p. A brane differs from its anti-brane by possessing the opposite Ramond–Ramond charge. Since the charge of a Dp-brane corresponds to an orientation, a $\bar{\mathrm{D}}p$-anti-brane is a Dp-brane rotated by π. We postulate that at very early times, high, as well as low, dimensional branes fill space.

We investigate the intersections between Dp-branes of various dimensionality p, embedded in a $(9+1)$-dimensional toroidal bulk. In Section 2, we first state the condition under which two Dp-branes intersect. For simplicity we disregard intersection between two branes of different dimensionality. We then argue that brane intersections will eventually lead to evaporation of Dp-branes with $p > 3$ into gravitons and dilatons, and a system of D3-branes (and possibly D2- and D1-branes). One of the D3-branes could become our Universe. In the process of 'evaporation' of the higher-dimensional branes, the entropy of the Universe increases. We state our conclusions as well as some remarks in Section 3.

2. Brane intersections

We consider a uniform distribution of Dp-branes embedded in a higher-dimensional bulk. We denote the spacetime dimension of the bulk by d. We want to set the condition for the intersection of two Dp-branes. A Dp-brane is p-dimensional with p taking any even value in the IIA theory, any odd value in the IIB theory, and the values 1, 5, and 9 in the type I theory. We assume that branes at macroscopic distances do not interact. Their intersection probability is then purely a question of dimensionality, and the following statements are true with probability 1, i.e., always except for branes which accidentally have one or several parallel directions. If $2p \geqslant d-1$, then the two Dp-branes intersect at all times on an intersection-manifold of dimension $2p-d+1$, while if $2p+1 = d-1$ then the two Dp-branes intersect in an 'event', i.e., they eventually intersect at some time t_c in a point. However, if $2p+1 < d-1$, the two Dp-branes will generically never intersect. Thus, the condition for generic intersection of two Dp-branes embedded in a d-dimensional spacetime (the bulk) is

$$2p+1 \geqslant d-1. \tag{1}$$

Let us consider the case of $d = 10$. Then according to the condition above, two Dp-branes will never intersect if $p \leqslant 3$, but they will eventually collide provided $p \geqslant 4$.

The simple dimensional condition (1) for the intersection of generic Dp-branes has also been mentioned in Refs. [9,10]. But there, the authors do not conclude that this fact leads to the disappearance of higher dimension branes. In Ref. [9] they argue that the density of higher-dimensional branes is exponentially suppressed for entropic reasons and in Ref. [10] they even say that due to the inter-brane potential, a simple dimensional argument is not valid.

We now want to argue that intersecting branes are unstable and eventually evaporate so that we are left with D3-branes (and any permitted lower-dimensional branes, D1-branes in type IIB theory). This is the main point of the present Letter. For our argument we need the following hypotheses:

(1) We assume that the 9 bulk coordinates are compactified on a torus. Closed branes which do not wind around the torus shrink and disappear emitting gravity waves (and dilatons or other closed string modes). We call this process evaporation.
(2) If a Dp-brane intersects with another Dp-brane on a hypersurface of dimension $p-1$ the branes reconnect, this means one side of the first brane reconnects with the other side of the second brane and vice versa (see Figs. 1 and 2). In this way their winding number is reduced until they finally do not wind anymore and thus can evaporate.
(3) If two Dp-branes intersect on a manifold of dimension lower than $p-1$ the open strings which switch between the branes lead to an alignment/anti-alignment of the directions with the smallest respective opening angle (see Fig. 3). This process continues until the intersection manifold has dimension $p-1$ and the branes can reconnect and separate again.
(4) We assume that the total winding number of all branes of a given dimensionality vanishes.

Let us briefly comment on each of these hypotheses. Point (1) is quite natural. A simple entropy argument implies that a state of many gravitons is entropically favored over a state with a brane. If it is not topologically forbidden, evaporation will therefore take place. This process leads to entropy production in the bulk.

Point (2) seems also unproblematic. It has been verified numerically for Nambu–Goto strings in [7]. More

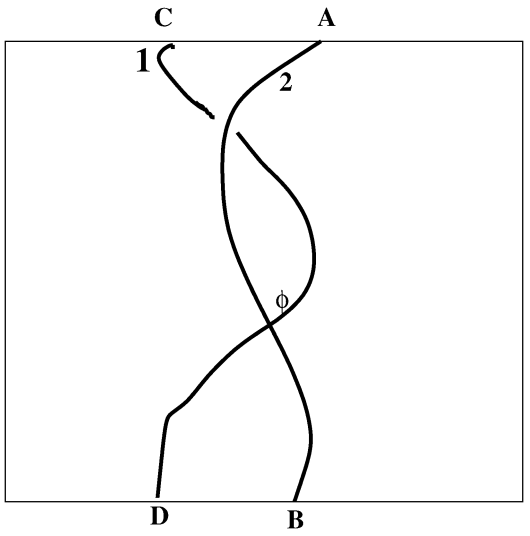

Fig. 1. The projection of two Dp-branes, denoted by 1 and 2, which intersect along a $p-1$ dimensional manifold along dimensions omitted in this figure. They intersect in the point Φ. We choose periodic boundary conditions. In a toroidal geometry, point A is identified with B, and point C with D.

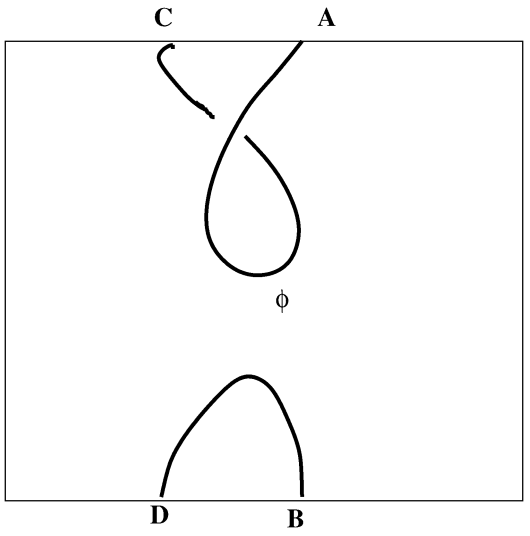

Fig. 2. The new Dp-brane which results from the intersection of the two D-branes shown in Fig. 1. With respect to the directions shown in the figure, it no longer winds around the torus.

realistically, the branes might have some reconnection probability $P < 1$ but this does not change our argument qualitatively. For intersecting D-strings at an angle ϕ, it has been shown analytically in the low energy limit [8], that there is a tachyon mode which represents

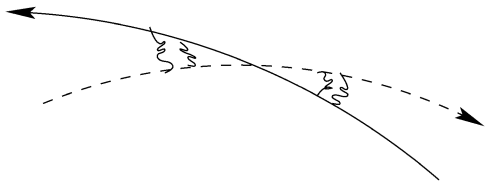

Fig. 3. Schematic representation of two intersecting branes and the open strings which are attached to both of them. They lead to an anti-alignment of the two branes.

the instability to reconnection. Dp-branes which intersect in $p-1$ directions can be reduced to this case by applying T-duality in the $p-1$ common directions.

Point (3) is a crucial assumption for our scenario to work. We sketch here the reasons for which we expect that this assumption does hold.

If branes are parallel and have vanishing relative velocity, some of the supersymmetries are preserved. In this case, the Ramond–Ramond repulsion cancels exactly the gravitational and dilaton attraction; the potential energy is zero. But in the general case, and even more importantly in the case of several dynamically moving branes, one expects that Dp-branes are at general angles to each other, implying that all supersymmetries are broken [1]. In this case, there will be an angle-dependent (or equivalently, velocity-dependent) force acting on the branes. This force corresponds to fundamental strings attached to both branes. For non-zero angles, and intersecting branes, these open strings will be confined to the region near the intersection (see Fig. 3).

At low energy, the interaction between branes can be described by a well-known interaction potential, which comes from the scattering amplitude of open strings which end on the two branes (which can also be viewed as closed strings travelling from one brane to the other, the cylinder amplitude). In this case, the force between the branes is simply the gradient of the interaction potential. In Ref. [1] the potential is given for the case of two D4-branes at some minimal separation y.

In the simplest example of only one non-vanishing angle between the two D4-branes, the lightest excitation has the energy (mass) [1]

$$m^2 = \frac{y^2}{4\pi^2\alpha'^2} - \frac{\phi}{2\pi\alpha'} \quad \text{with } 0 < \phi \leqslant \pi, \tag{2}$$

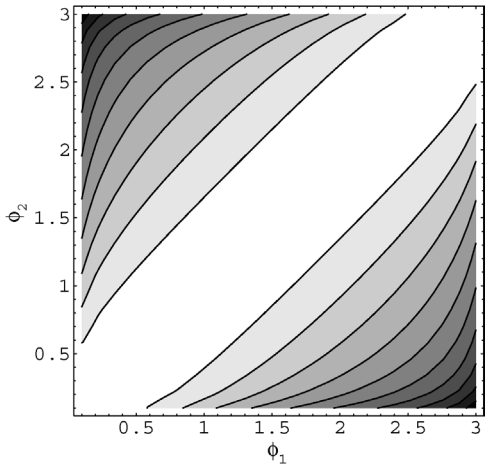

Fig. 4. The interaction potential $V(\phi_1, \phi_2)$ for two D4-branes which intersect on a plane and have two directions which are not aligned. The diagonals $\phi_1 = \phi_2$ and $\phi_1 = \pi - \phi_2$ are symmetry axes of the potential. The potential is exactly zero for $\phi_1 = \phi_2$ and negative everywhere else. A configuration initially in one of the four quadrants will always move to the closest boundary of the plot, which corresponds to an alignment or anti-alignment in one of the directions.

where y is the closest separation between the four branes. When the branes come close, this mode becomes tachyonic (negative m^2) indicating an instability. When $\phi = \pi$, the branes are anti-aligned and form a D4-brane/anti-D4-brane configuration which will annihilate. But even if the branes are nearly aligned, $\phi \ll \pi/2$, a tachyonic mode appears once their separation y is small enough, $y \ll \sqrt{\alpha'}$. The branes can lower their energy by reconnection, which will eventually lead to unwinding. This confirms again our point (2). In this case, the branes are already aligned in 3 directions, i.e., they intersect on a $3(= 4-1)$-dimensional sub-manifold and can thus reconnect.

The potential for the more interesting case of two angles is shown in Fig. 4. As one can see, a D4-brane initially at angles ϕ_1, ϕ_2 prefers to align the smaller of the two angles (or anti-align the one which is closer to π). Then, if the brane distance y is small enough, reconnection can take place.

We expect this alignment to proceed locally, and in a causal way. Once there is a region where the branes intersect in $p-1$ dimensions, they can reconnect there and we expect that a ‘wave of reconnection’ appears which moves outward until finally the branes have en-

tirely reconnected. We hope to give a detailed description of this picture in [11].

The lightest mass of two D4-branes at four arbitrary angles, $0 < \phi_i \leqslant \pi$, depends on the largest angle. If this is e.g. ϕ_1, the lightest mass is

$$m_1^2 = \frac{y^2}{4\pi^2\alpha'^2} + \frac{\phi_4+\phi_3+\phi_2}{2\pi\alpha'} - \frac{\phi_1}{2\pi\alpha'}. \tag{3}$$

Hence the energy can be lowered by aligning the angles ϕ_2 to ϕ_4. As soon as y and ϕ_2 to ϕ_4 are sufficiently small, a tachyonic mode appears which indicates an instability which probably leads to reconnection. Of course a corresponding mode exists for each angle ϕ_i by symmetry reasons, however, the lowest mass mode is the one determined by the largest angle.

The example discussed here, two D4-branes, is not relevant for type IIB string theory and we have chosen it, because it is treated in detail in Ref. [1]. Nevertheless, from the generality of the interaction potential it is clear, that the situation will be very similar for D5-branes. Apart from having one more angle which is readily incorporated, the main difference is that the minimal distance between D5-branes in 9-dimensional space vanishes. They generically intersect along a line, except in the special case when they are parallel in at least two directions. By applying T-duality along the intersecting direction we end up with D4-branes which intersect in a point, $y = 0$. The general expression for the potential given in Ref. [1] diverges for vanishing brane distance and we thus have to make a more thorough analysis in this case which we postpone to later work [11].

A detailed investigation of two D2-branes intersecting in a point at two angles is given in Ref. [12]. There it is found that tachyon condensation leads to 'local diffusion' of the two D2-branes near the intersection point. We shall argue that to the next order, tachyon condensation leads to a reduction of the smaller of the two angles, thereby rendering the two branes more parallel along this direction [11], leading finally to a region where we have $p-1$ nearly parallel directions and the branes can start to reconnect.

Point (4) is probably not very important. If it is not satisfied, then for topological reasons some Dp-branes may remain even if $p > 3$, but nevertheless they would be much rarer than D3-branes.

Our scenario takes place within the framework of ten-dimensional type IIB supersymmetric string theory. Hence the number of spatial dimensions of the bulk is $d-1 = 9$, and the possible dimensionality of the Dp-branes is $p = 1, 3, 5, 7$ or 9. We consider an initial state where the bulk is filled with a 'gas' of all allowed Dp-branes. Assuming the correctness of our hypotheses (1), (2), (3), and (4), after some time only D3- and D1-branes survive.

D9-branes are space-filling and the bulk coincides with their world-volume. For a D9-brane there is no partition of spacetime into Neumann and Dirichlet directions. Since D9-branes overlap entirely, they can immediately reconnect in a way that the winding number of each of them vanishes and thus evaporate.

Two D7-branes, or two D5-branes, will always intersect on manifolds of dimension 5 and 1, respectively. The D7-branes then have to align along one direction before they can reconnect and eventually unwind. The D5-branes have to align three directions before they can reconnect.

We therefore expect first the D9-branes to evaporate, then the D7- and the D5-branes last. At the end we are left with D3-branes and D1-branes and a background of closed string modes (gravitons and dilatons) in the bulk.

Neglecting the interaction of branes with different dimensionality is probably not a very good approximation. But since D3-branes generically only intersect with D7- and D9-branes, as soon as those have unwound and evaporated, the D3-branes are no longer affected and survive.

3. Remarks and conclusions

We have addressed the question of why we live in $3+1$ dimensions in the framework of braneworlds, where the standard model of strong and electroweak interactions is described by open string modes ending on branes, while gravitons and dilatons are the low energy closed string modes in the bulk. We consider IIB string theory and we claim that all Dp-branes with p greater than 3 disappear through brane collisions, emitting fundamental string loops. After some time we are left with Dp-branes of 1 or 3 dimensions and a collection of closed strings in the bulk.

It is interesting to note that the collision and evaporation of higher-dimensional Dp-branes generates entropy by populating the bulk with gravitons and dila-

tons. If their density is sufficiently high, we expect them to thermalize. These bulk modes also interact with the 3-brane representing our universe and can be converted there into a thermal bath of all modes living on the brane. This might explain the entropy of our Universe.

The mechanism discussed in this Letter does not address the question of how gravity gets localised to the brane.

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