Modeling radioactive decay

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Abstract

This study was prepared at the aim of teaching of the law of radioactive decay. The formula of the law of radioactive decay was tried to reach by using of 200 coins and with 200 procreated numbers in simulation to represent a radioactive element. With the data obtained, it was requested that the linear and exponential graphs be drawn connecting with the number of tails to the number of tosses. The students could match up the concepts used in the modeling to the concepts in nuclear physics and reached the law of radioactive decay’s formula in broad terms.

Keywords : Half-mean life, modeling, radioactive decay, radioactivity teaching, simulation

I. Introduction

Radioactive decay is the name given to the natural process by which an atomic nucleus spontaneously transforms itself (decays) into another, releasing dangerous high energy emissions (Hughes and Zalts, 2000).

If a sample consists of \(N_0\) radioactive nuclei at time \(t=0\), and if \(N(t)\) denotes the number of the nuclei remaining at any later time \(t\), then the function \(N(t)\) decreases exponentially with time, as given by eqs (1).

\[
N(t) = N_0 e^{-\lambda t}
\]  

(1)

There are several ways to characterize the rate at which a radioactive nucleus decays. One is to give the decay constant \(\lambda\), the probability that any one nucleus will decay in unit time. An alternative is to give the reciprocal, \(1/\lambda\), which is denoted by \(\tau\):

\[
\tau = \frac{1}{\lambda} \quad \text{(2)}
\]

\[
N(\tau) = \frac{N_0}{e} \quad \text{(3)}
\]

\(\tau\) is called the lifetime of the nucleus. With \(t=\tau=1/\lambda\), \(\tau\) is the time in which \(N\) drops to the fraction \(1/e\) of its original value. One can also show that \(\tau\) is the average time for which the nuclei in a sample survive, and \(\tau\) is therefore also called the mean life of the nucleus. Another way to characterize the decay rate of a nucleus is to give its half-life \((t_{1/2})\), the time in which the number of nuclei drops to half its original value (Taylor and Zafiratos, 1991):

\[
t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad \text{(4)}
\]
According to Hughes and Zalts (2000), in many cases, students are not used to exponential mathematics and lack the understanding necessary for interpreting eqs (1). Because of this class discussions of the persistence of radiation and its consequences may be quite limited. As the use of radioactive material in the classroom is not always practical or advisable, several alternative activities for modeling radioactive decay have been suggested (Edge, 1978; Hughes and Zalts, 2000; Jesse, 2003; Klein and Kagan, 2010; McGeachy, 1988; Schultz, 1997). For example, in Hughes and Zalts’ (2000) studies, an exponential decay graph was constructed quite easily with using a stiff paper, a scissor, a glue, a measuring tape, a pencil and strips. In addition, Klein and Kagan (2010) described in their study a visual and interactive use of dice to develop student understanding of radioactive decay. Jesse (2003) described a computer simulation, based on dice game, for identify radioactive decay.

In physics education research the word “model” is associated with David Hestenes and his colleagues, who advocated the use of models in physics instruction more than 20 years ago (Etkina et al, 2006). Hestenes (1987) defined a model in the following way: “A model is a surrogate object, a conceptual representation of a real thing. The models in physics are mathematical models, which is to say that physical properties are represented by quantitative variables in the models.”

Couch and Vaughn (1995) provided product information and data which would be helpful to instructors of introductory physics and physical science in explaining radioactivity demonstrations and experiments to students. Lapp (2010) presented examples of naturally radioactive items that were likely to be found in most communities in his study. Additionally, there was information provided on how to acquire many of these items inexpensively. He found that the presence of these materials in the classroom was not only useful for teaching about nuclear radiation and debunking the “nuclear free” myth, but also for helping students to understand the history of save of the commercial uses of radioactive materials since the early 20th century.

According to Crippen and Curtright (1998), graphing calculators have become common place in both secondary and post-secondary classroom and represent a practical resource for instruction. In their study, students used graphing calculators to develop concepts regarding nuclear decay.

This study aims teaching of the law of radioactive decay with modeling and simulation.

2. Method

It was carried out with 11 students of 4th grade in the Department of Physics Teaching, who are taking nuclear physics lecture, in a university of Turkey. In the research, the one-shot case study design among experimental designs was implemented.

Firstly, some questions are asked to the students with a view to designating their pre-knowledge about the radioactivity. Secondly, the formula of the law of radioactive decay is tried to reach. In the modeling of nuclear decay, in the first model, 200 coins (each of which is 5 kr) are used in an attempt to represent radioactive elements. (MEB, 2009) In here, it is important to lay on identical coins. And, in order to keep the cost low, we must been attentive to choose the lowest valuable coins (5 kr). While the 200 coins represent a radioactive element, each coin symbolizes one nucleus of the element. The head side of the coin stands for remaining nucleus of the atom and the tail side symbolizes disintegrated nucleus of the atom. It is only an acceptance and also it may convert.

After this analogy is established, 200 coins randomly throw and then the coins becoming tail take away. The heads throw again so as to show remaining atomic nucleus. Just all of the coins can be thrown by the same person; also they are carved up to groups according to the population of the team. In each groups, everyone shakes the coins in their hands and throws them onto the table. In every tests, it is determined the coins which are heads and which are tails. After all tosses, the figures of head sides count, and the tail sides are ousted out from the test. The new test is made with only the tail coins. So the test has been continued until all of coin will be taken out.

In the second model, the Microsoft Office Excel programmer is used to simulate the same radioactive element represented by 200 coins. In this programmer, randomly numbers between 0 and 1 are procreated in 200 cells. The 0 ones stand for remaining nucleus of the atom and the 1 symbolizes disintegrated nucleus of the atom. The simulation programmer is started and at every turn the numbers which are 0 are ousted out. For the 1 ones, the programmer is started again. It is continued until all of cells will be 0 (MEB, 2009).
After the implementations finished, some questions are asked as to which concepts in nuclear physics the concepts used in the modeling have exemplified. With the data obtained it is requested from the students that the linear and exponential graphs be drawn connecting with the number of tails to the number of tosses. Then some questions are asked representing the concepts of half life and mean life with reference to the application. And it is requested that these concepts’ numerical values were calculated from the graphs drawn by them.

2.1. Classroom Questions

1. What kind of an event brought about the discovery of the radioactivity?
2. Which event/events may have started and developed nuclear physics?
3. How do you describe the radiation and radioactivity?
4. When may have the event of the radioactivity begun in your opinion?
5. What is the meaning of 200 coins/200 procreated numbers, a coin/ a procreated number, heads/ the number 1, tails/ the number 0, throwing number / simulation number? What does it refer to?
6. What did you observe at the end of the tosses?
7. What do the half throwing, mean throwing, the probability of being head/tail, inclination mean?

3. Results and Discussion

At the end of the analysis of the study, it is determined that the students had pre-knowledge relevant to the discovery of radioactivity and its development process. As can be seen below, these are the answers of the students: (CQ refers to the classroom question)

(CQ1) “In 1896, in the experiment which Becquerel made with x rays, he observed the element used left trace on the photograph plaque when the tube was broken with an accident...” (CQ2) “Becquerel’s discovery of the radioactivity, Rutherford’s invention of the nucleus, Curie’s invention of the natural radioactivity, made up of the cyclotrons, artificial radioactivity...” (CQ3) “Radiation is radiance that the nucleus make to transform the steady state... Radioactivity is transform of the unsteady nucleus to the steady nucleus with alpha, beta and gamma radiance...” (CQ4) “... approximately 4.5 billion years ago, with the event of Big Bang...”

Values obtained during the sampling process of the radioactive decay were presented in Table 1.

<table>
<thead>
<tr>
<th>Toss number</th>
<th>Coins’ numbers</th>
<th>Procreated numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>105</td>
<td>101</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td>53</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

And the students’ answers of CQ5 were presented in Table 2.

<table>
<thead>
<tr>
<th>Radioactive element</th>
<th>200 coins</th>
<th>200 procreated numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic nucleus</td>
<td>1 coin</td>
<td>1 procreated number</td>
</tr>
<tr>
<td>Disintegrated nucleus</td>
<td>Head</td>
<td>The number 1</td>
</tr>
<tr>
<td>Remaining nucleus</td>
<td>Tail</td>
<td>The number 0</td>
</tr>
<tr>
<td>Time</td>
<td>Toss number</td>
<td>Simulation number</td>
</tr>
</tbody>
</table>
CQ6) “The parent material decayed and transformed to the steady state...”

It was seen that the students could match up the concepts used in the modeling the concepts in nuclear physics. (CQ7) “Half toss is referred to half life. Accordingly, it is the necessary time for the half of the atomic nuclei in a radioactive element to decay. Mean toss stands for mean life. It is the elapsing time until the radioactive element decays. The probability of being head refers to the probability of the number of nuclei to transform the steady state. The inclination of linear graph means decay rate (the number of decay in unit of time)”

In the graphs drawn by the students, there were great numbers of mistakes relevant to graph drawing. And they were asked to redraw the graphs after being given education on this subject. (Figure 1)

![Figure 1. The exponential and the linear graphs drawn connecting with the number of tails to the number of toss.](image)

It was observed that most of the students found approximately true values (drawn and counted with the same values in SPSS programmer) in the calculations done from their graphs. These values have indicated in Table 3.

<table>
<thead>
<tr>
<th>The Formula</th>
<th>Theoretical</th>
<th>Modeling</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(t) = 200e^{-0.7684t}$</td>
<td>$N(t) = 175.5e^{-0.6012t}$ (SPSS)</td>
<td>$N(t) = 232.7e^{-0.7684t}$ (SPSS)</td>
<td>$N(t) = 200e^{-0.76t}$ (by students)</td>
</tr>
<tr>
<td>Decay rate</td>
<td>0.693</td>
<td>0.66</td>
<td>0.76</td>
</tr>
<tr>
<td>Half life</td>
<td>1</td>
<td>1.05</td>
<td>0.98</td>
</tr>
<tr>
<td>Mean life</td>
<td>1.44</td>
<td>1.51</td>
<td>1.44</td>
</tr>
</tbody>
</table>
Edge (1978) explained a simple experiment which was an effective way to introduce students to the abstract concept of radioactive decay in his study. The experiment required beans or marbles of two different colors. At the end of his experiment, it was found that students’ results were usually quite close to the theoretical values.

Bevelacqua (2010) found that a discussion of radiological dispersion devices provides a vehicle to obtain a more comprehensive understanding of basic radiological science.

Marshall and Carrejo (2008) investigated university students’ development of mathematical models of motion in a physical science course for pre-service teachers and graduate students in science and mathematics education. And they found that although some students were familiar with the standard concepts of position, velocity and acceleration from physics classes, most students had difficulty using these concepts to characterize actual or hypothetical motions.

4. Conclusion

As a result, the students were enabled to better understand the subject by reifying the law of radioactive decay, which the students find it hard to understand, through modeling method. Brewe (2008) described the nature of modeling instruction at the university level and clarified the role of models in physics instruction in his paper. He defined a benefit of modeling instruction in the following way: “Modeling instruction provides students with a learning experience that is representative of the work of scientists.”

By varying the number of coin, it can easily be demonstrated that the radioactivity (rate constant) is independent of the number of coin. It is possible that dice may be used in the modeling in order to represent another radioactive element. According to Schultz (1997), the dice-shaking analogy, in addition to establishing an intuitive view of the concepts of radioactivity and half-life, provides a natural connection to the mathematical formalism and graphical treatment that describe radioactive decay. To use modeling or simulation in order to explain the subjects which are hazardous, costly, and hard to measure and practiced such as radioactive decay is considerably advantageous.

Although the modeling is good at explanation of the concepts or events which are notional and hard to understand, if it is not taken into consideration it will cause contradiction in terms and misconceptions. That’s why, the resemblances and differences should be stated before the modeling or simulation as to prevent the misconceptions.

References


