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Soft Set Theory

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Abstract—In this paper, the authors study the theory of soft sets initiated by Molodtsov. The authors define equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection are defined. De Morgan's laws and a number of results are verified in soft set theory. © 2003 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. However, there are many complicated problems in economics, engineering, environment, social science, medical science, etc., that involve data which are not always all crisp. We cannot successfully use classical methods because of various types of uncertainties present in these problems. There are theories, wiz., theory of probability, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2,3], theory of vague sets [4], theory of interval mathematics, [3,5], and theory of rough sets [6] which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties as pointed out in [7]. The reason for these difficulties is, possibly, the inadequency of the parametrization tool of the theories. Consequently, Molodtsov [7] initiated the concept of soft theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties. (We are aware of the soft sets defined by Pawlak [8], which is a different concept and useful to solve some other type of problems.) Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work [7]. In the present paper, we make a theoretical study of the "Soft Set Theory" in more detail.

2. THEORY OF SOFT SETS

Molodtsov [7] defined the soft set in the following way. Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U and $A \subset E$.

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Definition 2.1. (See [7].) A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F: A \rightarrow P(U)$$
.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ -approximate elements of the soft set (F,A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [7], one of which we present below.

EXAMPLE 2.1. Suppose the following.

U is the set of houses under consideration.

E is the set of parameters. Each parameter is a word or a sentence.

 $E = \{\text{expensive}; \text{ beautiful}; \text{ wooden}; \text{ cheap}; \text{ in the green surroundings}; \text{ modern}; \text{ in good repair}\}.$

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. The soft set (F, E) describes the "attractiveness of the houses" which Mr. X (say) is going to buy.

We consider below the same example in more detail for our next discussion.

Suppose that there are six houses in the universe U given by

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$$
 and $E = \{e_1, e_2, e_3, e_4, e_5\}$,

where

 e_1 stands for the parameter 'expensive',

 e_2 stands for the parameter 'beautiful',

e₃ stands for the parameter 'wooden',

 e_4 stands for the parameter 'cheap',

e₅ stands for the parameter 'in the green surroundings'.

Suppose that

$$F(e_1) = \{h_2, h_4\},$$

$$F(e_2) = \{h_1, h_3\},$$

$$F(e_3) = \{h_3, h_4, h_5\},$$

$$F(e_4) = \{h_1, h_3, h_5\},$$

$$F(e_5) = \{h_1\}.$$

The soft set (F, E) is a parametrized family $\{F(e_i), i = 1, 2, 3, ..., 8\}$ of subsets of the set U and gives us a collection of approximate descriptions of an object. Consider the mapping F which is "houses (.)" where dot (.) is to be filled up by a parameter $e \in E$. Therefore, $F(e_1)$ means "houses (expensive)" whose functional-value is the set $\{h_2, h_4\}$.

Thus, we can view the soft set (F, E) as a collection of approximations as below: $(F, E) = \{\text{expensive houses} = \{h_2, h_4\}, \text{ beautiful houses} = \{h_1, h_3\}, \text{ wooden houses} = \{h_3, h_4, h_5\}, \text{ cheap houses} = \{h_1, h_3, h_5\}, \text{ in the green surroundings} = \{h_1\}\}, \text{ where each approximation has two parts:}$

- (i) a predicate p; and
- (ii) an approximate value-set v (or simply to be called value-set v).

For example, for the approximation "expensive houses = $\{h_2, h_4\}$ ", we have the following:

- (i) the predicate name is expensive houses; and
- (ii) the approximate value set or value set is $\{h_2, h_4\}$.

U	'Expensive'	'Beautiful'	'Wooden'	'Cheap'	'In the green surroundings'
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	1	1	0
h_4	1	0	1	0	0
h_5	0	0	1	1	0
h_6	0	0	0	0	0

Table 1. Tabular representation of a soft set.

Thus, a soft set (F, E) can be viewed as a collection of approximations below:

$$(F,E) = \{p_1 = v_1, p_2 = v_2, \dots, p_n = v_n\}.$$

For the purpose of storing a soft set in a computer, we could represent a soft set in the form of Table 1, (corresponding to the soft set in the above example).

DEFINITION 2.2. The class of all value sets of a soft set (F, E) is called value-class of the soft set and is denoted by $C_{(F,E)}$.

For the above example, $C_{(F,E)} = \{v_1, v_2, \dots, v_n\}$. Clearly, $C_{(F,E)} \subseteq P(U)$.

DEFINITION 2.3. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

- (i) $A \subset B$, and
- (ii) $\forall \epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations.

We write $(F, A) \tilde{\subset} (G, B)$.

(F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by $(F,A) \tilde{\supset} (G,B)$.

DEFINITION 2.4. EQUALITY OF TWO SOFT SETS. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

EXAMPLE 2.2. Let $A = \{e_1, e_3, e_5\} \subset E$, and $B = \{e_1, e_2, e_3, e_5\} \subset E$. Clearly, $A \subset B$.

Let (F, A) and (G, B) be two soft sets over the same universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ such that

$$G(e_1) = \{h_2, h_4\},$$
 $G(e_2) = \{h_1, h_3\},$ $G(e_3) = \{h_3, h_4, h_5\},$ $G(e_5) = \{h_1\},$ and $F(e_1) = \{h_2, h_4\},$ $F(e_3) = \{h_3, h_4, h_5\},$ $F(e_5) = \{h_1\}.$

Therefore, $(F, A) \tilde{\subset} (G, B)$.

DEFINITION 2.5. NOT SET OF A SET OF PARAMETERS. Let $E = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of parameters. The NOT set of E denoted by E is defined by $E = \{e_1, e_2, e_3, \dots, e_n\}$ where $e_i = \text{not } e_i$, $\forall i$. (It may be noted that $e_i = \text{not } e_i$) and $e_i = \text{not } e_i$ are different operators.)

The following results are obvious.

Proposition 2.1.

- 1.](]A) = A.
- $2. \ \](A \cup B) = (]A \cup]B).$
- 3. $\rceil(A \cap B) = (\rceil A \cap \rceil B)$.

EXAMPLE 2.3. Consider the example as presented in Example 2.1.

Here, $E = \{\text{not expensive}; \text{ not beautiful}; \text{ not wooden}; \text{ not cheap}; \text{ not in the green surroundings}\}.$

DEFINITION 2.6. COMPLEMENT SOFT SET OF A SOFT SET. The complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,]A)$, where $F^c:]A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(]\alpha), \forall \alpha \in]A$.

Let us call F^c to be the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.

EXAMPLE 2.4. Consider Example 2.1.

Here $(F, E)^c = \{\text{not expensive houses} = \{h_1, h_3, h_5, h_6\}, \text{ not beautiful houses} = \{h_2, h_4, h_5, h_6\}, \text{ not wooden houses} = \{h_1, h_2, h_6\}, \text{ not cheap houses} = \{h_2, h_4, h_6\}, \text{ not in the green surroundings houses} = \{h_2, h_3, h_4, h_5, h_6\}\}.$

DEFINITION 2.7. NULL SOFT SET. A soft set (F, A) over U is said to be a NULL soft set denoted by Φ , if $\forall \epsilon \in A$, $F(\epsilon) = \phi$, (null-set).

EXAMPLE 2.5. Suppose that,

U is the set of wooden houses under consideration;

A is the set of parameters.

Let there be five houses in the universe U given by

$$U = \{h_1, h_2, h_3, h_4, h_5\}$$
 and $A = \{\text{brick; muddy; steel; stone}\}.$

The soft set (F, A) describes the "construction of the houses". The soft sets (F, A) is defined as

F (brick) means the brick built houses,

F (muddy) means the muddy houses,

F (steel) means the steel built houses,

F (stone) means the stone built houses.

The soft set (F, A) is the collection of approximations as below:

 $(F,A) = \{ \text{brick built houses} = \phi, \text{ muddy houses} = \phi, \text{ steel built houses} = \phi, \}$

stone built houses = ϕ }.

Here, (F, Λ) is NULL soft set.

DEFINITION 2.8. ABSOLUTE SOFT SET. A soft set (F, A) over U is said to be absolute soft set denoted by \tilde{A} , if $\forall \epsilon \in A$, $F(\epsilon) = U$.

Clearly, $\tilde{A}^c = \Phi$ and $\Phi^c = \tilde{A}$.

EXAMPLE 2.6. Suppose that,

U is the set of wooden houses under consideration;

B is the set of parameters.

Let there be five houses in the universe U given by

$$U = \{h_1, h_2, h_3, h_4, h_5\}$$
 and $B = \{\text{not brick}; \text{ not muddy}; \text{ not steel}; \text{ not stone}\}.$

The soft set (G, B) describes the "construction of the houses". The soft sets (G, B) is defined as

G (not brick) means the houses not built by brick,

G (not muddy) means the not muddy houses,

G (not steel) means the houses not built by steel,

G (not stone) means the houses not built by stone.

The soft set (G, B) is the collection of approximations as below:

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(G,B) = \{ \text{not brick built houses} = \{h_1, h_2, h_3, h_4, h_5 \}, \text{ not muddy houses} = \{h_1, h_2, h_3, h_4, h_5 \}, \}
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 $\{h_5\}$, not steel built house = $\{h_1, h_2, h_3, h_4, h_5\}$, not stone built house = $\{h_1, h_2, h_3, h_4, h_5\}$.

The soft set (G, B) is the absolute soft set.

By the suggestions given by Molodtsov in [7], we present the notion of AND and OR operations on two soft sets as below.

DEFINITION 2.9. AND OPERATION ON TWO SOFT SETS. If (F, A) and (G, B) are two soft sets then "(F, A) AND (G, B)" denoted by $(F, A) \land (G, B)$ is defined by $(F, A) \land (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall (\alpha, \beta) \in A \times B$.

EXAMPLE 2.7. Consider the soft set (F, A) which describes the "cost of the houses" and the soft set (G, B) which describes the "attractiveness of the houses".

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$, $A = \{\text{very costly; costly; cheap}\}$ and $B = \{\text{beautiful; in the green surroundings; cheap}\}$.

Let F (very costly) = $\{h_2, h_4, h_7, h_8\}$, F (costly) = $\{h_1, h_3, h_5\}$, F (cheap) = $\{h_6, h_9, h_{10}\}$, and G (beautiful) = $\{h_2, h_3, h_7\}$, G (in the green surroundings) = $\{h_5, h_6, h_8\}$, G (cheap) = $\{h_6, h_9, h_{10}\}$. Then $(F, A) \land (G, B) = (H, A \times B)$, where H (very costly, beautiful) = $\{h_2, h_7\}$, H (very costly, in the green surroundings) = $\{h_8\}$, H (very costly, cheap) = ϕ , H (costly, beautiful) = $\{h_3\}$, H (costly, in the green surroundings) = $\{h_5\}$, H (cheap, cheap) = $\{h_6, h_9, h_{10}\}$.

DEFINITION 2.10. OR OPERATION ON TWO SOFT SETS. If (F, A) and (G, B) be two soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (O, A \times B)$, where, $O(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall (\alpha, \beta) \in A \times B$.

EXAMPLE 2.8. Consider Example 2.7 above. We see that $(F, A) \vee (G, B) = (O, A \times B)$, where O (very costly, beautiful) = $\{h_2, h_3, h_4, h_7, h_8\}$, O (very costly, in the green surroundings) = $\{h_2, h_4, h_5, h_6, h_7, h_8\}$, O (very costly, cheap) = $\{h_2, h_4, h_6, h_7, h_8, h_9, h_{10}\}$, O (costly, beautiful) = $\{h_1, h_3, h_5, h_6, h_8\}$, O (costly, cheap) = $\{h_1, h_3, h_5, h_6, h_9, h_{10}\}$, O (cheap, beautiful) = $\{h_2, h_3, h_6, h_7, h_9, h_{10}\}$, O (cheap, in the green surroundings) = $\{h_5, h_6, h_8, h_9, h_{10}\}$, O (cheap, cheap) = $\{h_6, h_9, h_{10}\}$.

We see that the following De Morgan's types of results are true.

Proposition 2.2.

(i)
$$((F,A) \vee (G,B))^c = (F,A)^c \wedge (G,B)^c$$
.

(ii)
$$((F,A) \wedge (G,B))^c = (F,A)^c \vee (G,B)^c$$
.

PROOF.

(i) Suppose that $(F, A) \vee (G, B) = (O, A \times B)$. Therefore, $((F, A) \vee (G, B))^c = (O, A \times B)^c = (O^c, (A \times B))$. Now,

$$(F,A)^c \wedge (G,B)^c = (F^c, A) \wedge (G^c, B),$$

= $(J, A \times B),$ where $J(x,y) = F^c(x) \cap G^c(y),$
= $(J, A \times B).$

Now, take $({}_{\uparrow}\alpha,{}_{\uparrow}\beta) \in {}_{\uparrow}(A \times B)$. Therefore,

$$\begin{split} O^c\left({}_{\uparrow}\alpha,{}_{\uparrow}\beta\right) &= U - O(\alpha,\beta), \\ &= U - [F(\alpha \cup G(\beta)], \\ &= [U - F(\alpha)] \cap [U - G(\beta)], \\ &= F^c\left({}_{\uparrow}\alpha\right) \cap G^c\left({}_{\uparrow}\beta\right), \\ &= J\left({}_{\uparrow}\alpha,{}_{\uparrow}\beta\right). \end{split}$$

 $\Rightarrow O^c$ and J are same. Hence, proved.

(ii) Suppose that $(F,A) \wedge (G,B) = (H,A \times B)$. Therefore, $((F,A) \wedge (G,B))^c = (H,A \times B)^c = (H^c, \exists (A \times B))$. Now,

$$(F,A)^{c} \vee (G,B)^{c} = (F^{c}, \rceil A) \vee (G^{c}, \rceil B),$$

$$= (K, \rceil A \times \rceil B), \quad \text{where } K(x,y) = F^{c}(x) \cup G^{c}(y),$$

$$= (K, \rceil (A \times B)).$$

Now, take $(\alpha, \beta) \in (A \times B)$.

Therefore,

$$H^{c}(\gamma\alpha,\gamma\beta) = U - H(\alpha,\beta),$$

$$= U - [F(\alpha) \cap G(\beta)],$$

$$= [U - F(\alpha)] \cup [U - G(\beta)],$$

$$= F^{c}(\gamma\alpha) \cup G^{c}(\gamma\beta),$$

$$= K(\gamma\alpha,\gamma\beta).$$

 $\Rightarrow H^c$ and K are same. Hence, proved.

DEFINITION 2.11. Union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = F(e),$$
 if $e \in A - B$,
 $= G(e),$ if $e \in B - A$,
 $= F(e) \cup G(e),$ if $e \in A \cap B$.

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

In the above example, $(F, A)\tilde{\cup}(G, B) = (H, C)$, where H (very costly) = $\{h_2, h_4, h_7, h_8\}$, H (costly) = $\{h_1, h_3, h_5\}$, H (cheap) = $\{h_6, h_9, h_{10}\}$, H (beautiful) = $\{h_2, h_3, h_7\}$, and H (in the green surroundings) = $\{h_5, h_6, h_8\}$.

DEFINITION 2.12. Intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C=A\cap B$, and $\forall \epsilon\in C$, H(e)=F(e) or G(e), (as both are same set). We write $(F,A)\tilde{\cap}(G,B)=(H,C)$.

In the above example, intersection of two soft sets (F, A) and (G, B) is the soft set (H, C), where $C = \{cheap\}$ and $H(cheap) = \{h_6, h_9, h_{10}\}$.

The following results are obvious.

Proposition 2.3.

- (i) $(F, A)\tilde{\cup}(F, A) = (F, A)$.
- (ii) $(F, A) \cap (F, A) = (F, A)$.
- (iii) $(F, A)\tilde{\cup}\Phi = \Phi$, where Φ is the null soft set.
- (iv) $(F, A) \cap \Phi = \Phi$.
- (v) $(F, A)\tilde{\cup}\tilde{A} = \tilde{A}$, where \tilde{A} is the absolute soft set.
- (vi) $(F, A) \cap \tilde{A} = (F, A)$.

Proposition 2.4.

- (i) $((F,A)\tilde{\cup}(G,B))^c = (F,A)^c\tilde{\cup}(G,B)^c$.
- (ii) $((F,A)\tilde{\cap}(G,B))^c = (F,A)^c\tilde{\cap}(G,B)^c$.

PROOF.

(i) Suppose that $(F, A)\tilde{\cup}(G, B) = (H, A \cup B)$, where

$$H(\alpha) = F(\alpha),$$
 if $\alpha \in A - B,$
 $= G(\alpha),$ if $\alpha \in B - A,$
 $= F(\alpha) \cup G(\alpha),$ if $\alpha \in A \cap B.$

Therefore,

$$((F, A)\tilde{\cup}(G, B))^c = (H, A \cup B)^c,$$
$$= (H^c, A \cup B).$$

Now, $H^{c}(\alpha) = U - H(\alpha), \forall \alpha \in A \cup B$. Therefore,

$$\begin{split} H^{c}\left({\scriptstyle \uparrow}\alpha \right) &= F^{c}\left({\scriptstyle \uparrow}\alpha \right), & \text{if } {\scriptstyle \uparrow}\alpha \in \rbrack A - \rbrack B, \\ &= G^{c}\left({\scriptstyle \uparrow}\alpha \right), & \text{if } {\scriptstyle \uparrow}\alpha \in \rbrack B \dot{-} \rbrack A, \\ &= F^{c}\left({\scriptstyle \uparrow}\alpha \right) \cup G^{c}({\scriptstyle \uparrow}\alpha), & \text{if } {\scriptstyle \uparrow}\alpha \in \rbrack A \cap \rbrack B. \end{split}$$

Again,

$$(F,A)^{c}\tilde{\cup}(G,B)^{c} = (F^{c}, A)\tilde{\cup}(G^{c}, B),$$

= $(K, A\cup B),$ (say),

where,

$$K({}_{\!1}\alpha) = F^c({}_{\!1}\alpha), \qquad \text{if } {}_{\!1}\alpha \in]A -]B,$$

$$= G^c({}_{\!1}\alpha), \qquad \text{if } {}_{\!1}\alpha \in]B -]A,$$

$$= F^c({}_{\!1}\alpha) \cup G^c({}_{\!1}\alpha), \qquad \text{if } {}_{\!1}\alpha \in]A \cap]B.$$

 $\Rightarrow H^c$ and K are same. Hence, proved.

(ii) Suppose that $((F,A)\tilde{\cap}(G,B)) = (H,A\cap B)$. Therefore, $((F,A)\tilde{\cap}(G,B))^c = (H^c, \lceil A\cap \rceil B)$. Now,

$$(F,A)^{c} \tilde{\cap} (G,B)^{c} = (F^{c}, A) \tilde{\cap} (G^{c}, B),$$

= $(K, A) B$, say

where, $\forall_{1}\alpha \in (A \cap B)$, we have

$$egin{aligned} K(_1lpha) &= F^c\left(_1lpha
ight) & ext{ or } & G^c\left(_1lpha
ight), \ &= F(lpha) & ext{ or } & G(lpha), & ext{ where } lpha \in A\cap B, \ &= H(lpha), \ &= H^c\left(_1lpha
ight). \end{aligned}$$

 $\Rightarrow K$ and H^c are same function. Hence, proved.

The following propositions are straight forward and so stated below without proofs.

PROPOSITION 2.5. If (F, A), (G, B), and (H, C) are three soft sets over U, then

- (i) $(F, A)\tilde{\cup}((G, B)\tilde{\cup}(H, C)) = ((F, A)\tilde{\cup}(G, B))\tilde{\cup}(H, C),$
- (ii) $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C)) = ((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C),$
- (iii) $(F, A)\tilde{\cup}((G, B)\tilde{\cap}(H, C)) = ((F, A)\tilde{\cup}(G, B))\tilde{\cap}((F, A)\tilde{\cup}(H, C)),$
- (iv) $(F, A) \tilde{\cap} ((G, B) \tilde{\cup} (H, C)) = ((F, A) \tilde{\cap} (G, B)) \tilde{\cup} ((F, A) \tilde{\cap} (H, C)).$

PROPOSITION 2.6. If (F, A), (G, B), and (H, C) are three soft sets over U, then

- (i) $(F, A) \lor ((G, B) \lor (H, C)) = ((F, A) \lor (G, B)) \lor (H, C)$,
- (ii) $(F, A) \wedge ((G, B) \wedge (H, C)) = ((F, A) \wedge (G, B)) \wedge (H, C),$
- (iii) $(F, A) \lor ((G, B) \land (H, C)) = ((F, A) \lor (G, B)) \land ((F, A) \lor (H, C)),$
- (iv) $(F, A) \wedge ((G, B) \vee (H, C)) = ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C))$.

3. CONCLUSION

Molodtsov [7] initiated the concept of soft set theory, giving several applications in various directions. Our work in this paper is completely theoretical. We have defined the operations AND, OR, union, intersection of two soft sets, supported by examples. We have proved some propositions on soft set operations.

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