Dynamic pricing of seasonal goods with spot and forward purchase demands

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Abstract

This paper deals with the problem of jointly determining the order size and dynamic prices for a perishable inventory system over a finite time planning horizon when firms simultaneously provide customers with a prompt delivery option and a delivery schedule option. Customers are segmented into two types, namely spot purchase customers and the forward purchase customers. Demands for both types of customers are assumed to be time and price dependent. The decision-maker of the inventory system is assumed to apply pricing policies to stimulate demand to improve revenues under the condition that customers with forward purchases may cancel their orders. A mathematical model is developed to find the optimal number of price settings, the optimal dynamic prices and the order quantity. A solution procedure is found to determine the optimal decisions.

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1. Introduction

When firms dispatch a truck to deliver goods, a fixed delivery setup cost is incurred to issue a delivery. The delivery setup cost usually varies with the delivery frequency. The strategy of increasing delivery frequency will enhance the responsiveness to customers who purchase goods with the retailers’ delivery schedules. However, such a strategy will make the added delivery frequency increase the delivery setup costs. Thus, a company decision maker is facing a fundamental trade-off problem between efficiency and responsiveness, while the delivery frequency is to be determined. To trade off between efficiency and responsiveness and to improve revenues, the decision maker must determine how often to issue a truck to delivery goods to customers, along with the optimal delivery schedule.

With the advantages of information technology, customers are able to purchase goods anytime, anywhere through the internet. Also, compared to the virtual stores, the personnel expenses can be minimized in such online stores. Moreover, the pricing or promotion strategy can be easily applied in the e-market. Due to the technology advantages, nowadays, web marketing has been popularly employed by many firms. For instance, as one of the fruit producers, more and more fruit-retailers sale fruits through internet in Taiwan. In the beginning, the fruit-retailers will order a specified quantity of fruits from farmers at the start of sales season, and then resell those fruits to their customers.

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Based on the trading through web marketing approach, the purchasing behaviors of customers can be generally divided into two types, namely the spot purchase and forward purchase (advance purchase). The spot purchase arises in such cases where customers expect to receive fruits for themselves immediately, or have them delivered to friends as gifts as soon as possible. Conversely, the forward purchase may occur in cases such that customers are not hurry to receive their fruits right away, so that they are willing to wait and receive fruits based on the delivery schedule of a retailer. Due to the above two different purchasing behaviors, retailers are then to offer different options for delivering their goods and/or services to customers at different cost levels. Under such circumstance, customers have the alternatives of choosing between a regular delivery (the delivery time is pre-scheduled) with a normal charge, or a prompt delivery (the delivery time is right after the order is received) with a higher charge. From a marketing perspective, firms may try to attract both types of customers to enhance their revenues. Since pricing policy is a good vehicle to stimulate demand, firms then may seriously consider the problems of how to set the sales price for each type of demand.

It is noted that customers with advance purchases may withdraw their orders before receiving them. An inventory decision maker, without considering this phenomenon, thus may over-estimate the actual demand. From an economic point of view, the phenomenon of order cancellation cannot be disregarded when making inventory and pricing decisions. In this paper, we aim to develop a strategy for obtaining the optimal profit for a firm. Based on the proposed strategy, a periodic of review pricing policy can be employed to sell a perishable item under the condition that demand for the item is price dependent and declines with time, and the firm provide the options of prompt delivery and periodical delivery services to customers.

The existing related models concentrating on work pertaining to the ordering problem are the EOQ models with backorders and perishable inventory problems with price dependent demands. We refer the readers to the reviews of Khouja [1], Nahmias [2] and Silver [3] for references in the field of EOQ models, and the reviews of Petruzzi and Dada [4], Weatherford and Bodily [5] and Elmaghraby and Keskinocak [6] for references to the dynamic pricing inventory approach.

Gupta et al. [7] considered an inventory model when the demand intensity and reservation prices are time dependent. The demand function is assumed to be the form of \( D(p) = K e^{-\beta p \xi} \), where \( K \) is a measure of market size, \( \beta \) is a constant value to determine the sensitivity of demand to price \( p \), and \( \xi \) is a random variable.

Rakesh and Steinberg [8] simultaneously determined the pricing and ordering decisions for an inventory system in which the demand is assumed to be time and price dependent. Shinn et al. [9] simultaneously determined the pricing and lot-sizing decisions for an inventory system under conditions of permissible delay in payments. Urban and Baker [10] addressed a deterministic inventory model in which the demand is a multivariate function of price, time and inventory level, and extended the model to a case with a single price markdown. Chun [11] developed inventory models for determining the optimal list price and order quantity for a seasonal/perishable product which is sold for a limited period of time. The list price is posted at the start of a sales period.

In addition to the above literature, dynamic pricing problems have received much attention. Gallego and Ryzin [12] proposed a dynamic pricing model for selling a fixed number of items by a deadline. They found the optimal pricing policy in closed form under exponential demand. By considering the phenomenon of order cancellations, You [13] dealt with a dynamic pricing and lot sizing problem for seasonal products. You [14] dealt with an advance sales system with single price change.

Studies on perishable inventory models with pricing strategies mostly assume that either the advance or spot purchase customers exist. However, these phenomena may simultaneously exist. For solving the problem, an inventory model is to be developed by considering this phenomenon. In addition, many related research papers assumed that the number of price changes is given. This assumption implies that the price decision is determined under the assumption that the times of price changes are pre-specified. To relax this assumption, in the presented paper, the number of price setting (when to change price) and the corresponding prices are simultaneously determined. Taken together, we develop a perishable inventory model to resolve the optimal decisions for this problem. Generally, the purpose of this paper is to maximize the total profit through the simultaneous determination of (1) the order quantity, (2) the number of price settings, (3) the advance sales prices, and (4) the regular sales prices.

The preceding sections of this paper are organized as follows. Section 2 outlines all the assumptions made and formulates the problem as a mathematic model. Section 3 then analyzes the model and determines the optimal decisions. The analysis reveals that the Kuhn–Tucker saddle point theorems can be applied to develop an algorithm for finding the optimal decisions. Section 4 elucidates the features of the proposed algorithm, using a numerical example. Conclusions are finally drawn in Section 5.
2. Assumptions and formulation

Consider an inventory system where a firm purchases a perishable item at the start of a sales season \( L \) and sells it over that season. Suppose the firm provides the option of a prompt delivery and scheduled delivery service to customers. As mentioned previously, we refer to the former as the spot purchase demands and the latter as the forward purchase demands. We assume that regardless of the delivery size, an amount of fixed cost \( c_s \) is incurred to issue a delivery.

The spot and advance purchase demands for the item are assumed to be time and price dependent, and are respectively assumed to be the form of \( d_t^s(p) = a e^{-\alpha t} - \beta p \) and \( d_t^a(y) = a_2 e^{-\theta t} - g_2 y \), where \( p \) and \( y \) are respectively the spot sales price and advance sales price, and the values of \( \alpha, a, \beta, a_2, b_2 \) and \( g_2 \) are known and constant. Assume that the unit purchasing cost is \( c_p \) and the unit time inventory carrying cost per unit is \( h \).

In addition, we assume that in every equal time space \( T \), sales prices are reset and a delivery is issued. Let \( n \leq N_{\max} \) be the number of price settings; then \( T \) is given by \( T = L/n \). We use period \( j \) to represent the time interval between \( j \)-th and \( j + 1 \)-th price setting. Let \( p_j \) and \( y_j \) denote the advance and spot sales prices of period \( j, j = 1, 2, \ldots, n \). We assume that there is a cost \( c_o \) associated with a price setting.

Moreover, we assume that an advance purchase demands during a period are delivered at the end of that period. We make an assumption that forward purchase customers may cancel their orders with a constant rate \( \theta \) before receiving their orders. Customers who withdraw their orders are postulated to be charged a penalty. Also, the penalty is assumed to be linearly dependent with the time interval from the time of cancellation in a period up to the end of that period.

Let \( w_t \) denote the ratio of penalty to advance sales price when an advance purchase customer cancels his/her order at time \( t \) of a period. Then, we have \( w_t = t/T \). Finally, we assume that items remaining at the end of the sales season are disposed of zero.

The purpose of a firm is mainly to maximize the profit over the finite time interval \( L \) by simultaneously determining (1) the optimal order quantity \( Q \), (2) the number of prices setting \( n \), (3) the advance sales prices \( p = \{ p_j | 1 \leq j \leq n \} \), and (4) the advance sales prices \( y = \{ y_j | 1 \leq j \leq n \} \).

Now, we will develop the mathematical model. Suppose the firm orders \( q_j \) units of the item, set the number of price settings at \( n \), and set spot and advance sales prices at \( p^q = \{ p_j | 1 \leq j \leq n \} \) and \( y^q = \{ y_j | 1 \leq j \leq n \} \), respectively. Let \( D^q_t(j), 1 \leq j \leq n \) denote the cumulative advance purchase amount from the start of a period \( j \) up to the time \( t \) of that period. Then, for period \( j \) we have

\[
\frac{dD^q_t(j)}{dt} = d^q_{(j-1)T+t}(y_j) - \theta D^q_t(j), \quad 0 \leq t \leq T, \quad 1 \leq j \leq n. \tag{1}
\]

It is noted that advance sales purchases for any period are satisfied at the end of that period. Thus, the advance purchase amount at the start of period \( j \) is \( D^q_0(j) = 0 \). Using this condition to solve (1) gives

\[
D^q_t(j) = \frac{a_2 e^{-b_2 T} (e^{-b_2 T} - e^{-\theta t})}{b_2 - \theta} + \frac{g_2 (1 - e^{-\theta t}) y_j}{\theta}, \quad 0 \leq t \leq T, \quad 1 \leq j \leq n. \tag{2}
\]

Let \( S^s_i \) and \( S^a_i \) respectively represent the cumulative spot and advance sales amount from the start of the sales season up to the end of period \( i \). Then, we have

\[
S^s_i = \sum_{j=1}^{i} \int_{(j-1)T}^{jT} d^s_t(p_j) dt = \frac{\alpha}{a} \left( 1 - e^{-iTa} \right) - \beta T \sum_{j=1}^{i} p_j, \tag{2}
\]

\[
S^a_i = \sum_{j=1}^{i} D^a_t(j) = \frac{a_2 (1 - e^{-b_2 T}) (e^{-b_2 T} - e^{-\theta T})}{(b_2 - \theta)(1 - e^{-b_2 T})} + \frac{g_2 (1 - e^{-\theta T})}{\theta} \sum_{j=1}^{i} y_j. \tag{3}
\]

Let \( I_t(j), 1 \leq j \leq n \) denote the inventory level at time \( t \) of period \( j \). Then, the differential equation governing the system during the period \( j \) is given by

\[
\frac{dI_t(j)}{dt} = -d^q_{(j-1)T+t}(p_j). \tag{4}
\]
Since the advance sales amount up to period \( j \) is \( S^a_j - 1 \), and the cumulative spot sales up to the end of period \( j - 1 \) is \( S^s_j - 1 \), the inventory level at the start of period \( j \) is \( I^s_j = q - S^s_j - 1 - S^s_j - 1 \). Using this condition to solve (4), we get

\[
I_t(j) = q + \frac{\alpha}{a} e^{(T-t)a} e^{-jTa} + \beta p_j t - \frac{\alpha}{a} + \beta T \sum_{k=1}^{j-1} p_k + \frac{(1 - e^{(b_2-\theta)T}) e^{-jb_2T}}{(1 - e^{-b_2T})(b_2 - \theta)} a_2
+ \frac{(1 - e^{-\theta T})a_2}{(1 - e^{-b_2T})(b_2 - \theta)} - \frac{a_2}{b_2 - \theta} + \frac{g_2(1 - e^{-\theta T})}{\theta} \sum_{k=1}^{j-1} y_k.
\]

The objective function is comprised of the spot sales revenues \( R^s(n, p^n, y^n) \), advance sales revenues \( R^a(n, p^n, y^n) \), revenues from order cancellation \( R^c(n, p^n, y^n) \), inventory carrying cost \( C^h(n, p^n, y^n) \), purchasing cost \( C^p(n, p^n, y^n) \), and price setting cost \( C^p(n, p^n, y^n) \).

The spot sales revenues during period \( i \), \( R^s_i(n, p^n, y^n) \), and the total spot sales revenues, \( R^s(n, p^n, y^n) \), are respectively given by

\[
R^s_i(n, p^n, y^n) = \int_{(i-1)T}^{iT} d^s_i(t) p_i dt = \frac{\alpha e^{-iTa}(e^{aT} - 1)p_i}{a} - \beta T p_i^2,
\]

\[
R^s(n, p^n, y^n) = \alpha \frac{e^{aT} - 1}{a} \sum_{i=1}^{n} (e^{-iT} p_i) - \beta T \sum_{i=1}^{n} p_i^2.
\]

The advance purchase demand that should be delivered to customers at the end of period \( j \) is \( D^a_j \). Thus, the advance sales revenues during period \( i \), \( R^a_i(n, p^n, y^n) \) and the total advance sales revenues are respectively given by

\[
R^a_i(n, p^n, y^n) = D^a_i(T) y_i = \frac{a_2 e^{(b_2-\theta)T} - 1 e^{-ib_2T} y_i}{b_2 - \theta} - \frac{g_2(1 - e^{-\theta T})y_i^2}{\theta},
\]

\[
R^a(n, p^n, y^n) = \sum_{i=1}^{n} R^a_i(n, p^n, y)
= \frac{a_2 e^{(b_2-\theta)T} - 1 \sum_{i=1}^{n} (e^{-ib_2T} y_i)}{b_2 - \theta} - \frac{g_2(1 - e^{-\theta T}) \sum_{i=1}^{n} y_i^2}{\theta}.
\]

The number of order cancellation at time \( t \) of period \( j \) is given by \( \theta D^c_i(t) \). Thus, the revenues from order cancellation during period \( i \), \( R^c_i(n, p^n, y^n) \) and the total revenues from order cancellation are respectively given by

\[
R^c_i(n, p^n, y^n) = \int_{0}^{T} \theta D^c_i(t) w_i y_i dt
= \frac{a_2((b_2 - \theta^2)e^{b_2T} + \theta^2(1 + b_2T) - e^{(b_2-\theta)T} b_2^2(1 + \theta T)) e^{-ib_2T} y_i}{\theta b_2^2 T (b_2 - \theta)}
- \frac{g_2(T^2 \theta^2 - 2 + 2(1 + \theta T) e^{-\theta T}) y_i^2}{2 \theta^2 T},
\]

\[
R^c(n, p^n, y^n) = \sum_{i=1}^{n} R^c_i(n, p^n, y)
= \frac{a_2((b_2 - \theta^2)e^{b_2T} + \theta^2(1 + b_2T) - e^{(b_2-\theta)T} b_2^2(1 + \theta T)) \sum_{i=1}^{n} (e^{-ib_2T} y_i)}{\theta b_2^2 T (b_2 - \theta)}
- \frac{g_2(T^2 \theta^2 - 2 + 2(1 + \theta T) e^{-\theta T}) \sum_{i=1}^{n} y_i^2}{2 \theta^2 T}.
\]
The inventory holding cost during period $i$, $C^h_i(n, p^n, y^n)$, and the total inventory holding cost are respectively given by

$$C^h_i(n, p^n, y^n) = \int_{(i-1)T}^{iT} I_i(t)hdt = hTq + \frac{ha(e^{aT} - 1)e^{-iat}}{a^2} + 0.5h\beta T^2p_i - \frac{haT}{a} + h\beta T^2\sum_{j=1}^{i-1}p_j$$

$$- a_2hT(1 - e^{(b_2-\theta)T})e^{-ib_2T} \frac{(1 - e^{-b_2T})}{(1 - e^{-b_2T})} - a_2hT(1 - e^{-\theta T}) \frac{y_j}{b_2 - \theta} + \frac{hTg_2(1 - e^{-\theta T}) \sum_{j=1}^{i-1}y_j}{\theta},$$

$$C^h(n, p^n, y^n) = \sum_{i=1}^{n} H_i(n, p^n, y^n)$$

$$= nhTq + \frac{ha}{a^2}(1 - e^{-NTa}) + 0.5h\beta T^2\sum_{i=1}^{n} p_i - \frac{nhTa}{a} + h\beta T^2\sum_{i=1}^{n-1}(n-i)p_i$$

$$+ \frac{hT(e^{-\theta T} - e^{-b_2T})a_2(1 - e^{-nb_2T})}{(1 - e^{-b_2T})^2(b_2 - \theta)} + \frac{na_2hT(1 - e^{-\theta T})}{(1 - e^{-b_2T})}$$

$$- \frac{nhTa_2}{b_2 - \theta} + \frac{hTg_2(1 - e^{-\theta T}) \sum_{i=1}^{n-1}(n-i)y_i}{\theta}.$$  (5)

It is clear that the purchasing cost is $C^p = qc_p$, the price setting cost is $C^s = nc_s$ and the delivery setup cost is $(S^s_n + n)c_s$, since the number of deliveries issued for spot sales is $S^s_n$ and the number of deliveries issued for advance sales is $n$. Thus, our problem can be formulated as follows:

$$F(n, q, p^n, y^n) = R^s(n, p^n, y^n) + R^r(n, p^n, y^n) + R^c(n, p^n, y^n)$$

$$- C^h(n, p^n, y^n) - qc - nc_s - (s^s_n + n)c_s$$  (6)

subject to

$$p_i \leq \frac{ae^{-iTa}}{\beta},$$  (7)

$$y_i \leq \frac{a_2e^{-iTa}}{b_2},$$  (8)

$$n \leq N_{max}.$$  (9)

3. Analysis

In this section, we will develop an algorithm for finding the values of $q$, $n$, $p^n$ and $y^n$ optimally. Note any item that remains at the end of sales season is disposed of at zero, and shortages are lost sales. Thus the order quantity is expected to be the same as the total sales volume. Accordingly, from (2) and (3) we obtain

$$q = S^s_n + S^r_n$$

$$= \frac{a}{a}(1 - e^{-NTa}) + \frac{a_2(1 - e^{-nb_2T})(e^{-\theta T} - e^{-b_2T})}{(b_2 - \theta)(1 - e^{-b_2T})} - \frac{g_2(1 - e^{-\theta T}) \sum_{j=1}^{n}y_j}{\theta} - \beta T\sum_{j=1}^{n}p_j.$$  (10)

Substituting the value $q$ in (10) into $C^h(n, p^n, y^n)$ in (5), we can reduce the number of variables of the objective function in (6). Thus, our problem is reduced to the problem of finding the values of $n$, $p^n$ and $y^n$ that maximize
First, we have

\[ L = F(n, p^n, y^n) - \sum_{i=1}^{n} \lambda_i (p_i - \alpha e^{-iaT/\beta} + z_i^2) - \sum_{i=1}^{n} \eta_i (y_i - a_2 e^{-ib_2T/g_2 + u_i^2}). \]

Taking the partial derivatives of \( L \) with respect to \( p_i, y_i, z_i \) and \( u_i \), it is obtained that the Kuhn–Tucker necessary conditions for maximizing \( V_n \) are:

\[
\frac{\partial L}{\partial p_i} = \frac{\partial F}{\partial p_i} - \lambda_i = 0, \quad \frac{\partial L}{\partial y_i} = \frac{\partial F}{\partial y_i} - \eta_i = 0, \\
\frac{\partial L}{\partial \lambda_i} = -(p_i - \alpha e^{-iaT/\beta} + z_i^2) = 0, \\
\frac{\partial L}{\partial \eta_i} = -(y_i - a_2 e^{-ib_2T/g_2 + u_i^2}) = 0, \\
\frac{\partial L}{\partial z_i} = -2\lambda_i z_i = 0, \\
\frac{\partial L}{\partial u_i} = -2\eta_i u_i = 0. \tag{16}
\]

The restrictions on \( \lambda_i \)'s and \( \eta_i \)'s must hold as part of the Kuhn–Tucker necessary conditions are \( \lambda_i \geq 0 \) and \( \eta_i \geq 0 \) for all \( i \). In addition, the Kuhn–Tucker necessary conditions are sufficient if the profit function is concave. First, we have the following theorem.

**Theorem 3.1.** For any given \( n \), \( F(n, p^n, y^n) \) is a concave function of \( p^n \) and \( y^n \).

**Proof.** First, we have

\[
\frac{\partial F}{\partial p_i} = \frac{\partial F}{\partial y_i} = 0 \text{ for } i \neq j, \quad \frac{\partial^2 F}{\partial p_i \partial p_j} = -2\beta T, \quad \frac{\partial^2 F}{\partial p_i \partial y_j} = 0 \text{ for } i \neq j, \quad \frac{\partial^2 F}{\partial y_i \partial y_j} = 0.
\]

Thus, we have \( \frac{\partial^2 F}{\partial p_i^2} = -2\beta T, \frac{\partial^2 F}{\partial y_i^2} = 0 \) for \( i \neq j \), \( \frac{\partial^2 F}{\partial y_j^2} = -g_2 \frac{\phi(\theta T)}{\theta^2} \), \( \frac{\partial^2 F}{\partial p_i \partial y_j} = 0 \) for \( i \neq j \), and \( \frac{\partial^2 F}{\partial p_i \partial y_j} = 0 \). Then, we get the result that the value of \( k \) principal minor determinant of the Hessian matrix \( H \) has the sign of \((-1)^k, k = 1, 2, \ldots, n; \) thus \( H \) is negative-definite and the proof is completed.

For developing the optimal decisions, we will denote the following two functions:

\[ A_i = \frac{\beta T (hT (2i - 1) + 2(c + c_s))}{2} + \frac{\alpha e^{-iaT} (e^{aT} - 1 - 2aT)}{a}. \]
\[ B_i = \frac{a_2 e^{-ib_2 T} \phi(\theta T)}{\theta^2 T} + \frac{g_2 (1 - e^{-\theta T} (c + i h T))}{\theta} - \frac{a_2 e^{-((i-1)b_2 T (\theta^2 - b_2^2 (1 - e^{-\theta T})))}{\theta T (b_2 - \theta) b_2^2} + \frac{a_2 e^{-ib_2 T (\theta + b_2 \theta T - b_2^2 T)}}{(b_2 - \theta) b_2^2 T}. \]

**Lemma 3.1.** For fixed \( n \), \( p_i = p_i(1) \) for \( A_i > 0 \) and \( p_i = p_i(2) \) for \( A_i \leq 0 \) are feasible solutions to the KKT necessary conditions, where

\[
p_i(1) = \frac{ae^{-ita}}{\beta}, \tag{17}
\]
\[
p_i(2) = \frac{ae^{-iaT(e^{T} - 1) - 1 + (2(c + c_i) + (2i - 1)hT)}{2\beta aT} + \frac{2(c + c_i) + (2i - 1)hT}{4}. \tag{18}
\]

**Proof.** Case I: \( A_i > 0 \). In this case, let \( z_i^2 = 0 \) and we obtain that the value of \( p_i(1) \) in (17) is the solution to (13). Substituting \( p_i(1) \) into (11) for solving \( \lambda_i = A_i > 0 \). Case II: \( A_i \leq 0 \). In this case, let \( \lambda_i = 0 \), and we can obtain that the value of \( p_i(2) \) in (18) is the solution to (11). Substituting \( p_i(2) \) into (13) for solving \( z_i^2 \) yields \( z_i^2 = -A_i/(2\beta T) > 0. \]

**Lemma 3.2.** For fixed \( n \), \( y_i = y_i(1) \) for \( B_i > 0 \) and \( y_i = y_i(2) \) for \( B_i \leq 0 \) are feasible solutions to the KKT necessary conditions where

\[
y_i(1) = \frac{a_2 e^{-ib_2 T}}{g_2}, \tag{19}
\]
\[
y_i(2) = \frac{-a_2 \theta^T e^{-ib_2 T}}{g_2(\theta)^T \phi(\theta T)} + \frac{\theta a_2 e^{-(i-1)b_2 T (1 - e^{-\theta T})} - \theta^3 a_2 e^{-(i-1)b_2 T (1 - e^{-b_2 T})}}{g_2(b_2 - \theta) \phi(\theta T)} - \frac{b_2^2 g_2(b_2 - \theta) \phi(\theta T)}{\phi(\theta T)} + \frac{\theta T c(1 - e^{-\theta T})}{\phi(\theta T)} + \frac{i \theta h T^2 (1 - e^{-\theta T})}{\phi(\theta T)}. \tag{20}
\]

**Proof.** Case I: \( B_i > 0 \). In this case, let \( u_i^2 = 0 \) and we obtain that the value of \( y_i(1) \) in (19) is the solution to (14). Substituting \( y_i(1) \) into (12) for solving \( \eta_i = B_i > 0 \). Case II: \( B_i \leq 0 \). In this case, let \( \eta_i = 0 \) and we obtain that the value of \( y_i(2) \) in (20) is the solution to (12). Substituting \( y_i(2) \) into (14) for solving \( u_i^2 \) yields \( u_i^2 = -\theta^2 T B_i/(g_2 \phi(\beta T)) > 0. \]

From Lemmas 3.1 and 3.2, we see that the stationary points \( p_i(1), p_i(2), y_i(1) \) and \( y_i(2) \) are feasible solutions to the equation system (11)–(16) and yield a global constrained maximum of Problem \( V_n \), since \( F(n, P, y) \) is concave from Theorem 3.1 and the solution space is convex (linear restrictions). Let \( \lambda^*(n) \) and \( y^*(n) \) be the optimal spot and advance prices for period \( i \) for problem \( V_n \). Then, we have the following theorem.

**Theorem 3.2.** For fixed \( n \), \( \lambda^*(n) = p_i(1) \) for \( A_i > 0 \), \( \lambda^*(n) = p_i(2) \) for \( A_i \leq 0 \), \( y^*(n) = y_i(1) \) for \( B_i > 0 \) and \( y^*(n) = y_i(2) \) for \( B_i \leq 0 \) are optimal prices.

Now, Theorem 3.2 can be applied to generate a solution procedure for finding the optimal values of \( N, P, y \) and \( Q \).

**Solution procedure**
1. Set \( n = 1, N = 1, \) and \( F^* = 0. \)
2. While \( n \leq N_{\text{max}} \) do Steps 3–5.
3. While \( i \leq n \) do Steps 3.1 and 3.2.
   3.1. Calculate \( A_i \) and \( B_i \)
   3.2. Calculate \( p_i^*(n) \) and \( y_i^*(n) \) according to Lemmas 3.1 and 3.2, respectively.
4. Calculate \( q \) by (10) and \( F(n, P^*, y^*) \) by (6).
5. If \( F(n, P^*, y^*) > F^* \), then let \( F^* = F(n, P^*, y^*), N = n, P = P^*, y = y^* \) and \( Q = q. \)
6. Output.
4. An numerical example

Suppose a retailer purchases a seasonal product at $c = $3 per unit at the start of a sales season, and sells the purchased product over the sales season. The length of the sales season is assumed to be $L = 1200$ units of time (e.g., day). Inventory holding, price setting and delivery setup cost are assumed to be $h = $0.003, $c_o = $100, and $c_s = 1$, respectively. Spot demand and forward demand for the product are assumed to be in the forms of $d_s(p) = \alpha e^{-\beta p}$ and $d_r(y) = a_2 e^{-b_2 y} - g_2 y = 9e^{-0.001t} - 0.8y$, respectively. The cancellation rate is assumed to be $\theta = 0.01$. The maximum permissible number of price setting is restricted to $N_{\text{max}} = 10$.

For different values of $n$, by the solution procedure, we can obtain $F(n, p^n, y^n)$ (see Table 1).

We see from Table 1 that the profit of $7721.1 is maximum for $n = 4$, and the corresponding optimal order quantity is 2124.39 units. Additionally, $n = 4$ implies that price changes are set 3 times, at time points 300, 600 and 900, respectively. The optimal spot prices 8.40, 7.25, 5.81 and 4.30 as well as the optimal advance prices 5.59, 4.55, 3.84 and 3.36, can be computed and are set during time intervals $[0, 300]$, $[301, 600]$, $[601, 900]$ and $[901, 1200]$ respectively.

We also observe from Table 1 that the optimal profit obtaining from the static pricing strategy ($n = 1$) is $F^*_1 = 3417.76$. Comparing this value with that of the dynamic pricing strategy, it can be concluded that the dynamic pricing strategy outperforms the static pricing strategy.

5. Conclusion

This paper deals with a perishable inventory models with time and price dependent demand. Most dynamic pricing inventory works assume that customers are either the advance purchase type or spot purchase type. This implies that the demands of advance and spot purchase customers do not occur at the same time. However, in real world practice, demands of advance and spot purchase customers may simultaneously exist. Taking this factor into account, a mathematical model to deal with this problem has been developed in this paper. We show that for any number of price settings, the profit function is a concave function of the sales prices, and the advance and spot sales prices can be obtained optimally. From this characteristic, we then developed a procedure to find the order size, number of price settings, advance sales prices and spot sales prices optimally. Numerical examples are provided to illustrate how to solve this complicated problem optimally by using the proposed method. For possible further research, the proposed model can be extended to cases with deteriorating factor-extensions.

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