Characteristic size and mass of galaxies in the Bose–Einstein condensate dark matter model

Jae-Weon Lee [a,b,*]

[a] *Department of Physics, North Carolina State University, Raleigh, NC 27695, USA*
[b] *Department of Renewable Energy, Jungwon University, 85 Munmu-ro, Goesan-eup, Goesan-gun, Chungcheongbuk-do, 367-805, Republic of Korea*

A B S T R A C T

We study the characteristic length scale of galactic halos in the Bose–Einstein condensate (or scalar field) dark matter model. Considering the evolution of the density perturbation we show that the average background matter density determines the quantum Jeans mass and hence the spatial size of galaxies at a given epoch. In this model the minimum size of galaxies increases while the minimum mass of the galaxies decreases as the universe expands. The observed values of the mass and the size of the dwarf galaxies are successfully reproduced with the dark matter particle mass $m \simeq 5 \times 10^{-22}$ eV. The minimum size is about $6 \times 10^{-3}\sqrt{m/H}\lambda_c$ and the typical rotation velocity of the dwarf galaxies is $O(\sqrt{H/m})$ c, where H is the Hubble parameter and λ_c is the Compton wave length of the particle. We also suggest that ultra compact dwarf galaxies are the remnants of the dwarf galaxies formed in the early universe.

One of the long standing questions in astronomy is what determines the size of galaxies. In this paper we show that the Bose–Einstein condensate (BEC) dark matter (DM) or the scalar field dark matter (SFDM) can explain the minimum size and the mass of galaxies in a unified way.

DM remains a great mystery in astrophysics, particle physics and cosmology. The cold dark matter (CDM) model is very successful in explaining the large scale structures in the universe, but has many problems in explaining galactic structures. For example, one of the early evidences for the DM presence is the flatness of galactic rotation curves [1], however the CDM is not so successful in explaining the rotation curves in galaxy cores. Numerical studies with ΛCDM model predict a cusped halo central density and many subhalos, which are also in discord with observational data [2–5]. On the other hand, the BEC/SFDM [6–9] can be a good alternative to the CDM, because the BEC/SFDM plays the role of the CDM at super-galactic scales and suppresses sub-galactic structures. In this model the DM is a BEC of the scalar particles with the ultra-light mass $m \simeq 5 \times 10^{-22}$ eV, whose quantum nature prevents the formation of the structures smaller than a galaxy due to the long Compton wavelength $\lambda_c = 2\pi\hbar/mc \simeq 0.08$ pc.

There are two other difficulties the CDM models encounter. First, the studies on satellite dwarf spheroidal (dSph) galaxies of the Milky Way [10,11] indicate that a typical dSph never has a size $<$ kpc, and that the mass enclosed within the radius of 300 pc in dwarf galaxies is approximately constant ($\sim 10^7 M_\odot$) regardless of their luminosity [12]. This result implies the existence of a minimum mass scale in addition to the minimum length scale for DM dominated objects [10,13]. However, without introducing the roles of visible matter the CDM models usually predict DM dominated structures down to $10^{-6} M_\odot$. Second, the observations [14–16] of the size evolution of the most massive galaxies imply that these galaxies rapidly grow their size about 5 times since $z \sim 2$ while in the CDM models we expect compact early galaxies having smaller masses.

In Ref. [17] we showed that BEC/SFDM can explain the minimum mass of dwarf galaxies, if there is a minimum length scale. We also proposed that the size evolution of the massive galaxies can be attributed to the evolution of a length scale ξ of BEC DM [18]. In these works we considered the various length scales for ξ such as λ_c, a thermal de Broglie wavelength or a self-interaction scale. For all galaxies $\xi \gg \lambda_c$, and we need to find the exact physical origin of this long scale, which is the main subject of this paper.

The conjecture that DM is in BEC has a long history. (See Refs. [19–24] for a review.) Baldeschi et al. [6] studied the galactic halos of self-gravitating bosons, and Membrado et al. [7] calculated the rotation curves of self-gravitating boson halos. Sin [8]

* Correspondence to: Department of Renewable Energy, Jungwon University, 85 Munmu-ro, Goesan-eup, Goesan-gun, Chungcheongbuk-do, 367-805, Republic of Korea.
E-mail address: scikid@jwu.ac.kr.

suggested that the halos are like atoms made of ultra-light BEC DM. Lee and Koh [9] suggested that the DM halos are the giant boson stars described by the relativistic scalar field theory. Similar ideas were suggested by many authors [25–43]. In literature it has been shown that BEC/SFDM could explain the many observed aspects such as rotation curves [28,44–46], the large scale structures of the universe [47], the cosmic background radiation, and spiral arms [48].

In this paper, we show that BEC/SFDM has the natural length scale and the mass scale determined only by background matter density and the DM particle mass m. In the BEC DM model [8] a galactic DM halo is described with the wave function $\psi(\mathbf{r})$, which is the solution of the Gross–Pitaevskii equation (GPE)

$$i\hbar\partial_t\psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}, t) + m\Phi\psi(\mathbf{r}, t) \tag{1}$$

with a self-gravitation potential Φ. This equation could be obtained from the mean field approximation of a BEC Hamiltonian or the non-relativistic approximation of SFDM action [9]. For simplicity, we consider the spherical symmetric case with

$$\Phi(r) = \int_0^r dr' \frac{1}{r'^2} \int_0^{r'} dr'' 4\pi r''^2 (GmM|\psi(r)|^2 + \rho_v), \tag{2}$$

where M is the mass of the halo, and ρ_v is the mass density of visible matter. We do not consider a particle self-interaction term in this paper.

The Madelung representation [20,23]

$$\psi(r, t) = \sqrt{A(r, t)} e^{iS(r, t)} \tag{3}$$

is useful for studying the cosmological structure formation in the fluid approach. Here the amplitude A and DM density have a relation $\rho = mA$. Substituting Eq. (3) into GPE, one can obtain a continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\mathbf{v}) = 0 \tag{4}$$

and a modified Euler equation

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla\Phi + \frac{\nabla p}{\rho} - \frac{\nabla Q}{m} = 0 \tag{5}$$

with a quantum potential $Q \equiv \frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}$, a fluid velocity $\mathbf{v} \equiv \nabla S/2m$, and the pressure from a self-interaction pressure p (if there is). Here, Δ is the Laplacian. The quantum pressure term $\nabla Q/m$ is the key difference between the CDM and the BEC DM. Perturbing the equations (4) and (5) around $\rho = \bar{\rho}$, $\mathbf{v} = 0$, and $\Phi = 0$ and then combining the two perturbed equations gives a differential equation for density perturbation $\delta\rho \equiv \rho - \bar{\rho}$,

$$\frac{\partial^2\delta\rho}{\partial t^2} + \frac{\hbar^2}{4m^2}\nabla^2(\nabla^2\delta\rho) - c_s^2\nabla^2\delta\rho - 4\pi G\bar{\rho}\delta\rho = 0, \tag{6}$$

where c_s is the sound velocity from p, and $\bar{\rho}$ is the average background matter density (see, for example, Ref. [49] for details). We have ignored the effect of the cosmic expansion in this equation for simplicity. We can rewrite this equation into the Fourier transformed equation of the density contrast $\delta \equiv \delta\rho/\bar{\rho} = \delta_k e^{ik\cdot r}$ with a wave vector k,

$$\frac{d^2\delta_k}{dt^2} + \left[(c_q^2 + c_s^2)k^2 - 4\pi G\bar{\rho}\right]\delta_k = 0, \tag{7}$$

where $c_q = \hbar k/2m$ is a quantum velocity. Note that the k^4 dependent term (the c_q dependent term) came from the perturbation of

the quantum pressure term. From this equation we can see that the BEC DM behaves like the CDM for a small k (for a large scale) while for a large k (at a small scale) the quantum pressure disturbs the structure formation. If the self-interaction is negligible we can ignore the c_s term. Equating $c_q^2 k^2$ with $4\pi G\bar{\rho}$ defines the time dependent quantum Jeans length scale [30],

$$\lambda_Q(z) = \frac{2\pi}{k} = \left(\frac{\pi^3\hbar^2}{m^2 G\bar{\rho}(z)}\right)^{1/4}$$

$$\simeq 55593\left(\frac{\rho_b}{m_{22}^2\Omega_m h^2\bar{\rho}(z)}\right)^{1/4} \text{ pc}, \tag{8}$$

where the current matter density $\rho_b = 2.775 \times 10^{11}\Omega_m h^2 \ M_\odot/$Mpc^3, the (dark + visible) matter density parameter $\Omega_m = 0.315$ [50], $h = 0.673$ and $m_{22} = m/10^{-22}$ eV. The quantum Jeans mass can be defined as

$$M_J(z) = \frac{4\pi}{3}\bar{\rho}(z)\lambda_Q^3 = \frac{4}{3}\pi^{\frac{13}{4}}\left(\frac{\hbar}{G^{\frac{1}{2}}m}\right)^{\frac{3}{2}}\bar{\rho}(z)^{\frac{1}{4}}, \tag{9}$$

which is the minimum mass of the DM structures at z. Note that the only time dependent term in the righthand side is the average density.

Though λ_Q is related to the minimum length scale of DM dominated objects [51,52], λ_Q alone does not determine the actual size of galaxies. Usually, $\lambda_Q > \xi > \lambda_c$. We need a governing equation for stable configurations of the DM dominated objects. To find the characteristic length ξ we study the ground state of the GPE. In the BEC/SFDM model, $\xi \sim \hbar/m\Delta v$ due to the uncertainty principle, where Δv is the velocity dispersion of DM in a halo. However, we have not been able to derive Δv from any theory so far. From Eq. (1) the energy E of the halo can be approximated as

$$E(\xi) \simeq \frac{\hbar^2}{2m\xi^2} + \int_0^\xi dr' \frac{Gm}{r'^2} \int_0^{r'} dr'' 4\pi r''^2(\rho(r'') + \rho_v(r'')), \tag{10}$$

as a function of the halo length scale ξ. The ground state can be found by extremizing it by ξ [53];

$$\frac{dE(\xi)}{d\xi} \simeq -\frac{\hbar^2}{m\xi^3} + \frac{GMm}{\xi^2} = 0, \tag{11}$$

where, $M \equiv \int_0^\xi dr'' 4\pi r''^2(\rho(r'') + \rho_v(r''))$ is the total mass within ξ. Solving Eq. (11) gives [8,53]

$$\xi = \frac{\hbar^2}{GMm^2} = \frac{c^2\lambda_c^2}{4\pi^2 GM}. \tag{12}$$

The quantum Jeans mass represents the smallest amount of the DM having enough self-gravity to overcome the quantum velocity, so M_J in Eq. (9) can be identified to be M of the smallest galaxies. Therefore, from Eq. (9) the smallest galaxy formed at z has a size (the gravitational Bohr radius)

$$\xi(z) = \frac{\hbar^2}{GM_J(z)m^2} = \frac{3\hbar^{1/2}}{4\pi^{13/4}(Gm^2\bar{\rho}(z))^{1/4}} \propto \bar{\rho}(z)^{-1/4}, \tag{13}$$

which is a quantum mechanical relation absent in the CDM models. Therefore, $M_J(z)$ and $\xi(z)$ represent the time dependent mass and size of the smallest galaxies at the redshift z. Recall that M (and M_J) is the total mass including DM and visible matter, which explains the universal minimum mass of dwarf galaxies independent of visible matter fraction [18].

Once we fix one of ξ and M_J, the other is fixed automatically. In the previous works it was uncertain which one comes first, and
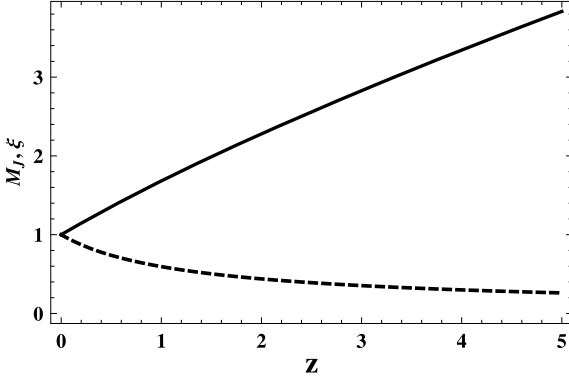
Fig. 1. The quantum Jeans mass $M_J(z)/M_J(0)$ (solid line) and the DM halo length scale $\xi(z)/\xi(0)$ (dashed line) as functions of the redshift z. For $m = 5 \times 10^{-22}$ eV, $M_J(0) = 1.1 \times 10^7 M_\odot$ and $\xi(0) = 311.5$ pc.

Fig. 2. (Color online.) The observed size evolution $r(z)$ of the massive galaxies versus the redshift z for spheroid-like galaxies (the red squares), disc-like galaxies (the blue circles) (data from [16]), and a typical compact galaxy (the black dot) (data from [15]). The line represents the predicted size evolution of the visible parts of the galaxies $r_*(z) = 1/(1 + z)^{9/8}$ in our model, which agrees with the observational data. We have set $r_*(0) = 1$.

several length scales including a thermal de Broglie wavelength or self-interaction scale were considered for ξ. In Ref. [18] the thermal de Broglie wavelength was proposed as ξ. Now, considering the evolution of the density perturbation leading to Eq. (9) it is reasonable to think that $\bar{\rho}(z)$ determines $M_J(z)$ first, and $M_J(z)$ determines $\xi(z)$ in turn. Therefore, the most natural length scale for galaxies is $\xi(z)$ given by Eq. (13), if there is no self-interaction.

This simple argument leads to many interesting predictions. Most of all, the smallest galaxies have the mass and the size determined by the epoch when the galaxies were formed. Since $\bar{\rho} \propto a(z)^{-3}$, from Eq. (9) we see

$$M_J(z) = M_J(0)a^{-3/4}(z) = M_J(0)(1 + z)^{3/4},$$
$$\xi(z) = \xi(0)a^{3/4}(z) = \xi(0)(1 + z)^{-3/4}, \quad (14)$$

which means that the minimum mass of galaxies decreases and the size of the halos increases as the time flows. (See Fig. 1.) Note that this does not mean the mass of typical galaxies decreases. Obviously, galaxies can be heavier during hierarchical merging processes. For the mass evolution above, we are considering only the smallest DM dominated galaxies formed at a given epoch.

The particle mass $m = O(10^{-22})$ eV is required to solve the cusp problem and the missing satellite problem [30,34]. For $m = 5 \times 10^{-22}$ eV, Eq. (9) gives $M_J(0) = 1.1 \times 10^7 M_\odot$ and $\xi(0) = 311.5$ pc. Interestingly, these values are similar to the minimum mass and the size of dSph galaxies nearby obtained from astronomical data, respectively. This is an interesting coincidence.

The average mass density of the dwarf galaxies evolves as

$$\rho_g(z) \sim \frac{M_J}{\xi(z)^3} \propto a^{-3}(z) = (1 + z)^3. \quad (15)$$

Thus, another prediction of our model is that early dwarf galaxies are more compact than present ones. For example, $M_J(z) = 4.2 \times 10^7 M_\odot$ and $\xi(z) = 81.2$ pc at $z = 5$. The high resolution numerical study with the BEC/SFDM [47,54] found that early galaxies were compact, which also supports our model. If these early compact dwarf galaxies are found in the sky, it could be another evidence for the BEC/SFDM. Interestingly, we already have similar galaxies. The ultra-compact dwarf galaxies (UCD) are very compact galaxies with high stellar populations. They are generally very old (> 8 Gyr), small (< 100 pc) and they have mass $M \simeq 2 - 9 \times 10^7 M_\odot$, which are similar to the predicted parameters of the early dwarf galaxies in our model. Therefore, we conjecture that the UCD are remnants of these old dwarfs which have not experienced major mergers since their formation, and have kept an initial DM distribution.
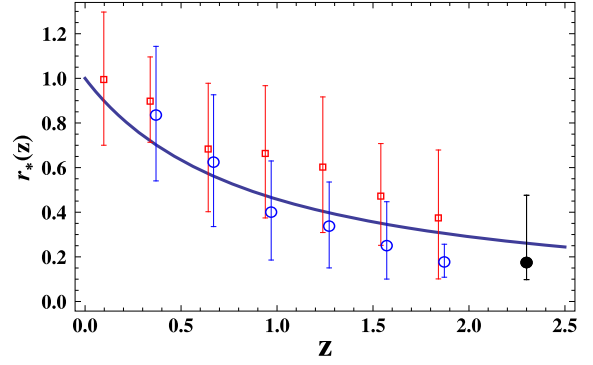
Owing to the scaling properties of the BEC DM halos, we can assume that this size evolution happens also to the most massive galaxies. The visible part of a galaxy seems to scale as $r_*(z) \propto \xi(z)^{3/2}$ [18], so we expect the size of the very massive and compact galaxies evolves as

$$\frac{r_*(z)}{r_*(0)} = a^{9/8}(z) = (1 + z)^{-1.125}, \quad (16)$$

which is shown in Fig. 2. The predicted size of the visible parts of the galaxies turns out to be similar to the average value of disc-like galaxies and spheroid-like galaxies, and to be in a good agreement with the observational data. Furthermore, HST /WFC3 IR images were analyzed to study evolution of quiescent galaxies [55], and it was found that the size evolution of the massive galaxies follows $r_* \propto (1 + z)^{-\alpha}$ with $\alpha = 1.06 \pm 0.19$, which is also consistent with our prediction. (Our prediction for α value is different from that of the previous work [18] because of the choice of ξ.)

One can see the ratio $\lambda_Q(z)/\xi(z) = 4\pi^4/3 \simeq 129.8$ is a constant independent of the time from Eq. (8) and Eq. (13). This number is the contraction factor during the DM collapse to form a halo. The quantum Jeans length indeed decides the length scale but the actual galaxy scale is much smaller by this factor.

On the other hand, the ratio λ_c/ξ represents how relativistic a given halo is. Using Eq. (13) we find

$$\frac{\lambda_c}{\xi} = \frac{4\pi^2 G M_J}{c^2 \lambda_c} = \frac{2^{9/4}\pi^4 \hbar^{1/2} \Omega_m^{1/4}}{3^{3/4} c} \sqrt{\frac{H}{m}} > \frac{4\pi^2 G M_J}{c^2 \xi}, \quad (17)$$

which is proportional to the ratio of gravitational potential energy and the self-energy of the halo DM particles. Here, $H \simeq 10^{-33}$ eV is the Hubble parameter and we have used the Friedmann equation $H^2 = 8\pi G \rho_c/3$, where $\bar{\rho} = \Omega_m \rho_c$. From this we can understand why galaxies are non-relativistic ($\lambda_c/\xi \ll 1$). It is due to the small M_J, or the small matter density $\bar{\rho}$ at the epoch of galaxy formations (see Eq. (9)). More fundamentally, this is caused by the small ratio of the Hubble parameter to the mass m, i.e., $\sqrt{H/m} \sim 10^{-6}$. From Eq. (13) we obtain

$$\xi = \frac{3^{3/4}\hbar^{1/2}}{2^{5/4}\pi^3 \Omega_m^{1/4}\sqrt{Hm}} \simeq 0.00656\sqrt{\frac{m}{H}}\lambda_c. \quad (18)$$

It is now clear that the rotation velocity $V_{rot} \simeq \Delta v$ of typical dwarf galaxies is

$$V_{rot} = \sqrt{\frac{G M_J}{\xi}} = \frac{2^{5/4}\pi^3 \hbar^{1/2} \Omega_m^{1/4}}{3^{3/4}}\sqrt{\frac{H}{m}} \simeq 5 \times 10^{-5} c, \quad (19)$$

which is similar to the observed value $V_{rot} = O(10)$ km/s. Thus, the characteristic rotation velocity of galaxies has a quantum origin and is $O(\sqrt{H/m})c$, which is somewhat surprising.

In summary, the BEC/SFDM can explain not only the problems of the CDM but also the observed minimum size and the mass of galaxies with a few parameters. The background matter density (or equivalently the Hubble parameter) and m decide the coherence length of the BEC halos, and this length in turn decides the rotation velocity. In this model the minimum size of galaxies increases, while the minimum mass of the galaxies decreases as the universe evolves. This scenario seems to explain the observed size evolution of massive galaxies and gives a new hint to the origin of the UCD.

Acknowledgement

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