Evaluating transit operator efficiency: An enhanced DEA model with constrained fuzzy-AHP cones

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Abstract

This study addresses efforts to combine the Analytic Hierarchy Process (AHP) with Data Envelopment Analysis (DEA) to deliver a robust enhanced DEA model for transit operator efficiency assessment. The proposed model is designed to better capture inherent preferences information over input and output indicators by adding constraint cones to the conventional DEA model. A revised fuzzy-AHP model is employed to generate cones, where the proposed model features the integration of the fuzzy logic with a hierarchical AHP structure to: 1) normalize the scales of different evaluation indicators, 2) construct the matrix of pairwise comparisons with fuzzy set, and 3) optimize the weight of each criterion with a nonlinear programming model. With introduction of cone-based constraints, the new system offers accounting advantages in the interaction among indicators when evaluating the performance of transit operators. To illustrate the applicability of the proposed approach, a real case in Nanjing City, the capital of China’s Jiangsu Province, has been selected to assess the efficiencies of seven bus companies based on 2009 and 2010 datasets. A comparison between conventional DEA and enhanced DEA was also conducted to clarify the new system’s superiority. Results reveal that the proposed model is more applicable in evaluating transit operator’s efficiency thus encouraging a broader range of applications.

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1. Introduction

1.1. Background

Over the past several decades, traffic congestion and air pollution has emerged as imperative issues across the world. Development of a transit-oriented urban transport system has been realized by an increasing number of countries and administrations as one of the most effective strategies for mitigating congestion and pollution problems. Despite the rapid development of public transportation system, doubts regarding the efficiency of the system and financial sustainability have arisen. A significant amount of public resources...
have been invested into public transportation. However, complaints about low service quality and unreliable transit system performance have increasingly arisen as well. Evaluating transit operational efficiency from various levels has become one of the most crucial challenges faced by responsible authorities to sustain the public transport system development and improve its performance and service.

1.2. Evaluation of transit system performance

A transit system performance evaluation is an essential task for transit service providers to capture the passenger demand trends, operational constraints, stakeholders concerns, and evolving service needs. It also allows the responsible authorities to achieve better economic performance assessments, organize their administration, and plan and finance transportation service.

In view of literature, previous studies on transit performance evaluation focus on the service level and fall into three different categories (Hassan et al., 2013), namely the user perception/satisfaction approach (Zboli and Mazzulla, 2011; Nathanael, 2008; Tyrinopoulos and Antoniou, 2008), the efficiency indicator approach (Badami and Haider, 2007; Lao and Liu, 2009), and the integrated approach based on both user opinions and efficiency indicators (Sheth et al., 2007).

To better promote public transport development, some countries and transit associates have enacted a series of national standards or codes to offer best-practice guideline for evaluating transit performance. The International Association of Public Transport (UITP) has set up a group of indicators, including population of transit users; services coverage; number of bus routes; stations, vehicles, and vehicle mileage; patronage; average trip distance; and fare. As ever-increasing applications of DEA in the transit efficiency assessment, some critical issues are deserved further investigation. Halme et al. (1999) has pointed out that DEA

1.3. Literature review


Other researchers have assumed transit system as production lines, evaluating the efficiency of such lines by comparing multiple inputs and outputs (Barnum et al., 2007; Boile, 2001; Fare and Grosskopf, 1996, 2000; Hwang and Kao, 2006; Kao and Hwang, 2008; Karlaftis, 2004; Lao and Liu, 2009; Nakanishi and Falccoci, 2004; Nolan et al., 2002; Sanchez, 2009; Seiford and Zhu, 1999; Sexton and Lewis, 2003; Sheth et al., 2007; Tsamboulas, 2006; Yu and Fan, 2009; Zhao et al., 2011; Zhu, 2002). Most of these researchers used the Data Envelopment Analysis (DEA), a non-parametric method introduced by Farrell (1957) and popularized by Charnes et al. (1978). It is a managerial approach to assess relative performance/efficiency for evaluating decision making units (DMUs). Each DMU selects its best set of corresponding weights to consider inputs and outputs and the values of weights may thus vary from one DMU to another. The DEA models then calculate each DMU’s performance score ranging between 0 and 1 that represents its relative degree of efficiency (Wei and Chang, 2011). The basic relative performance model of DMU0, as perceived by DMU0 itself, can be formulated, following the CCR model (Charnes et al., 1978)

$$\max \quad p_0 = \left( \frac{\bar{r} Y_{i0}}{C_{i0}} \right)$$

s.t. \quad W^T X_{ij} - r^T Y_{ij} \geq 0 \quad j = 1, \ldots, J; \quad i = 1, \ldots, N; \quad k = 1, \ldots, M

$$W^T X_{i0} = 1$$

$$W \geq 0, \quad r \geq 0$$

where j is a decision making unit (DMU) index, \(j = 1, \ldots, J\), i is an input index, \(i = 1, \ldots, N\), k is an output index, \(k = 1, \ldots, M\), \(X_{ij}\) is the ith input for the jth DMU, \(Y_{ij}\) is the kth input for the jth DMU, \(r^T\) and \(W^T\) are two non-negative scalars (weights) for the kth output and the ith input, \(p_0\) is the efficiency/effectiveness ratio of DMU0.

Recently, Arman et al. (2014) presented a DEA-based framework to comparatively assess the operational productivity and efficiency of transit agencies. In their study, input indicators were selected for annual operating expenses, number of employees, and total fuel consumption. Outputs include the total ridership and total vehicle miles traveled during an 8-year period (2002–2009) for public transit agencies in Indiana. Both datasets were used to construct relative efficiency scores through data envelopment analysis.
calculations are traditionally value-free and the underlying assumption is that no output or input is more important than the rest. However, in the real-world, there generally exists a Decision Maker (DM) who has preferences over outputs and inputs. Nevertheless, the different importance of different input or output indicators is an obvious case that one cannot ignore when reviewing system efficiency. Andersen and Petersen (1993) stated that DEA evaluated the relative efficiency of decision-making units but did not allow for ranking of the efficient units themselves. Both issues are constraints to widely and extensively apply DEA in system efficiency assessment.

To remedy such limitations, some efforts of combining the analytic hierarchy process (AHP), a subjective method developed by Saaty (1980) to support multi-criteria decision making, with the DEA have been made to complement each other. Bowen (1990) suggested a two-step process in site selection, where the first step is to apply DEA to exclude numerically inefficient sites. The second step is to apply AHP for further ranking DEA-efficient sites. A similar method was also applied to manage investments in various parts (sub-systems) of the State Economic Information System (SEIS) of China by Zhang and Cui (1999). Comparing it with the above method, Shang and Sueyoshi (1995) proposed a reversal process to select the most appropriate and flexible alternative, which firstly used AHP to quantify all the alternatives and then used DEA to determine the most suitable one. Additionally, Sinuany-Stern et al. (2000) presented an interesting AHP/DEA methodology for the facilities layout design was proposed by Yang and Kuo (2003) and Ertay et al. (2006). Ramanathan (2006) developed a DEAHP model, which uses DEA to generate local weights of alternatives from pair-wise comparison matrices and AHP to aggregate the local weights of alternatives over all the criteria.

Despite the constructive efforts in combing AHP and DEA, most existing studies use AHP and DEA separately rather than inherently integrating them into a unified model.

1.4. Research motivation

To contend with critical issues, the objective of this research is to develop an enhanced DEA model with sufficient flexibility to capture the inherent preference information over input and output indicators, and further apply the proposed model to evaluate the efficiency transit operators. The paper will focus on the following critical research tasks.

(1) Proposes a robust enhanced DEA model to effectively take the preferences over indicators into account, which features the integration of a Fuzzy-AHP model to generate cone constraints for the conventional DEA;
(2) Offers the advantage in breaking the tie between those efficient units under the conventional DEA;
(3) Apply the proposed model into a real world case to demonstrate the model’s potential application.

2. Modeling framework

2.1. Notation of the proposed model

To facilitate the model presentation, all definitions and notations used hereafter are summarized in Table 1.

2.2. Selection of input and output indicators

The proposed model is based on the concept of evaluating performance according to selected criteria. Thus, a set of representative indicators associated with transit operator performance is recommended to select data for the proposed model. In accordance with the theory of DEA models, the targeted indicators are classified into two groups: the input group and the output group. The input group includes the indicators that allocate passenger service resources, for example, cost structure, bus fleet, human resources, etc. Meanwhile, the output indicators reflect resource allocation based on goals, such as passenger volume, operating mileage and customer satisfaction. Normally, the selected indicators are widely available, easily collected, and customized to fit the local situation.

### Table 1 – Notation of key parameters used in the proposed model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Index corresponding to input indicator group (i = 1, …, m)</td>
</tr>
<tr>
<td>k</td>
<td>Index corresponding to output indicator group (k = 1, …, s)</td>
</tr>
<tr>
<td>j</td>
<td>Index corresponding to DMU (j = 0, …, n)</td>
</tr>
<tr>
<td>vi</td>
<td>The weight of input indicator (i = 1, …, m)</td>
</tr>
<tr>
<td>Wk</td>
<td>The weight of output indicator (k = 1, …, s)</td>
</tr>
<tr>
<td>pij</td>
<td>The efficiency of DMU j (j = 0, …, n)</td>
</tr>
<tr>
<td>xij</td>
<td>The value of input indicator i corresponding to DMU j</td>
</tr>
<tr>
<td>yij</td>
<td>The value of output indicator k corresponding to DMU j</td>
</tr>
<tr>
<td>μij</td>
<td>Fuzzy membership value corresponding to xij</td>
</tr>
<tr>
<td>πi</td>
<td>Average fuzzy membership value for indicator i</td>
</tr>
<tr>
<td>ximin</td>
<td>The minimal crisp value for input indicator i</td>
</tr>
<tr>
<td>ximid</td>
<td>The medium crisp value for input indicator i</td>
</tr>
<tr>
<td>ximax</td>
<td>The maximal crisp value for input indicator i</td>
</tr>
<tr>
<td>Sij</td>
<td>Standard deviation of indicator values corresponding to input indicator i</td>
</tr>
<tr>
<td>Smin</td>
<td>[ \min { Sij \mid i = 1, \ldots, n } ]</td>
</tr>
<tr>
<td>Smax</td>
<td>[ \max { Sij \mid i = 1, \ldots, n } ]</td>
</tr>
<tr>
<td>( A = {a_{ij}}_{n \times n} )</td>
<td>Pair-wise comparison matrix</td>
</tr>
<tr>
<td>( a_{im} )</td>
<td>Comparison scale for the pair-wise comparison matrix</td>
</tr>
<tr>
<td>u_i</td>
<td>Weight of criterion i</td>
</tr>
<tr>
<td>( Y = {y_{ij}}_{n \times n} )</td>
<td>Consistency judgment matrix</td>
</tr>
<tr>
<td>CIC(n)</td>
<td>Consistency index coefficient</td>
</tr>
<tr>
<td>( (y_{input})_{m \times m} )</td>
<td>The input group pair-wise matrix</td>
</tr>
<tr>
<td>( (y_{output})_{s \times s} )</td>
<td>The output group pair-wise matrix</td>
</tr>
<tr>
<td>( \lambda_{input} )</td>
<td>The max eigenvalue of input pair-wise matrix</td>
</tr>
<tr>
<td>( \lambda_{output} )</td>
<td>The max eigenvalue of output pair-wise matrix</td>
</tr>
</tbody>
</table>
2.3. **Introduction of the constraint cone into DEA**

Wu et al. (1999) firstly introduced a concept of AHP restraint cone to be utilized by conventional DEA model. The model maintains characteristics of the conventional DEA model, as well as reflects preferences of the decision makers by adding the constraint cones.

Along the line of Wu's work, this study has developed and added two constraint cones, \( y_{\text{input}}m \times m \) and \( y_{\text{output}}s \times s \), contained weights information over indicators in input and output group, are added into the constraints of conventional DEA model.

\[
\begin{align*}
\text{max } & \quad p_0 = \langle r^T Y_{10} \rangle \\
\text{s.t. } & \quad W_j^T X_{i0} - r^T Y_{j0} \geq 0 \quad j = 1, \ldots, J; \quad i = 1, \ldots, N; \quad k = 1, \ldots, M \\
W^T X_0 &= 1 \\
W^T \left[ \left( y_{\text{input}} \right)_{m \times m} - \lambda_{\text{input}} E_n \right] &\geq 0 \quad W \geq 0 \\
\end{align*}
\]

The main limitation of Wu's enhanced DEA model is to employ the conventional AHP model to generate constraint cones, where some critical issues deserved further investigation, specifically, 1) how to handle the very unbalanced scale of judgment, 2) how to properly construct the pair-wise comparison matrix subject to the biased impacts from the objective judgment, selection and preference of decision-makers.

To resolve those problems, this paper proposes a robust Fuzzy-AHP model to generate constraint cones. The proposed model features the integration of the fuzzy logic with a hierarchical AHP structure to: 1) normalize the scales of different evaluation indicators, 2) construct a matrix of pair-wise comparisons with the fuzzy set, and 3) optimize the weight of each criterion with a non-linear programming model to maximize consistency.

A diagram below illustrates the logical relationship between DEA and Fuzzy-AHP in the proposed model (Fig. 1).

2.4. **Construction of the Fuzzy-AHP constraint cones**

**Step 1: Fuzzy scaling**

Considering the difficulty in comparing various criteria with different units, this step have employed a set of fuzzy membership functions to normalize the scales of different indicators, based on the characteristics of selected criterion. In this case, the min–max normalization is introduced to scales the data from \((x_{\min}, x_{\max})\) to \((0, 1)\) in proportion. The advantages of this method can be concluded as: 1) it preserves all relationships of the data values exactly as it carries out a linear normalization; 2) it does not introduce any potential bias into the data, and 3) it functions to nondimensionalize different indicator, further making them comparable (Li and Liu, 2011; Yu et al., 2011). Two types of indicators, i.e. “the-lower-the-better” and “the-higher-the-better”, are identified to normalize \(x_{ik}\) with their fuzzy sets given by:

- **The-lower-the-better indicators:**
  \[
  m_{ik} = \frac{x_{ik} - x_{\min}}{x_{\max} - x_{\min}} 
  \]

- **The-higher-the-better indicators:**
  \[
  m_{ik} = \frac{x_{\max} - x_{ik}}{x_{\max} - x_{\min}} 
  \]

**Step 2: Pair-wise comparison**

After the normalization of all the indicators by fuzzy scaling, it is noticeable that, if the variation of an indicator for all operators \(\mu_{ik} | k = 1, \ldots, m, i = 1, \ldots, n\) is larger than that of

![Fig. 1 – Proposed model structure.](image-url)
the other indicator $a_{ijk}$, criterion $i$ is more influential than criterion $j$ when evaluating operator $k$. The calculation of standard deviation ($S_j$) is given by the following equation:

$$S_j = \sqrt{\frac{\sum_{i=1}^{m} (\mu_{ik} - \mu)^2}{m-1}}$$  \hspace{1cm} (12)

Then, a pair-wise comparison matrix $A = (a_{ij})_{n \times n}$ is calculated to measure the relative importance of criterion $i$ over criterion $j$.

$$a_{ij} = \frac{S_i - S_j}{\max(S_i, S_j)} \times (a_{mm} - 1) \quad S_i \geq S_j$$  \hspace{1cm} (13)

$$a_{ij} = \frac{1}{\max(S_i, S_j)} \times (a_{mm} - 1) + 1 \quad S_i < S_j$$  \hspace{1cm} (14)

where $a_{mm} = \min \left(9 \times \frac{\sum w_i}{\sum n} + 0.5 \right)$ is a comparison scale for all criteria recommended by Jin et al. (2004).

### Step 3: Consistency maximization

According to theory of AHP analysis, if $a_{ij}$ can consistently or correctly reflect the importance of technical criterion $i$ over criterion $j$, we will have $a_{ij} = w_i/w_j$. Then, the following three laws will hold: (1) $a_{ij} = w_i/w_j = 1$; (2) $a_{ij} = w_i/w_j = 1/a_{ji}$; (3) $a_{ij}a_{jk} = (w_i/w_j) \times (w_j/w_k) = w_i/w_k = a_{ik}$. Therefore, one can obtain the weight for each criterion by solving the following linear equations:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}| w_j = w_i$$  \hspace{1cm} (15)

$$w_i > 0 \quad i = 1, \cdots, n$$  \hspace{1cm} (16)

$$\sum_{i=1}^{n} w_i = 1$$  \hspace{1cm} (17)

However, as mentioned in many previous studies (Bryson, 1995; Jin et al., 2004; Saaty, 1980), it is usually difficult in practice to obtain a completely consistent pair-wise comparison matrix that satisfies the aforementioned three laws. Thus, this study has proposed the following non-linear optimization model to estimate the weights $[w_i; i = 1, \cdots, n]$ from the inconsistent $a_{ij}$.

$$\min \text{CIC}(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{y_{ij} - a_{ij}}{2} \right| + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|y_{ij}w_i - w_j|}{2p^2}$$  \hspace{1cm} (18)

$$y_{ij} = 1 \quad i = 1, \cdots, n$$  \hspace{1cm} (19)

$$\frac{1}{y_{ij}} = y_{ij} \in [a_{ij} - da_{ij}; a_{ij} + da_{ij}] \quad i = 1, \cdots, n; \quad j = i + 1, \cdots, n$$  \hspace{1cm} (20)

$$w_i > 0 \quad i = 1, \cdots, n$$  \hspace{1cm} (21)

$$\sum_{i=1}^{n} w_i = 1$$  \hspace{1cm} (22)

In the above equations, $Y = (y_{ij})_{n \times n}$ is defined as the consistency judgment matrix which is adjusted based on $A = (a_{ij})_{n \times n}$ during the minimization process of the consistency index coefficient, denoted by CIC(n). It consists of the following two parts:

1. Minimization of $\sum_{i=1}^{n} \sum_{j=1}^{n} |y_{ij} - a_{ij}|$ to match the judgment matrix $Y = (y_{ij})_{n \times n}$ with the original comparison matrix $A = (a_{ij})_{n \times n}$ as closely as possible so that $Y = (y_{ij})_{n \times n}$ can reflect the original comparison information to the maximum extent;
2. Minimization of $\sum_{i=1}^{n} \sum_{j=1}^{n} |y_{ij} - a_{ij}|$, functions to ensure that $Y = (y_{ij})_{n \times n}$ is as consistent as possible to satisfy Eqs. (15)–(17).

Constraints in Eqs. (19) and (20) limit that all the elements in $A = (a_{ij})_{n \times n}$, which should satisfy the first two aforementioned laws. Note that the third law is not included in the constraints since it is considered by the second part of the objective function. In addition, constraint in Eq. (20) introduces a non-negative parameter $d$ to measure the deviation degree between $Y = (y_{ij})_{n \times n}$ and $A = (a_{ij})_{n \times n}$. Constraint in Eq. (21) ensures the non-negative weights, and constraint in Eq. (22) limits the sum of all weights equal to 1.

Solving the proposed optimization model yields two types of information: 1) the judgment matrix $Y = (y_{ij})_{n \times n}$, and 2) the vector of weights for different technical criteria $[w_i; i = 1, \cdots, n]$. However, the global optimal solutions are not assured for the proposed optimization model due to its non-convexity attribute. Thus, this study has employed the convergence criterion of CIC(n) $\leq 0.1$ to ensure that the obtained judgment matrix $Y = (y_{ij})_{n \times n}$ is consistent based on extensive numerical experiments.

By processing the Fuzzy-AHP model for input and output group respectively, two optimized consistent pair-wise matrices, $(y_{\text{input}})_{m \times m}$ and $(y_{\text{output}})_{s \times s}$, are obtained to represent the constraint cones and ready to be utilized by conventional DEA.

### 2.5. Derivation of the proposed model

To prove formulation’s validity and reliability, the derivation is given as following. Here, we take the constraint cones to the input group as an example:

**Definition 1.** The solution domains of $W^T$:

$$[y_{\text{input}}]_{m \times m} - \lambda_{\text{input}}E_m \geq 0 \quad \text{and} \quad W^T[y_{\text{input}}]_{m \times m} - \lambda_{\text{output}}E_m = 0$$

are the same when the pair-wise matrix $(y_{\text{input}})_{m \times m}$ satisfies the consistency check of AHP requirement.

It is required to calculate the maximum eigenvalue $\lambda_{\text{input}}$ of matrix $(y_{\text{input}})_{m \times m}$.

Set $C = (y_{\text{input}})_{m \times m} - \lambda_{\text{input}}E_m$, where $E_m$ is an $m$ order unit matrix;

Since $[y_{\text{input}}]_{m \times m} - E_m \geq 0 \quad \text{and} \quad [y_{\text{input}}]_{m \times m} - \lambda_{\text{input}}E_m$;

$$W = CW \geq 0;$$

Then $[y_{\text{input}}]_{m \times m} - E_m \geq 0; \quad [y_{\text{input}}]_{m \times m} - \lambda_{\text{input}}E_m \geq 0;$

and $\lambda_{\text{input}}E_m$.

$$W \geq 0;$$
Since \( \langle y(\text{input}) \rangle_{m_1 m_2} = \langle y_j \rangle_{m_1 m_2} \) satisfies the consistency-check of AHP process;

Then \( y_{j} = y_{j} y_{k} \) and \( \langle y(\text{input}) \rangle_{m_1 m_2} = y_{j} y_{k} = \sum_{k=1}^{n} L_{y_{j} y_{k}} \)

\[ m(\langle y(\text{input}) \rangle_{m_1 m_2}) = \]

\[ (\langle y(\text{input}) \rangle_{m_1 m_2} - y_{j} y_{k}) W = C W \leq 0; \]

\[ (\langle y(\text{input}) \rangle_{m_1 m_2} - y_{j} y_{k}) W (\langle y(\text{input}) \rangle_{m_1 m_2} - y_{j} y_{k}) = \]

Since \( (\langle y(\text{input}) \rangle_{m_1 m_2} - y_{j} y_{k}) W \approx C W \geq 0 ; \]

\[ \text{(23)} \]

[Definition 2. The efficiency of the selected DMU obtained from enhanced DEA model is equal to the weighted average of the selected DMU obtained from AHP process, given by:

\[ p_0^* = \frac{\sum_{k=1}^{n} w_{j} x_{j} x_{k}}{\sum_{k=1}^{n} w_{j}} \]

where T is a parameter, \( x_{j} x_{k} \) is the value of output indicator k of DMU 0, and \( x_{j} x_{k} \) is the value of output indicator k of DMU 0.

According to definition 1, \( \langle y(\text{input}) \rangle_{m_1 m_2} - y_{j} y_{k} W = C W = 0 \)

Then, we have \( W = y_{j} y_{k} \). The enhanced DEA model could be rewritten as

\[ \text{max} p_0 = K^{-1} y_0 \]

s.t. \( K w x_0 - K x_0 W = 0 \quad j = 1, \ldots, n \]

\[ K w x_0 = 1 \]

(24)

(25)

(26)

Then, the max value is equal to \( p_0^* = K^{-1} y_0 \), where \( K = \max_{y_j} \frac{w_x}{y_j} \) and \( K = \frac{1}{w_x y_0} \).

so \( p_0^* = \frac{\text{T} y_0}{\text{T} y_j} \max_{y_j} \frac{w_x}{y_j} \) where \( T = \max_{y_j} \frac{w_x}{y_j} \).

3. Case study

In this section, the application of the proposed model to evaluate the efficiencies of seven bus operators in Nanjing City, China is described. The area of municipal district is 6598 square kilometers with over 7.4 million permanent residents. This study evaluates the efficiency of seven bus companies in both 2009 and 2010 with the proposed model and the conventional DEA. Comparison analysis of results between the conventional DEA and the proposed model is also performed.

In the case study, fuel cost, labor cost, depreciation expense and other cost have been collected as indicators in the input group, whereas the volume of patronage, mileage and satisfaction index have been selected as the indicators in output group. Tables 2 and 3 show the data in 2009 and 2010.

3.1. Construction of constraint cones

Step 1: Fuzzy scaling

This step has employed a set of fuzzy membership functions to normalize the scales of different indicators, based on

<p>| Table 2 – Data used for evaluation in 2009. |</p>
<table>
<thead>
<tr>
<th>Indicator</th>
<th>Fuel cost (Yuan)</th>
<th>Labor cost (Yuan)</th>
<th>Depreciation expense (Yuan)</th>
<th>Other cost (Yuan)</th>
<th>Patronage volume (Trips)</th>
<th>Mileage (km)</th>
<th>Satisfaction index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nanjing bus</td>
<td>27,728.101</td>
<td>44,930.800</td>
<td>12,484.722</td>
<td>2437.661</td>
<td>51,428.510</td>
<td>17,979.921</td>
<td>59.716</td>
</tr>
<tr>
<td>Zhongbei bus</td>
<td>10,712.022</td>
<td>14,625.681</td>
<td>4218.512</td>
<td>1470.032</td>
<td>21,505.310</td>
<td>7363.795</td>
<td>62.790</td>
</tr>
<tr>
<td>Yagao bus</td>
<td>4779.875</td>
<td>4942.479</td>
<td>1813.263</td>
<td>757.199</td>
<td>7914.638</td>
<td>2823.705</td>
<td>53.588</td>
</tr>
<tr>
<td>Xinzheng bus</td>
<td>6116.101</td>
<td>8402.902</td>
<td>2230.166</td>
<td>600.331</td>
<td>10,086.515</td>
<td>4896.807</td>
<td>50.794</td>
</tr>
<tr>
<td>Xinmingpu bus</td>
<td>2487.872</td>
<td>2355.404</td>
<td>635.507</td>
<td>565.992</td>
<td>4082.552</td>
<td>1600.342</td>
<td>56.675</td>
</tr>
<tr>
<td>Pukou bus</td>
<td>1621.567</td>
<td>2541.051</td>
<td>515.642</td>
<td>209.532</td>
<td>2820.611</td>
<td>1618.651</td>
<td>60.492</td>
</tr>
<tr>
<td>Liuhe bus</td>
<td>2898.059</td>
<td>3454.670</td>
<td>587.771</td>
<td>244.863</td>
<td>2856.341</td>
<td>2831.942</td>
<td>62.292</td>
</tr>
</tbody>
</table>

<p>| Table 3 – Data used for evaluation in 2010. |</p>
<table>
<thead>
<tr>
<th>Indicator</th>
<th>Fuel cost (Yuan)</th>
<th>Labor cost (Yuan)</th>
<th>Depreciation expense (Yuan)</th>
<th>Other cost (Yuan)</th>
<th>Patronage volume (Trips)</th>
<th>Mileage (km)</th>
<th>Satisfaction index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nanjing bus</td>
<td>32,674.731</td>
<td>53,715.450</td>
<td>13,470.471</td>
<td>3368.88</td>
<td>50,057.131</td>
<td>18,581.940</td>
<td>60.870</td>
</tr>
<tr>
<td>Zhongbei bus</td>
<td>12,614.802</td>
<td>17,051.800</td>
<td>4792.801</td>
<td>1293.802</td>
<td>20,852.120</td>
<td>7381.976</td>
<td>63.770</td>
</tr>
<tr>
<td>Yagao bus</td>
<td>5684.551</td>
<td>5897.242</td>
<td>2360.456</td>
<td>850.487</td>
<td>7951.782</td>
<td>2939.931</td>
<td>55.560</td>
</tr>
<tr>
<td>Xinzheng bus</td>
<td>7037.315</td>
<td>9621.091</td>
<td>2471.733</td>
<td>1399.178</td>
<td>8511.252</td>
<td>4754.776</td>
<td>50.480</td>
</tr>
<tr>
<td>Xinmingpu bus</td>
<td>2786.802</td>
<td>3058.442</td>
<td>822.224</td>
<td>560.712</td>
<td>4364.703</td>
<td>1755.605</td>
<td>56.680</td>
</tr>
<tr>
<td>Pukou bus</td>
<td>2246.119</td>
<td>3119.237</td>
<td>620.478</td>
<td>288.791</td>
<td>3127.535</td>
<td>1878.909</td>
<td>61.830</td>
</tr>
<tr>
<td>Liuhe bus</td>
<td>3706.318</td>
<td>4265.043</td>
<td>760.267</td>
<td>273.742</td>
<td>2947.177</td>
<td>2938.805</td>
<td>61.930</td>
</tr>
</tbody>
</table>
the characteristics of each criterion. According to the definitions, all the input indicators here are considered as “the-lower-the-better”, which will be processed with Eq. (13) while the output indicators are taken as the higher-the-better ones, and thus computed by Eq. (14). Further, the deviation of each technical criterion was calculated by Eq. (15). All of the fuzzy values and the standard deviations for 2009 and 2010, denoted as \( \mu_i \) and \( \sigma_i \) (\( i = 1, \ldots, 7 \) and \( j = 1, \ldots, 7 \)), are listed in Tables 4 and 5.

**Step 2: Pair-wise comparison**

After normalization of all indicators with the fuzzy sets, the pair-wise comparison matrices corresponding to the input and output groups are constructed respectively with Eqs. (16) and (17), each measuring the relative importance of indicator \( j \) over indicator \( i \).

The pair-wise matrix of “fuel cost”, “labor cost”, “depreciation expense” and “other cost” in 2009 input group is as follow:

\[
A_{\text{input}} = \begin{bmatrix}
1.000 & 0.759 & 0.710 & 1.332 \\
1.318 & 1.000 & 0.916 & 1.649 \\
1.409 & 1.091 & 1.000 & 1.741 \\
0.751 & 0.606 & 0.575 & 1.000
\end{bmatrix}
\]

The pair-wise matrix of “the volume of patronage”, “mileage” and “satisfaction index” in 2009 output group is as follow:

\[
A_{\text{output}} = \begin{bmatrix}
1.000 & 1.688 & 7.919 \\
0.592 & 1.000 & 7.231 \\
0.126 & 0.138 & 1.000
\end{bmatrix}
\]

The pair-wise matrix of “fuel cost”, “labor cost”, “depreciation expense” and “other cost” in 2010 input group is as follow:

The pair-wise matrix of “the volume of patronage”, “mileage” and “satisfaction index” in 2010 output group is as follow:

\[
A_{\text{output}} = \begin{bmatrix}
1.000 & 0.661 & 0.713 & 1.354 \\
1.513 & 1.000 & 0.918 & 1.652 \\
1.403 & 1.091 & 1.000 & 1.763 \\
0.739 & 0.605 & 0.567 & 1.000
\end{bmatrix}
\]

The pair-wise matrix of “the volume of patronage”, “mileage” and “satisfaction index” in 2010 output group is as follow:

\[
A_{\text{output}} = \begin{bmatrix}
1.000 & 1.606 & 7.223 \\
0.623 & 1.000 & 7.005 \\
0.138 & 0.143 & 1.000
\end{bmatrix}
\]

**Step 3: Consistency maximization**

After the construction of two original pair-wise matrices, the non-linear optimization model, as described in Eqs. (21)–(26), is then solved for each comparison matrix to maximize its judgment consistency. Eventually, two optimized pair-wise matrices corresponding to the input and output groups are obtained as the constraint cones for the DEA model.

The optimized pair-wise matrix of 2009 input indicator group (the input indicator group constraint cone) is given as following:

\[
y_{\text{input}} = \begin{bmatrix}
1.000 & 0.789 & 0.738 & 1.385 \\
1.267 & 1.000 & 0.953 & 1.715 \\
1.355 & 1.049 & 1.000 & 1.810 \\
0.722 & 0.583 & 0.552 & 1.000
\end{bmatrix}
\]

The optimized pair-wise matrix of 2009 output indicator group (the output indicator group constraint cone) is given as following:

\[
y_{\text{output}} = \begin{bmatrix}
1.000 & 0.799 & 0.915 \\
0.945 & 1.000 & 0.950 \\
0.946 & 0.956 & 1.000 \\
0.925 & 0.946 & 0.950
\end{bmatrix}
\]

**Table 4 – Fuzzy scaling for 2009 data.**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Nanjing bus</th>
<th>Zhongbei bus</th>
<th>Yagao bus</th>
<th>Xincheng bus</th>
<th>Xinningpu bus</th>
<th>Fukou bus</th>
<th>Liuhe bus</th>
<th>Sj</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel cost</td>
<td>0.055</td>
<td>0.635</td>
<td>0.837</td>
<td>0.792</td>
<td>0.915</td>
<td>0.945</td>
<td>0.901</td>
<td>0.313</td>
</tr>
<tr>
<td>Labor cost</td>
<td>0.050</td>
<td>0.691</td>
<td>0.895</td>
<td>0.822</td>
<td>0.950</td>
<td>0.946</td>
<td>0.927</td>
<td>0.324</td>
</tr>
<tr>
<td>Depreciation expense</td>
<td>0.040</td>
<td>0.676</td>
<td>0.861</td>
<td>0.828</td>
<td>0.950</td>
<td>0.960</td>
<td>0.955</td>
<td>0.330</td>
</tr>
<tr>
<td>Other cost</td>
<td>0.079</td>
<td>0.445</td>
<td>0.714</td>
<td>0.773</td>
<td>0.786</td>
<td>0.921</td>
<td>0.908</td>
<td>0.301</td>
</tr>
<tr>
<td>Patronage volume</td>
<td>0.948</td>
<td>0.396</td>
<td>0.146</td>
<td>0.186</td>
<td>0.075</td>
<td>0.052</td>
<td>0.053</td>
<td>0.324</td>
</tr>
<tr>
<td>Mileage</td>
<td>0.918</td>
<td>0.376</td>
<td>0.144</td>
<td>0.250</td>
<td>0.082</td>
<td>0.083</td>
<td>0.145</td>
<td>0.298</td>
</tr>
<tr>
<td>Satisfaction index</td>
<td>0.526</td>
<td>0.553</td>
<td>0.472</td>
<td>0.447</td>
<td>0.499</td>
<td>0.533</td>
<td>0.548</td>
<td>0.040</td>
</tr>
</tbody>
</table>

**Table 5 – Fuzzy scaling for 2010 data.**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Nanjing bus</th>
<th>Zhongbei bus</th>
<th>Yagao bus</th>
<th>Xincheng bus</th>
<th>Xinningpu bus</th>
<th>Fukou bus</th>
<th>Liuhe bus</th>
<th>Sj</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel cost</td>
<td>0.064</td>
<td>0.639</td>
<td>0.837</td>
<td>0.799</td>
<td>0.920</td>
<td>0.936</td>
<td>0.894</td>
<td>0.309</td>
</tr>
<tr>
<td>Labor cost</td>
<td>0.054</td>
<td>0.700</td>
<td>0.896</td>
<td>0.831</td>
<td>0.946</td>
<td>0.945</td>
<td>0.925</td>
<td>0.322</td>
</tr>
<tr>
<td>Depreciation expense</td>
<td>0.044</td>
<td>0.660</td>
<td>0.833</td>
<td>0.825</td>
<td>0.942</td>
<td>0.956</td>
<td>0.946</td>
<td>0.326</td>
</tr>
<tr>
<td>Other cost</td>
<td>0.075</td>
<td>0.645</td>
<td>0.767</td>
<td>0.616</td>
<td>0.846</td>
<td>0.921</td>
<td>0.925</td>
<td>0.296</td>
</tr>
<tr>
<td>Patronage volume</td>
<td>0.944</td>
<td>0.393</td>
<td>0.143</td>
<td>0.161</td>
<td>0.082</td>
<td>0.059</td>
<td>0.056</td>
<td>0.322</td>
</tr>
<tr>
<td>Mileage</td>
<td>0.914</td>
<td>0.363</td>
<td>0.145</td>
<td>0.234</td>
<td>0.086</td>
<td>0.092</td>
<td>0.145</td>
<td>0.294</td>
</tr>
<tr>
<td>Satisfaction index</td>
<td>0.533</td>
<td>0.558</td>
<td>0.486</td>
<td>0.442</td>
<td>0.496</td>
<td>0.541</td>
<td>0.542</td>
<td>0.041</td>
</tr>
</tbody>
</table>
Weights of indicators in 2009 and 2010.

The optimized pair-wise matrix of 2010 input indicator group (the input indicator group constraint cone) is given as following:

\[
y_{\text{input}} = \begin{bmatrix}
1.000 & 0.664 & 0.713 & 1.354 \\
1.507 & 1.000 & 0.198 & 1.652 \\
1.040 & 1.089 & 1.000 & 1.763 \\
0.739 & 0.605 & 0.567 & 1.000
\end{bmatrix}
\]

The optimized pair-wise matrix of 2010 output indicator group, also known as the output indicator group constraint cone, is given as following:

\[
y_{\text{output}} = \begin{bmatrix}
1.000 & 1.606 & 7.223 \\
0.623 & 1.000 & 7.005 \\
0.138 & 0.143 & 1.000
\end{bmatrix}
\]

The weights of indicators for year 2009 and year 2010 are summarized in Table 6.

As shown in Table 6, in both 2009 and 2010 the depreciation expense is assigned the largest weight whereas the indicator “other cost” gets the lowest weight. For the output group, the “patronage volume” has the highest weight, while “satisfaction index” is assigned the lowest weight. Noticeably, there isn’t a remarkable difference between the weights from 2009 to 2010. With constraint cones generated from Fuzzy-AHP model, the revised DEA model has the capacity to reflect the different importance of input or output indicators, and, furthermore to show the preference of indicators. The efforts of adding cones make DEA more reliable and consistent with the actual conditions.

3.2. Efficiency evaluation with the constrained cones

Through the aforementioned steps, two optimized input and output pair-wise matrices with their max eigenvalues can be obtained to generate the constraint cones, which are added to the DEA model. Evaluation results are summarized in Table 7.

3.3. Comparison and discussion

Table 8 shows the comparison results between the proposed model and the conventional DEA model for bus operator efficiency evaluation in the case study.

As shown in Table 8, all companies are assessed to be efficient using the conventional DEA model, as represented by each value of “1” in the second column in 2009 failing to identify the difference in performance of bus operators. In contrast, results from proposed model shows that only Xinningpu remains efficient when preferences over indicators are taken into account. Liuhe undergoes a greatest change from 1.000 to 0.769 because of a relatively poorer performance in patronage volume and mileages whose weights are 0.575 and 0.352 in output group respectively. Pukou is another interesting case for which the enhanced DEA has modified its efficiency from 1 to 0.894. The modification of Pukou is a result of a poor performance in patronage volume, which becomes a dragger, although Pukou does an excellent job in fuel cost control, which exerts a less impact on efficiency assessment than “patronage volume”.

In 2010, although Zhongbei, Xinningpu, Pukou, and Liuhe are evaluated as efficient units by the conventional DEA model, three of them, Zhongbei, Pukou and Liuhe, are assessed to be not efficient anymore by enhanced DEA model. There is a reason to believe the change is caused by the add-in of constraint cones. In this case, the labor cost and the depreciation expenses in input group (0.287 and 0.305) as well as the patronage (0.582) in output group show higher weights over others, suggesting that those three indicators should have more contributions to efficiency evaluation. Consequently, because of a relatively poorer performance in those three aspects, Zhongbei, Pukou and Liuhe are assessed as inefficient units via enhanced model. Meanwhile, the result also further reveals that both companies should improve their performances in terms of labor cost, depreciation expense and patronage.

Regarding the case of Xinningpu which is evaluated to be efficient unit by both models in both years, the operator

| Table 6 – Weights of indicators in 2009 and 2010. |
|--------------------|---------------|---------------|
| Input              |               |               |
| Fuel cost          | 0.234         | 0.223         |
| Labor cost         | 0.277         | 0.289         |
| Depreciation cost  | 0.308         | 0.312         |
| Other cost         | 0.181         | 0.175         |
| Output             |               |               |
| Patronage volume   | 0.575         | 0.474         |
| Mileage            | 0.352         | 0.460         |
| Satisfaction index | 0.073         | 0.066         |

<p>| Table 7 – Proposed model results. |</p>
<table>
<thead>
<tr>
<th>Bus operator</th>
<th>Efficiency in 2009</th>
<th>Ranking in 2009</th>
<th>Efficiency in 2010</th>
<th>Ranking in 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xinningpu</td>
<td>1.000</td>
<td>1.000</td>
<td>0.769</td>
<td>5</td>
</tr>
<tr>
<td>Liuhe</td>
<td>1.000</td>
<td>1.000</td>
<td>0.702</td>
<td>7</td>
</tr>
<tr>
<td>Zhongbei</td>
<td>0.966</td>
<td>2</td>
<td>0.915</td>
<td>2</td>
</tr>
<tr>
<td>Yagao</td>
<td>0.916</td>
<td>3</td>
<td>0.832</td>
<td>4</td>
</tr>
<tr>
<td>Xinningpu</td>
<td>0.874</td>
<td>5</td>
<td>0.732</td>
<td>6</td>
</tr>
<tr>
<td>Pukou</td>
<td>0.911</td>
<td>4</td>
<td>0.894</td>
<td>3</td>
</tr>
<tr>
<td>Nanjing</td>
<td>0.810</td>
<td>6</td>
<td>0.760</td>
<td>5</td>
</tr>
</tbody>
</table>

<p>| Table 8 – Comparison between proposed model and conventional DEA. |</p>
<table>
<thead>
<tr>
<th>Bus operator</th>
<th>2009 DEA</th>
<th>2009 Enhanced DEA</th>
<th>2010 DEA</th>
<th>2010 Enhanced DEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xinningpu</td>
<td>1.000</td>
<td>0.810</td>
<td>0.955</td>
<td>0.760</td>
</tr>
<tr>
<td>Liuhe</td>
<td>1.000</td>
<td>0.966</td>
<td>1.000</td>
<td>0.915</td>
</tr>
<tr>
<td>Zhongbei</td>
<td>1.000</td>
<td>0.916</td>
<td>0.953</td>
<td>0.832</td>
</tr>
<tr>
<td>Yagao</td>
<td>1.000</td>
<td>0.874</td>
<td>0.845</td>
<td>0.732</td>
</tr>
<tr>
<td>Xinningpu</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Pukou</td>
<td>1.000</td>
<td>0.911</td>
<td>1.000</td>
<td>0.894</td>
</tr>
<tr>
<td>Nanjing</td>
<td>1.000</td>
<td>0.769</td>
<td>1.000</td>
<td>0.702</td>
</tr>
</tbody>
</table>
demonstrated a relative balanced and outstanding results in all selected criteria with no obvious dragger.

By comparing the performance of seven bus operators in 2009 and 2010 (Fig. 2), both conventional DEA and enhanced DEA reveal that Nanjing, Yagao and Xincheng become weaker with a decrease in efficiency. However, by contrast to a decline in efficiency identified by proposed model, Zhongbei, Xinningpu Fukou and Liuhe are suggested to remain their efficient positions by conventional DEA. The reason can also be attributed to the introduction of preference of weights. Taking Zhongbei as an example, its depreciation expense has increased from 42.19 million RMB in 2009 to 47.93 million RMB in 2010 while the patronage volume decreased from 215.05 million to 208.52 million. The conventional DEA is unable to detect those changes because of a weight-free assumption while the proposed model targets those changes and takes them into consideration by using constraint cones.

In addition to yielding the overall ranking for all cities, the implementation of the Fuzzy-AHP model can also generate scores for each operator corresponding to any specific indicator. All the companies are expected to identify their operational weakness, which will directly help them to improve performance.

4. Conclusions

This paper presents an enhanced Data Envelop Analysis (DEA) model, which modified conventional DEA model by adding the constraint cones generated from the Fuzzy-AHP model to evaluate transit operator’s efficiency. The proposed model aims at including preference information over indicators into DEA process. The new model is designed to effectively solve a biased assumption of conventional DEA that no output or input is more important than the others as well as offering the advantages in ranking those efficient units. An extended Fuzzy-AHP model is employed to generate the constraint cones, which could prevent the vagueness and uncertainty. The characters of new system are applicable to help bus company identify its technical efficiency of input resource utilization.

To illustrate the applicability of the proposed approach, a real case in Nanjing City, the capital of Jiangsu Province has been selected, where the efficiencies of seven bus companies are assessed based on 2009 and 2010 dataset. A comparison between conventional DEA and enhanced DEA is also unfolded to clarify the new system’s dominance. Results reveal that the proposed model is more applicable in evaluating the transit operator’s efficiency and encouraging a boarder range of applications.

Acknowledgments

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REFERENCES


