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Goldstone modes in the neutron star core

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ABSTRACT

We studied the effect of the diverse Goldstone boson modes in the transport properties of dense neutron matter. The two Goldstone bosons associated with density oscillations of the neutron and electron + proton fluids, called *superfluid phonons*, mix and couple strongly to electrons reducing their mean free time and suppressing their contribution to transport coefficients. For typical neutron star temperatures in the range $T = 10^6$ – 10^9 K, the Goldstone modes associated with rotational symmetry, called *angulons*, couple weakly to each other and to electrons and, consequently, have anomalously large mean free paths and can contribute to both diffusive and ballistic transport of heat and momentum. Still, their contribution is smaller than coming from the electrons on account of their smaller density. We speculate that long-wavelength superfluid phonons and *angulons* can play a role in neutron star seismology, and lead to interesting phenomenology especially since angulons couple to magnetic fields and have anisotropic dispersion relations.

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1. Introduction

It is likely that neutron-rich matter encountered in the neutron star core is both superfluid and superconducting [1]. At the densities realized in the core, attractive s-wave interactions between protons, and p-wave interactions between neutrons naturally lead to the formation of Cooper pairs of nucleons at the Fermi surface. Neutrons are expected to form a 3P_2 superfluid and protons form a 1S_0 superconductor. Model calculations indicate that the typical pairing energy or equivalently the energy gap, denoted by Δ are small ($\Delta \lesssim 1$ MeV $\simeq k_B 10^{10}$ K) compared to the Fermi energy, but are large compared the typical temperatures $T \simeq 10^7$ – 10^8 K encountered in old and cold neutron stars. Hence, it has been known for some time that while pairing will not affect substantially the equation of state, it can dramatically alter key transport and cooling properties of neutron stars.

When $k_B T \ll \Delta$, the excitation of neutrons and protons is exponentially suppressed by the factor $\exp(-\Delta/k_B T)$ and the only relevant excitations are electrons and Goldstone bosons (GBs) associated with the symmetry breaking in the superfluid and superconducting ground states. Since nucleon excitations are gapped, the theoretical description of transport properties is dominated by GBs and electrons, and we show here that they are weakly interacting and that kinetic theory applies. As a consequence, long-wavelength oscillations, shear viscosity and thermal conductivity,

which play a role in interpreting various transport phenomena in neutron stars, can be calculated systematically in terms of just a handful of low-energy constants.

The letter is organized as follows. First, we use general symmetry arguments and simple dynamical considerations to discuss the structure of the low energy theory of GBs. Then, we use this effective theory to calculate the dispersion relations and mean free paths of all the GBs to show that some of them are especially weakly coupled. We conclude by estimating neutron star oscillation frequencies, and the GB mode contribution to thermal conductivity and shear viscosity, and discuss its implications for neutron stars. Through out this letter, unless we explicitly note otherwise, we use natural units with $\hbar = 1$, $c = 1$.

2. Superfluidity and superconductivity in the core

At the high densities encountered in the core, s-wave interactions between neutrons are repulsive and attractive p-wave interactions favor spin-one Cooper pairs with angular momentum in the 3P_2 channel. The resulting condensate, which we discuss in more detail below, breaks rotational invariance as well as the global $U(1)$ symmetry associated with rotations of the phase of the neutron wave function. Because the proton fraction is smaller, typically less than 10%, s-wave interactions between protons remain attractive even in the core, and this leads to the formation of spin-zero proton Cooper pairs in the 1S_0 channel. The resulting superconducting condensate breaks the $U(1)$ symmetry associated

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with rotations of the phase of the proton wave function. In contrast, due to their large Fermi energy ($\gtrsim 100$ MeV) electrons form a nearly ideal, relativistic and degenerate Fermi gas.

In the neutron superfluid there are Goldstone modes associated with breaking of rotational symmetry by the 3P_2 condensate labeled as *angulons* in [2], and one *superfluid phonon* mode associated with the neutron number $U(1)$ symmetry. The relevance of the *superfluid phonon* mode to transport phenomena in the neutron star core was recently studied in [3]. Our findings show that the *angulon* contribution is more relevant. Naively, one may expect that the Goldstone mode associated with the proton superconductor will not be massless due to long-range Coulomb interactions. However, due to efficient electron screening, a massless Goldstone does exist, and it corresponds to a charge neutral oscillation of the proton condensate and the electron fluid [4]. We now discuss in detail the theory needed to describe the propagation of these massless modes.

3. Low energy theory of phonons and angulons

Neutron pairing results in a non-vanishing spin-2 condensate given by

$$(N^T \sigma_2 \sigma^i \overleftrightarrow{\nabla}^j N) = \Delta_{ij}^0 e^{i\phi}, \quad (1)$$

where Δ_{ij}^0 is a symmetric traceless tensor and ϕ is a scalar. Different symmetric traceless tensors break the rotation group in different ways so there are several possible 3P_2 phases. Around the critical temperature one can rely on BCS and strong coupling estimates of the parameters of the Ginzburg–Landau free energy to conclude that the ground state is of the form $\Delta_{ij}^0 \sim \text{diag}(1, 1, -2)$ (or, of course, any rotation of this matrix) [5,6]. The structure of the gap equations are such that, at least within the BCS framework, the relative order of the different states is not changed as temperature, density or microscopic interactions change [7] so it is reasonable to assume that the ground state of neutron matter is in a phase characterized by the $\Delta_{ij}^0 \sim \text{diag}(1, 1, -2)$ form of the condensate. This assumption underlies our analysis, however our main qualitative conclusions are independent of it.

The presence of the condensate $\Delta_{ij}^0 \sim \text{diag}(1, 1, -2)$ breaks spontaneously the symmetry of the system under rotations, except for those around the z -axis. Thus, as first realized in [2] we expect the presence of two gapless excitations above the ground state, named “angulons”, corresponding to rotations of the condensate around the x and y axis. Angulons were then studied in more detail in [8] where, with mild assumptions, their properties were quantitatively estimated.

The low energy dynamics is succinctly described by the lagrangian

$$\begin{aligned} \mathcal{L}_{\text{ang}} = & \sum_{i=1,2} \left[\frac{1}{2} (\partial_0 \beta_i)^2 - \frac{1}{2} v_{\perp}^2 ((\partial_x \beta_i)^2 + (\partial_y \beta_i)^2) + v_{\parallel}^2 (\partial_z \beta_i)^2 \right] \\ & + \frac{e g_n f_{\beta}}{2M \sqrt{-\nabla_{\perp}^2}} [\mathbf{B}_1 \partial_0 (\partial_y \beta_1 + \partial_x \beta_2) + \mathbf{B}_2 \partial_0 (\partial_x \beta_1 - \partial_y \beta_2)] \\ & + \mathcal{O} \left(\beta^2 \frac{(\partial \beta)^2}{f_{\beta}^2} \right), \end{aligned}$$

where

$$\begin{aligned} v_{\perp}^2 &= \frac{24\pi \sqrt{3}}{18(9 + \pi \sqrt{3})} v_F^2, & v_{\perp}^2 &= \frac{81 - 4\pi \sqrt{3}}{18(9 + \pi \sqrt{3})} v_F^2, \\ v_{\parallel}^2 &= \frac{81 - 2\pi \sqrt{3}}{18(9 + \pi \sqrt{3})} v_F^2, & f_{\beta}^2 &= \frac{M k_F}{\pi^2} \left(1 + \frac{\pi}{3\sqrt{3}} \right), \end{aligned}$$

Table 1

Ambient conditions, low energy constants and eigenmode velocities v_1 and v_2 in units of the velocity of light for the equation of state from [13].

n_n (fm $^{-3}$)	0.08	0.16	0.20	0.24	0.32
x_p	0.024	0.043	0.050	0.057	0.070
v_p^2	0.029	0.049	0.060	0.072	0.104
v_n^2	0.015	0.070	0.128	0.210	0.430
v_{np}^2	−0.034	−0.016	0.024	0.086	0.268
v_1	0.12	0.21	0.23	0.25	0.28
v_2	0.17	0.26	0.36	0.46	0.71

$g_n \approx -1.91$ is the neutron magnetic moment in units of the nuclear Bohr magneton, \mathbf{B} is the magnetic field, k_{Fn} the neutron Fermi momentum, M the nucleon mass, $v_F = k_{Fn}/M$ is the neutron Fermi velocity, and $e = \sqrt{\alpha_{em}/4\pi^2}$ the electron charge. The fields $\beta_{1,2}$ are linear combinations of the fields describing rotations of the condensate around the x and y axis which mix among themselves; in terms of the original fields the lagrangian is analytic at small momenta.

We now discuss the two remaining massless modes, these now being associated with density fluctuations. The first mode is one that would exist in a pure 3P_2 (and also a 1S_0) neutron superfluid and it corresponds to the fluctuations of ϕ – the overall isotropic phase of the condensate. The other mode, which we call the *ep* phonon, is related to density fluctuations of proton condensate + the electron gas and is denoted by the scalar field ξ . The general low energy effective field theory of these scalar modes is well studied [9–11] and the low energy lagrangian density is given by

$$\begin{aligned} \mathcal{L}_{\text{phn}} = & \frac{1}{2} (\partial_0 \phi)^2 - \frac{v_n^2}{2} (\partial_i \phi)^2 + \frac{1}{2} (\partial_0 \xi)^2 - \frac{v_p^2}{2} (\partial_i \xi)^2 \\ & + v_{np}^2 \partial_0 \phi \partial_0 \xi + \frac{1}{f_{ep}} \partial_0 \xi \psi_e^\dagger \psi_e + \dots, \end{aligned}$$

where we have also included the coupling to the electron field ψ_e . The coefficients of the leading order terms in the derivative expansion are related to simple thermodynamic derivatives and can be obtained from the equation of state. They are given by

$$v_n^2 = \frac{n_n}{m} E_{nn}, \quad v_p^2 = \frac{n_p}{m} E_{pp}, \quad v_{np}^2 = \frac{1}{2} \frac{k_{Fn}^2}{\pi^2} \sqrt{\frac{k_{Fn}}{k_{Fp}}} E_{np} \quad (2)$$

where $E_{ij} = \partial^2 E(n_n, n_p) / (\partial n_i \partial n_j)$ and $E(n_n, n_p)$ is the energy density of the neutron–proton system. The effective coupling between the *ep* phonons and electron-hole states is calculated as in the jellium model and is given by $f_{ep} = \sqrt{m_p k_{Fp} / \pi^2}$ [12]. E_{np} arises solely due to nucleon–nucleon interactions and its value depends on the density, the equilibrium proton fraction and the equation of state model chosen. The low energy constants calculated using a representative microscopic equation of state from [13] and the eigenmode velocities in units of the speed of light are shown in Table 1.

4. Mean free paths of Goldstone bosons

The propagation of *angulons* and *superfluid phonons* can be damped by several processes. In the following we estimate the superfluid phonon and angulon mean free paths at low temperature $k_B T \ll \Delta$ and find that dominant decay mechanism is through the excitation of electron-hole states. First, we analyze the mean free paths of the two longitudinal *superfluid phonons*. In the absence of any mixing between these modes the *ep* mode couples strongly to the electron-hole excitations and its damping rate and the mean free paths are given by

$$\Gamma_{ep}(\omega = v_p q) = \frac{3\pi}{2} v_p^3 q,$$

$$\lambda_{ep}(\omega = v_p q) = \frac{c}{\Gamma_{ep}(\omega)} \simeq 1.4 \times 10^{-9} \left(\frac{10 \text{ keV}}{\omega} \right) \left(\frac{0.3}{v_p} \right) \text{ cm},$$

respectively [14]. The thermal average

$$\langle \lambda_{ep}(T) \rangle = \pi / (18\zeta(3) v_p T) \approx 10^{-9} \text{ cm}$$

at $T = 10^8$ K and $v_p = 0.3$.

The term proportional to v_{np} mixes the ep mode with the neutron *superfluid phonon* mode. We find that both eigenmodes decay predominantly by coupling to electron-hole excitations (Landau damping). This mixing is similar to the mixing between the longitudinal phonons of the nuclear lattice and the neutron *superfluid phonons* in the inner crust of the neutron star [11]. The velocity and damping rates of the two longitudinal eigenmodes can be obtained as solutions to the equation

$$(\omega^2 - v_n^2 q^2)(\omega^2 - v_p^2 q^2 - 2i\omega \Gamma_{e-p}(\omega)) - 2v_{np}^4 \omega^4 = 0 \quad (3)$$

In the limit of weak mixing the scattering rate of the predominantly ep -mode is $\approx \Gamma_{ep}(\omega = v_p q)$ given in Eq. (3), and the scattering rate of the predominantly neutron superfluid mode is

$$\Gamma_\phi(\omega = v_n q) \approx \frac{v_{np}^4 \Gamma_{ep}(\omega)}{(1 - (v_p/v_n)^2)^2 + 9\pi^2 v_p^4}, \quad (4)$$

and when $v_n \gg v_p$ and $v_p \ll 1$, $\Gamma_\phi(\omega = v q) \approx v_{np}^4 \Gamma_{ep}(\omega)$. Since typical values of v_{np}^4 are in the range 10^{-4} – 10^{-1} , we can conclude that the mean free path of the predominantly neutron superfluid mode will be in the range $\lambda_\phi \approx 10^{-5}$ – $(0.3/v) T_8^{-1}$ cm to $\lambda_\phi \approx 10^{-8}$ – $(0.3/v) T_8^{-1}$ cm. Here we note that this result is several orders of magnitude smaller than the estimate of the neutron *superfluid phonon* mean free path made in [3] where the coupling to electrons is neglected. Although $\lambda_\phi \gg \lambda_{ep}$ we shall see later that it is still too small to contribute significantly to transport phenomena in the neutron star core.

We now turn to the calculation of the *angulon* mean free path. Angulon–angulon scattering contributions are very small. Indeed, the angulon–angulon scattering amplitude is $\propto p^2/f_\beta^2$ since the powers of p are fixed by dimensional analysis. Its contribution to the mean free path can then be easily estimated and we find $\lambda_{\text{ang-ang}} \approx v_\beta^3 f_\beta^4 / T^5$. For $T \lesssim 10^9$ K, $\lambda_{\text{ang-ang}} \gg R$ where $R \simeq 10$ km is the radius of the neutron star, and implies that angulon–angulon processes are irrelevant. Angulon can scatter from electrons only in an indirect way through their mixing to *magnetic* photons. Angulons mix with the magnetic photons due to two processes. One mixing mechanism is due to the magnetic moment of the neutron and is described by the lagrangian in Eq. (2). The other is mediated by protons which, as charged particles, couple to photons. These two processes are depicted in Fig. 1. The latter indirect coupling necessarily involves a spin flip of both neutrons (on account of form the angulon–neutron coupling) and protons. Thus, only the *magnetic* photon mixes with the angulon and this mixing is suppressed by a power of the proton velocity change $\sim p/M$, the same suppression appearing in the magnetic moment process. We find that the proton mediated mixing is smaller than the mixing generated by the neutron magnetic moment. For the estimates we present here, we will neglect the proton mediated mixing.

Since magnetic photons are damped by electron-hole excitations, mixing ensures that angulons are also damped. The angulon scattering rate off electrons is given by

$$\Gamma_{\text{ang}}(\omega, q) \simeq \frac{1}{\omega} \left(\frac{ef_\beta g_n}{2M} q \right)^2 \text{Im} D(\omega, q), \quad (5)$$

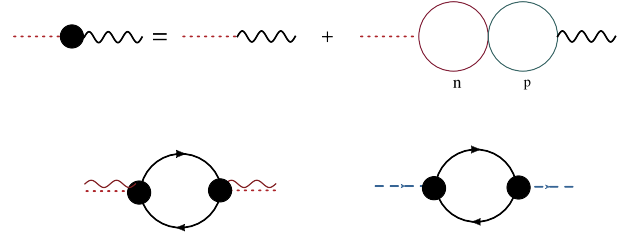


Fig. 1. The top line shows the magnetic moment and proton mediated mixing processes, respectively. Angulon, photon, electron, neutron and proton propagators are shown as a dotted, wavy and solid black, solid red and solid blue lines, respectively and nucleon loops include both normal and anomalous diagrams. The lower graphs contribute to the imaginary part of the self-energy of the angulon–magnetic photon mixed mode (left) and the electron–proton–neutron phonon mode (right). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $D(\omega, q) = (\omega^2 - q^2 - \omega_p^2 - iq_{\text{TFe}}^2(\omega/q))^{-1}$ is the dressed photon propagator, and $q_{\text{TFe}} = \sqrt{4\pi e^2 \partial n_e / \partial \mu_e}$ is the electron Debye screening momentum, and $\omega_p = \sqrt{4\pi e^2 n_p / m}$ is the proton plasma frequency. Since $q_{\text{TFe}}^2 \gg \omega_p^2$ we can write

$$\Gamma_{\text{ang}}(\omega, q) \simeq e^2 f_\beta^2 \frac{g_n^2}{4M^2 q_{\text{TFe}}^2} q^3 = \frac{\pi f_\beta^2 g_n^2}{16M^2 k_{\text{Fe}}^2} q^3. \quad (6)$$

From the angulon width estimated above we can determine the angulon mean free path

$$\begin{aligned} \lambda_{\text{ang}}(\omega) &= \frac{v_\beta}{\Gamma(\omega = v_\beta q)} = \frac{16M^2 k_{\text{Fe}}^2 v_\beta^4}{\pi f_\beta^2 g_n^2} \frac{1}{\omega^3} \\ &\approx 1.7 \gamma^4 v_F^3 \left(\frac{k_{\text{Fe}}}{100 \text{ MeV}} \right)^2 \left(\frac{10 \text{ keV}}{\omega} \right)^3 \text{ cm}, \end{aligned} \quad (7)$$

where $\gamma = v_\beta / v_F$ and v_β is the mean velocity of the angulon.

Now that we have identified relevant scattering processes, transport properties like the heat conductivity κ and the shear viscosity η can be computed by solving the Boltzmann equation for phonons and angulons. A quicker estimate of κ and η can be obtained as follows. In kinetic theory, the thermal conductivity and shear viscosity are given by $\kappa \sim (1/3) C_V v \langle \lambda \rangle$, $\eta \sim (1/3) n \langle p \rangle \langle \lambda \rangle$, respectively, where $C_V \simeq 2\pi^2 T^3 / (15v^3)$ is the specific heat, $n \simeq \zeta(3) T^3 / (\pi^2 v^3)$ is the number density, v is the velocity, $\langle p \rangle \simeq T/v$ is the average thermal momentum, $\zeta(3) \approx 1.2$, and $\langle \lambda \rangle$ is an appropriate thermal average mean free path of the phonon/angulon.

In cgs units the phonon/angulon contribution can be written as

$$\kappa_{\text{phn/ang}} = 1.7 \times 10^{21} T_8^3 \left(\frac{0.3}{v} \right)^3 \left(\frac{\langle \lambda \rangle}{\text{cm}} \right) \frac{\text{erg}}{\text{cm s K}} \quad (8)$$

The electron contribution to thermal conductivity has been calculated in earlier work and was found to be in the range 10^{22} – 10^{24} erg/cm/s/K for typical ambient conditions in the neutron star core [15]. Since $\langle \lambda_{ep} \rangle \ll \langle \lambda_\phi \rangle \lesssim 10^{-5}$ cm, we can safely neglect the contribution from both longitudinal phonons to thermal conductivity.

Estimating the angulon contribution is bit trickier because $\lambda_{\text{ang}}(\omega) \propto 1/\omega^3$ and the naive thermal average mean free path diverges. Here it is appropriate to write the thermal conductivity as

$$\begin{aligned} \kappa_{\text{ang}} &= \frac{1}{3} \int \frac{d^3 q}{(2\pi)^3} \frac{d}{dT} \left(\frac{\bar{v} q}{e^{\beta \bar{v} q} - 1} \right) \bar{v} \lambda_{\text{ang}}(q) \\ &\simeq \frac{8\gamma^2 v_F k_{\text{Fe}}^2}{3\pi g_n^2} \int_{\bar{x}}^{\infty} \frac{x e^x}{(e^x - 1)^2} \\ &\simeq 2.5 \times 10^{19} \left(\frac{k_{\text{Fe}}}{100 \text{ MeV}} \right)^2 \gamma^2 v_F (1 - \ln(\bar{x})) \frac{\text{erg}}{\text{cm s K}} \end{aligned}$$

where $\beta = 1/k_B T$ and \bar{v} is the angle average velocity. The lower limit $\bar{\lambda} = \beta \bar{v} q_c \ll 1$ is introduced because the Bose distribution function is meaningful for low energy angulons only when the mean free path $\lambda_{\text{ang}}(\omega = \bar{v} q_c) \ll R_c$ where $R_c \simeq 5\text{--}10$ km is radius of the core. Angulons with larger mean free path will not thermalize inside the star. For relevant neutron star temperatures we find that $\bar{\lambda}$ is in the range $10^{-4}\text{--}10^{-3}$. κ_{ang} is nearly independent of temperature and for typical values $\gamma \simeq 0.5\text{--}0.7$, $v_F \simeq 1/3$ and $\bar{\lambda} \simeq 10^{-3}$, $\kappa_{\text{ang}} \approx 1.6 \times 10^{19}$ erg/(cm s K). This is a few orders of magnitude smaller than earlier estimates of the electron thermal conductivity. Similarly, an estimate of the shear viscosity shows that it too is small compared to the electronic contribution. The elementary process between electrons and GBs could nevertheless play role in coupling the dynamics of the multicomponent core of the neutron star. In addition, long wavelength modes with $q < q_c$ are ballistic, and despite their low production rates, could transport energy and momentum and warrants further study.

5. Global oscillations of neutron stars

Perhaps the most consequential finding is the long lifetime of the long wavelength angulon and phonon modes. This implies that they would play a central role in neutron star seismology. If excited these modes will have different characteristic frequencies given by $\Omega = 2\pi v/R$ where v is the phonon/angulon velocity, and damping timescale $\tau = \lambda(q \simeq 2\pi/R)/v$ where λ is the corresponding mean free path and are likely to produce a unique spectrum and time evolution that may be observationally discernible. The low frequency longitudinal mode will damp quickly in the core, the superfluid phonon modes will also damp in the core but on a longer time scale. The angulon modes, in contrast, will likely be damped only at the crust–core interface. The long decay constant of angulons is a unique property among oscillations modes of neutron stars and it can be traced back to the fact that angulon interactions with electrons are mediated by magnetic forces suppressed by powers of q/M . Further, the anisotropy of the angulon modes, and the fact that they couple to the magnetic field suggests unique observable consequences that need to be explored. For example, catastrophic processes such as giant flares or tidal perturbations that occur prior to binary neutron star mergers could trigger seismic activity in the core leading to coupled dynamics of the angulon fields and the large scale magnetic field anchored in the star.

We hope that our calculation of the mode velocities and damping rates for the elementary excitations with macroscopic and microscopic wavelengths in the neutron star core will motivate further study. There are several issues that warrant a more detailed analysis. These include the role of ballistic angulon modes, dissipation of long wavelength angulons at the crust–core boundary, and

the role of phonon-emission and absorption reactions on the electronic transport properties.

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References

- [1] D. Dean, M. Hjorth-Jensen, Pairing in nuclear systems: from neutron stars to finite nuclei, *Rev. Mod. Phys.* 75 (2003) 607–656, <http://dx.doi.org/10.1103/RevModPhys.75.607>, arXiv:nucl-th/0210033.
- [2] P.F. Bedaque, G. Rupak, M.J. Savage, Goldstone bosons in the 3P(Z) superfluid phase of neutron matter and neutrino emission, *Phys. Rev. C* 68 (2003) 065802, <http://dx.doi.org/10.1103/PhysRevC.68.065802>, arXiv:nucl-th/0305032.
- [3] C. Manuel, L. Tolos, Shear viscosity due to phonons in superfluid neutron stars, *Phys. Rev. D* 84 (2011) 123007, <http://dx.doi.org/10.1103/PhysRevD.84.123007>, arXiv:1110.0669.
- [4] M. Baldo, C. Ducoin, Elementary excitations in homogeneous superfluid neutron matter: role of the proton component, *Phys. Rev. C* 84 (2011) 035806, <http://dx.doi.org/10.1103/PhysRevC.84.035806>, arXiv:1105.1311.
- [5] J.A. Sauls, J.W. Serene, 3p_2 pairing near the transition temperature in neutron-star matter, *Phys. Rev. D* 17 (1978) 1524–1528, <http://dx.doi.org/10.1103/PhysRevD.17.1524>.
- [6] R. Richardson, Ginzburg–Landau theory of anisotropic superfluid neutron-star matter, *Phys. Rev. D* 5 (1972) 1883–1896, <http://dx.doi.org/10.1103/PhysRevD.5.1883>.
- [7] V. Khodel, V. Khodel, J.W. Clark, Universalities of triplet pairing in neutron star matter, *Phys. Rev. Lett.* 81 (1998) 3828–3831, <http://dx.doi.org/10.1103/PhysRevLett.81.3828>, arXiv:nucl-th/9807034.
- [8] P.F. Bedaque, A.N. Nicholson, The low lying modes of triplet-condensed neutron matter and their effective theory, arXiv e-prints arXiv:1212.1122.
- [9] D. Son, M. Wingate, General coordinate invariance and conformal invariance in nonrelativistic physics: unitary Fermi gas, *Ann. Phys.* 321 (2006) 197–224, <http://dx.doi.org/10.1016/j.aop.2005.11.001>, arXiv:cond-mat/0509786.
- [10] C. Pethick, N. Chamel, S. Reddy, Superfluid dynamics in neutron star crusts, *Prog. Theor. Phys. Suppl.* 186 (2010) 9–16, <http://dx.doi.org/10.1143/PTPS.186.9>, arXiv:1009.2303.
- [11] V. Cirigliano, S. Reddy, R. Sharma, A low energy theory for superfluid and solid matter and its application to the neutron star crust, *Phys. Rev. C* 84 (2011) 045809, <http://dx.doi.org/10.1103/PhysRevC.84.045809>, arXiv:1102.5379.
- [12] A. Fetter, J. Walecka, *Quantum Theory of Many-Particle Systems*, Dover Books on Physics, Dover Publications Inc., 2003, <http://books.google.com/books?id=Owekf1s83b0C>.
- [13] S. Gandolfi, A.Y. Illarionov, S. Fantoni, J. Miller, F. Pederiva, et al., Microscopic calculation of the equation of state of nuclear matter and neutron star structure, *Mon. Not. R. Astron. Soc.* 404 (2010) L35–L39, arXiv:0909.3487.
- [14] D.N. Aguilera, V. Cirigliano, J.A. Pons, S. Reddy, R. Sharma, Superfluid heat conduction and the cooling of magnetized neutron stars, *Phys. Rev. Lett.* 102 (9) (2009) 091101, <http://dx.doi.org/10.1103/PhysRevLett.102.091101>, arXiv:0807.4754.
- [15] P. Shternin, D. Yakovlev, Electron–muon heat conduction in neutron star cores via the exchange of transverse plasmons, *Phys. Rev. D* 75 (2007) 103004, <http://dx.doi.org/10.1103/PhysRevD.75.103004>, arXiv:0705.1963.