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Anthropic distribution for cosmological constant and primordial density perturbations

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Abstract

The Anthropic Principle has been proposed as an explanation for the observed value of the cosmological constant. Here we revisit this proposal by allowing for variation between universes in the amplitude of the scale-invariant primordial cosmological density perturbations. We derive a priori probability distributions for this amplitude from toy inflationary models in which the parameter of the inflaton potential is smoothly distributed over possible universes. We find that for such probability distributions, the likelihood that we live in a typical, anthropically-allowed universe is generally quite small.

The Anthropic Principle has been proposed as a possible solution to the two cosmological constant problems: why the cosmological constant Λ is orders of magnitude smaller than any theoretical expectation, and why it is non-zero and comparable today to the energy density in other forms of matter [1–3]. This anthropic argument, which predates direct cosmological evidence of the dark energy, is the only theoretical prediction for a small, non-zero Λ [3,4]. It is based on the observation that the existence of life capable of measuring Λ requires a universe with cosmological structures such as galaxies or clusters of stars. A universe with too large a cosmological constant either does not develop any structure, since perturbations that could lead to clustering have not gone non-linear before the universe becomes dominated by Λ, or else has a very low probability of exhibiting structure-forming perturbations, because such perturbations would have to be so large that they would lie in the far tail-end of the cosmic variance. The existence of the string theory landscape, in which causally disconnected regions can have different cosmological and particle physics properties, adds support to the notion of an anthropic rule for selecting a vacuum.

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How well does this principle explain the observed value of Λ in our universe? Careful analysis by [4] finds that 5% to 12% of universes would have a cosmological constant smaller than our own. In everyday experience we encounter events at this level of confidence,[1] so as an explanation this is not unreasonable.

If the value of Λ is not fixed a priori, then one might expect other fundamental parameters to vary between universes as well. This is the case if one sums over wormhole configurations in the path integral for quantum gravity [5], as well as in the string theory landscape [6–9]. In [9] it was emphasized that all the parameters of the low energy theory would vary over the space of vacua ("the landscape"). Douglas [7] has initiated a program to quantify the statistical properties of these vacua, with additional contributions by others [8].

In [10], Aguirre stressed that life might be possible in universes for which some of the cosmological parameters are orders of magnitude different from those of our own universe. The point is that large changes in one parameter can be compensated by changes in another in such a way that life remains possible. Anthropic predictions for a *particular* parameter value will therefore be weakened if other parameters are allowed to vary between universes. One cosmological parameter that may significantly affect the anthropic argument is Q, the standard deviation of the amplitude of primordial cosmological density perturbations. Rees [11] and Tegmark and Rees [12] have pointed out that if the anthropic argument is applied to universes where Q is not fixed but randomly distributed, then our own universe becomes less likely because universes with both Λ and Q larger than our own are anthropically allowed. The purpose of this Letter is to quantify this expectation within a broad class of inflationary models. Restrictions on the a priori probability distribution for Q necessary for obtaining a successful anthropic prediction for Λ, were considered in [13,14].

In our analysis we let both Λ and Q vary between universes and then quantify the anthropic likelihood of a positive cosmological constant less than or equal to that observed in our own universe. We offer a class of toy inflationary models that allow us to restrict the a priori probability distribution for Q, making only modest assumptions about the behavior of the a priori distribution for the parameter of the inflaton potential in the anthropically-allowed range. Cosmological and particle physics parameters other than Λ and Q are held fixed as initial conditions at recombination. We provisionally adopt Tegmark and Rees's anthropic bound on Q: a factor of 10 above and below the value measured in our universe. Even though this interval is small, we find that the likelihood that our universe has a typical cosmological constant is drastically reduced. The likelihood tends to decrease further if larger intervals are considered.

Weinberg determined in [3] that, in order for an overdense region to go non-linear before the energy density of the universe becomes dominated by Λ, the value of the overdensity $\delta \equiv \delta\rho/\rho$ must satisfy

$$\delta > \left(\frac{729\Lambda}{500\bar{\rho}}\right)^{1/3}. \tag{1}$$

In a matter-dominated universe this relation has no explicit time dependence. Here $\bar{\rho}$ is the energy density in non-relativistic matter. Perturbations not satisfying the bound cease to grow once the universe becomes dominated by the cosmological constant. For a fixed amplitude of perturbations, this observation provides an upper bound on the cosmological constant compatible with the formation of structure. Throughout our analysis we assume that at recombination $\Lambda \ll \bar{\rho}$.

To quantify whether our universe is a typical, anthropically-allowed universe, additional assumptions about the distribution of cosmological parameters and the spectrum of density perturbations across the ensemble of universes are needed.

A given slow-roll inflationary model with reheating leads to a Friedman–Roberston–Walker universe with a (late-time) cosmological constant Λ and a spectrum of perturbations that is approximately scale-invariant and Gaussian with a variance

$$Q^2 \equiv \langle\tilde{\delta}^2\rangle_{\mathrm{HC}}. \tag{2}$$

The expectation value is computed using the ground state in the inflationary era and perturbations are evaluated at horizon-crossing. The variance is fixed by the parameters of the inflationary model together with some initial conditions. Typically, for single-field ϕ

[1] For instance, drawing two pairs in a poker hand.

slow-roll inflationary models,

$$Q^2 \sim \left.\frac{H^4}{\dot{\phi}^2}\right|_{\mathrm{HC}}. \tag{3}$$

This leads to spatially separated over- or under-dense regions with an amplitude δ that for a scale-invariant spectrum are distributed (at recombination) according to

$$\mathcal{N}(\sigma, \delta) = \sqrt{\frac{2}{\pi}}\frac{1}{\sigma}e^{-\delta^2/2\sigma^2}. \tag{4}$$

(The linear relation between Q and the filtered σ in Eq. (4) is discussed below.)

By Bayes's theorem, the probability for an anthropically-allowed universe (i.e., the probability that the cosmological parameters should take certain values, given that life has evolved to measure them) is proportional to the product of the a priori probability distribution P for the cosmological parameters, times the probability that intelligent life would evolve given that choice of parameter values. Following [4], we estimate that second factor as being proportional to the mean fraction $\mathcal{F}(\sigma, \Lambda)$ of matter that collapses into galaxies. The latter is obtained in a universe with cosmological parameters Λ and σ by spatially averaging over all over- or under-dense regions, so that [4]

$$\mathcal{F}(\sigma, \Lambda) = \int_{\delta_{\min}}^{\infty} d\delta\, N(\sigma, \delta)\mathcal{F}(\delta, \Lambda). \tag{5}$$

The lower limit of integration is provided by the anthropic bound of Eq. (1), which gives $\delta_{\min} \equiv (729\Lambda/500\bar{\rho})^{1/3}$. The anthropic probability distribution is

$$\mathcal{P}(\sigma, \Lambda) = P(\Lambda, \sigma)\mathcal{F}(\sigma, \Lambda)\, d\Lambda\, d\sigma. \tag{6}$$

Computing the mean fraction of matter collapsed into structures requires a model for the growth and collapse of inhomogeneities. The Gunn–Gott model [15,16] describes the growth and collapse of an overdense spherical region surrounded by a compensating underdense shell. The weighting function $\mathcal{F}(\delta, \Lambda)$ gives the fraction of mass in the inhomogeneous region of density contrast δ that eventually collapses (and then forms galaxies). To a good approximation it is given by [4]

$$\mathcal{F}(\delta, \Lambda) = \delta\frac{1}{\delta + \delta_{\min}}. \tag{7}$$

Additional model-dependence occurs in the introduction of the parameter s given by the ratio of the volume of overdense sphere to the volume of the underdense shell surrounding the sphere. We will set $s = 1$ throughout.

Since the anthropically allowed values for Λ are so much smaller than any other mass scale in particle physics, and since we assume that $\Lambda = 0$ is not a special point in the landscape, we follow [4,17] in using the approximation $P(\Lambda) \simeq P(\Lambda = 0)$ for Λ within the anthropically allowed window.[2] The requirement that the universe not recollapse before intelligent life has had time to evolve anthropically rules out large negative Λ [2,19]. We will assume that the anthropic cutoff for negative Λ is close enough to $\Lambda = 0$ that all $\Lambda < 0$ may be ignored in our calculations.

As an example of a concrete model for the variation in Q between universes, we consider inflaton potentials of the form (see, for example, [20])

$$V = \Lambda + \lambda\phi^{2p}, \tag{8}$$

where p is a positive integer.[3] We assume there are additional couplings that provide an efficient reheating mechanism, but are unimportant for the evolution of ϕ during the inflationary epoch. The standard deviation of the amplitude of perturbations gives

$$Q = A\sqrt{\lambda}\,\frac{\phi_{\mathrm{HC}}^{p+1}}{M_{\mathrm{Pl}}^3}, \tag{9}$$

where A is a constant, and ϕ_{HC} is the value of the field when the mode of wave number k leaves the horizon. This ϕ_{HC} has logarithmic dependence on λ and k, which we neglect. Randomness in the initial value for ϕ affects only those modes that are (exponentially) well outside our horizon. Throughout this Letter, we will set the spectral index to one and ignore its running. Eq. (9) then gives $\lambda \propto Q^2$.

Next, suppose that the fundamental parameters of the Lagrangian are not fixed, but vary between universes, as might be expected if one sums over wormhole configurations in the path integral for quantum

[2] Garriga and Vilenkin point to examples of quintessence models in which the approximation $P(\Lambda) \simeq P(\Lambda = 0)$ in the anthropically-allowed range is not valid [18].

[3] Recent analysis of astronomical data disfavors the $\lambda\phi^4$ inflationary model [21], but for generality we will consider an arbitrary p in Eq. (8).

gravity [5] or in the string theory landscape [6–9]. To obtain the correct normalization for the density perturbations observed in our universe, the self-coupling must be extremely small. As the standard deviation Q will be allowed to vary by an order of magnitude around 10^{-5}, for this model the self-coupling in alternate universes will be very small as well.

We may then perform an expansion about $\lambda = 0$ for the a priori probability distribution of λ. The smallness of λ suggests that we may keep only the leading term in that expansion. If the a priori probability distribution extends to negative values of λ (which are anthropically excluded due to the instability of the resulting action for ϕ), we expect it to be smooth near $\lambda = 0$, and the leading term in the power series expansion to be zeroth order in λ (i.e., a constant). Therefore we expect a flat a priori probability distribution for λ. The a priori probability distribution for Q is then

$$P(Q) \propto \frac{d\lambda}{dQ} \sim Q, \tag{10}$$

where the normalization constant is determined by the range of integration in Q. Note that this distribution favors large Q. On the other hand, if the a priori probability distribution for the coupling λ only has support for $\lambda > 0$ then $\lambda = 0$ is a special point and we cannot argue that $P(Q) \propto Q$. However, since the anthropically-allowed values of λ are very small, the a priori distribution for λ should be dominated, in the anthropically-allowed window, by a leading term such as $P(\lambda) \sim \lambda^q$. Normalizability requires $q > -1$. Using $\lambda \propto Q^2$, this gives $P(Q) \sim Q^{2q+1}$.

Before proceeding, it is convenient to transform to the new variables:

$$y \equiv \frac{\Lambda}{\rho_*}, \qquad \hat{\sigma} \equiv \sigma \left(\frac{\bar{\rho}}{\rho_*}\right)^{1/3}. \tag{11}$$

Here $\bar{\rho}$ is the energy-density in non-relativistic matter at recombination, which we take to be fixed in all universes, and ρ_* is the value for the present-day energy density of non-relativistic matter in our own universe. For a matter-dominated universe $\hat{\sigma}$ is time-independent, whereas y is constant for any era. Here and throughout this Letter, a subscript $*$ denotes the value that is observed in our universe for the corresponding quantity. The only quantities whose variation from universe to universe we will consider are y and $\hat{\sigma}$.

In terms of these variables and following [4], the probability distribution of Eq. (6) is found to be

$$\mathcal{P} = N\, d\hat{\sigma}\, dy\, P(\hat{\sigma}) \int_{\beta}^{\infty} dx \frac{e^{-x}}{\beta^{1/2} + x^{1/2}}, \tag{12}$$

where

$$\beta \equiv \frac{1}{2\hat{\sigma}^2} \left(\frac{729y}{500}\right)^{2/3}, \tag{13}$$

and N is the normalization constant.

Notice that, since $x \geqslant \beta$, large β implies that $\mathcal{P} \sim e^{-\beta} \ll 1$. For a fixed $\hat{\sigma}$, large y implies large β. Thus, for fixed $\hat{\sigma}$, large cosmological constants are anthropically disfavored. But if $\hat{\sigma}$ is allowed to increase, then $\beta \sim \mathcal{O}(1)$ may be maintained at larger y. Garriga and Vilenkin have pointed out that the distribution in Eq. (12) may be rewritten using the change of variables $(\hat{\sigma}, y) \mapsto (\hat{\sigma}, \beta)$ [14]. The Jacobian for that transformation is a function only of $\hat{\sigma}$. Eq. (12) then factorizes into two parts: one depending only on $\hat{\sigma}$, the other only on β. Integration over $\hat{\sigma}$ produces an overall multiplicative factor that cancels out after normalization, so that any choice of $P(\hat{\sigma})$ will give the same distribution for the dimensionless parameter β. In that sense, even in a scenario where $\hat{\sigma}$ is randomly distributed, the computation in [4] may be seen as an anthropic prediction for β.[4] The measured value of β is, indeed, typical of anthropically-allowed universes, but an anthropic explanation for β alone does not address the problem of why both Λ and Q should be so small in our universe.

Implementing the Anthropic Principle requires making an assumption about the minimum mass of "stuff", collapsed into stars, galaxies, or clusters of galaxies, that is needed for the formation of life. It is more convenient to express the minimum mass $M_{\min}$ in terms of a comoving scale R: $M_{\min} = 4\pi \bar{\rho} a_{\mathrm{eq}}^3 R^3/3$ (by convention $a = 1$ today, so R is a physical scale). We do not know the precise value of R. A better understanding of biology would in principle determine its value, which should only depend on chemistry, the fraction of matter in the form of baryons, and Newton's constant. In our analysis these are all fixed initial conditions at recombination. In particular, we would

[4] We thank Garriga and Vilenkin for explaining this point to us.

not expect $M_{\min}$ to depend on Λ or Q.[5] Therefore, even though the relation between $M_{\min}$ and R depends on present-day cosmological parameters, the value of this threshold will be constant between universes because it depends only on parameters that we are treating as fixed initial conditions. Thus, in computing the probability distribution over universes, we will fix R. Since we do not know what is the correct anthropic value for R, we will present our results for both $R = 1$ and 2 Mpc. (R on the order of a few Mpc corresponds to requiring that structures as large as our galaxy be necessary for life.)

We then proceed to filter out perturbations with wavelength smaller than R, leading to a variance σ^2 that depends on the filtering scale. Expressed in terms of the power spectrum evaluated at recombination,

$$\sigma^2 = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 P(k) W^2(kR) \tag{14}$$

where W is the filter function, which we take to be a Gaussian $W(x) = e^{-x^2/2}$. $P(k)$ is the power spectrum, which we assume to be scale-invariant. (For $P(k)$ we use Eq. (39) of [4], setting $n = 1$.)

Evaluating (14) at recombination gives, for our universe,

$$\hat{\sigma}_* = C_* Q_*. \tag{15}$$

The number C_* contains the growth factor and transfer function evaluated from horizon crossing to recombination and only depends on physics from that era. We assume Λ is small enough so that at recombination it can be ignored and thus we take the variation in $\hat{\sigma}$ between universes to come solely from its explicit dependence on Q.

We may then use observations of Q_* and σ_* to determine $\hat{\sigma} = C_* Q$, valid for all universes. We use the explicit expression for C_* that is obtained from Eqs. (39)–(43) and (48)–(51) in Ref. [4]. This takes as inputs the Hubble parameter $H_0 \equiv 100 h_*$ km/s, the energy density in non-relativistic matter Ω_*, the cosmological constant $\lambda_* = 1 - \Omega_*$, the baryon fraction $\Omega_b = 0.023 h_*^{-2}$, the smoothing scale R, and the COBE normalized amplitude of fluctuations at horizon crossing, $Q_* = 1.94 \times 10^{-5} \Omega_*^{-0.785-0.05*\ln\Omega_*}$.

As we have argued, the dependence of C_* on the cosmological constant is not relevant for our purposes. For our calculations we use $\Omega_* = 0.134 h_*^{-2}$, and $h_* = 0.73$ (consistent with their observed best-fit values [22]). The smoothing scale R will be taken to be either 1 Mpc or 2 Mpc, and the corresponding values for C_* are 5.2×10^4 and 3.8×10^4.

The values chosen for the range of Q are motivated by the discussion in [12] about anthropic limits on the amplitude of the primordial density perturbations. The authors of [12] argue that Q between 10^{-3} and 10^{-1} leads to the formation of numerous supermassive black holes which might obstruct the emergence of life.[6] They then claim that universes with Q less than 10^{-6} are less likely to form stars, or if star clusters do form, that they would not be bound strongly enough to retain supernova remnants. Since there is considerable uncertainty in these limits, we carry out calculations using both the range indicated by [12] as well as a range that is somewhat broader.[7]

Previous work on applying the Anthropic Principle to variable Λ and Q has assumed a priori distributions $P(Q)$ that fall off as $1/Q^k$ for large Q, with $k \geqslant 3$ [13,14]. Such distributions were chosen in order to keep the anthropic probability $\mathcal{P}(y, Q)$ normalizable, and they usually yield anthropic predictions for the cosmological constant similar to those that were obtained in [4] by fixing Q to its observed value, because they naturally favor a Q as small as its observed value in our universe. For instance, for $P(Q) \propto 1/Q^3$ in the range $Q_*/10 < Q < 10 Q_*$, $P(y < y_*) = 5\%$ for $R = 1$ Mpc, while $P(y < y_*) = 7\%$ for $R = 2$ Mpc.)

However, if we accept the argument of Tegmark and Rees in [12] that there are natural anthropic cutoffs on Q, it follows that the behavior of $P(Q)$ is irrel-

[5] Note, however, that requiring life to last for billions of years (long enough for it to develop intelligence and the ability to do astronomy) might place bounds on Q. See [12].

[6] They also note that for $Q > 10^{-4}$ formation of life is possible, but planetary disruptions caused by flybys may make it unlikely for planetary life to last billions of years.

[7] Notice that we are using the ranges indicated in [12] as absolute anthropic cutoffs. Arguments like those made in [12] introduce some correction to the approximation made in [4] that the probability of life is proportional to the amount of matter that collapses into compact structures. Since we are largely ignorant of what the form of this correction is, we have approximated it as a simple window function.

Table 1
Anthropically determined properties of the cosmological constant

	$P(Q) \propto 1/Q^{0.9}$ in the range				$P(Q) \propto Q$ in the range			
	$Q_*/10 < Q < 10Q_*$		$Q_*/15 < Q < 15Q_*$		$Q_*/10 < Q < 10Q_*$		$Q_*/15 < Q < 15Q_*$	
	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc
$P(y < y_*)$	1×10^{-3}	3×10^{-3}	4×10^{-4}	1×10^{-3}	5×10^{-4}	1×10^{-3}	1×10^{-4}	4×10^{-4}
$\langle y \rangle / y_*$	1×10^{4}	4×10^{3}	4×10^{4}	1×10^{4}	1×10^{4}	5×10^{3}	4×10^{4}	2×10^{4}
$y_{5\%}/y_*$	9×10	4×10	3×10^{2}	1×10^{2}	2×10^{2}	7×10	6×10^{2}	2×10^{2}

Table 2
Anthropically determined properties of the amplitude for density perturbations

	$P(Q) \propto 1/Q^{0.9}$ in the range				$P(Q) \propto Q$ in the range			
	$Q_*/10 < Q < 10Q_*$		$Q_*/15 < Q < 15Q_*$		$Q_*/10 < Q < 10Q_*$		$Q_*/15 < Q < 15Q_*$	
	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc	$R = 1$ Mpc	$R = 2$ Mpc
$P(Q < Q_*)$	8×10^{-4}	8×10^{-4}	2×10^{-4}	2×10^{-4}	1×10^{-5}	1×10^{-5}	1×10^{-6}	1×10^{-6}
$\langle Q \rangle / Q_*$	8	8	11	11	8	8	13	13
$Q_{5\%}/Q_*$	4	4	6	6	5	5	8	8

evant to the normalizability of $\mathcal{P}(y, Q)$. Furthermore, $P(Q) \sim 1/Q^k$ in the neighborhood of $Q = 0$ for $k \geqslant 1$ leads to an unnormalizable distribution, since the integral $\int P(Q)\,dQ$ blows up. In what follows we shall consider two a priori distributions: $P(Q) \propto Q$, and $P(Q) \propto 1/Q^{0.9}$ inside the anthropic window, motivated by the inflationary models we have discussed.

The results are summarized in Table 1, where $P(y < y_*)$ is the anthropic probability that the value y be no greater than what is observed in our own universe, $\langle y \rangle$ is the anthropically-weighed mean value of y, and $y_{5\%}$ is the value of y such that the anthropic probability of obtaining a value no greater than that is 5%.

By comparison, for this choice of cosmological parameters, the authors of [4] find that, for Q fixed (or measured), the probability of a universe having a cosmological constant no greater than our own is much higher: $P(y < 0.7/0.3) = 0.05$ and 0.1, for $R = 1$ Mpc and $R = 2$ Mpc, respectively.[8]

One can also ask what is the probability of observing a value for Q in the range $Q_*/10 < Q < Q_*$, after averaging over all possible cosmological constants. Table 2 summarizes the resulting distribution in Q.

[8] These numbers are taken from Table 1 in the published version of Ref. [4].

In summary, inflation and a landscape of anthropically determined coupling constants provides (in some scenarios) a conceptually clean framework for variation between universes in the magnitude of Q. Since increasing Q allows the probability of structure to remain non-negligible for Λ considerably larger than in our own universe, anthropic solutions to the cosmological constant problem are weakened by allowing Q as well as Λ to vary from one universe to another.

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