Effects of thermal radiation, Soret and Dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation

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Abstract

In this paper, the influence of some thermo-physical properties of fluid on heat and mass transfer flow past semi-infinite moving vertical plate is considered. The fluid considered is optically thin such that the thermal radiative heat loss on the fluid is modeled using Rosseland approximation. The governing equations representing the physical model is a system of partial differential equations which are transformed into systems of coupled non-linear partial differential equation by introducing non-dimensional variables. The resulting equations are solved using the spectral relaxation method (SRM). The result shows that an increase in Eckert number of the fluid actually increases the velocity and temperature profiles of the flow. Whereas an increase in thermal radiation parameter reduces the temperature distribution when the plate is being cooled. The computational results for velocity, temperature and the concentration profiles are displayed graphically for various flow pertinent parameters.

Keywords: Soret and Dufour effect; Heat transfer; Viscous dissipation; Thermal radiation; Chemical reaction; Spectral relaxation method

1. Introduction

The study of natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, drying process etc. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. Prasad and Reddy [1] have studied the radiation and mass transfer effects on an unsteady MHD convective flow past a heated vertical plate in a porous medium with viscous dissipation. Ferdows et al. [2] studied soret and dufour effects on natural convection heat and mass transfer flow in a porous medium considering internal heat generation. While analyzing the heat and mass transfer characteristic of flow using exponential form of internal heat generation, they suggested that the velocity, temperature and concentration flow fields are appreciably influenced by dufour and soret...
effects. Worth note in their analysis is the fact that with increasing Dufour number and decreasing Soret number the velocity and concentration distributions reduced significantly, while temperature distribution increases along the flow field when IHG is present or absent. Motsa and Shateri [3] studied the effects of soret and dufour on steady MHD natural convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation in the presence of a chemical reaction. In the analysis of the model, they remarked that an increase in soret and dufour parameters increase significantly the velocity and concentration profiles of the flow but noted that dufour effect enhances flow velocity much more than soret. In many chemical engineering processes, there occurred chemical reaction between a foreign mass and the fluid in which the plate is moving. Rajesh and Vijaya [4] investigated radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Gnaneswara and Bhaskar [5] studied the effects of soret and dufour on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Vempati and Laksh-Narayana-Gari [6] investigated the effects of soret and dufour on unsteady MHD flow past an infinite vertical porous plate with thermal radiation. Gbadeye et al. [7] studied the heat and mass transfer for soret and dufour effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field. soret and dufour effects on transient MHD flow past a semi-infinite vertical porous plate with chemical reaction was investigated by Shivaiah and Anand [8].

Heat and mass transfer (or double diffusion) finds applications in a variety of engineering process such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, nuclear waste disposal and others. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the energy fluxes generated by the transverse action of both temperature and composition gradients and the driving potentials are more complicated. The energy flux caused by a composition gradient is called Dufour or diffusion-thermal effect. Temperature gradient can also creates mass fluxes, and this is the called Soret or thermal-diffusion effect. Generally, the thermal-diffusion and the diffusion thermo effects are of smaller order of magnitude than the effects prescribed by Fick’s laws and are often neglected in heat and mass transfer processes by many researchers. The effects of soret for instance has been utilized for isotope separation. Subbakar and Gangadhar [9] investigated the effects of soret and dufour on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink. Olarawaju [10] studied similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation. In the like manner, Makinde and Mutuku [11] examined the effect of the complex interaction between the electrical conductivity of the conventional base fluid and that of the nanoparticles under the influence of magnetic field in a boundary layer flow with heat transfer over a convectively heated flat surface using numerical approach called Runge–Kutta–Fehlberg method with shooting technique. Hayat et al. [12] investigated soret and dufour effects on magnetohydrodynamic (MHD) flow of casson fluid. Progress has been considerably made in the study of heat and mass transfer on magnetohydrodynamic (MHD) flows due to its application in many devices such as the MHD power generators and hall accelerators. Madhusudhana and Reddy [13] studied the effects of soret and dufour on hydromagnetic heat and mass transfer over a vertical plate in a porous medium with a convective surface boundary condition and chemical reaction. Bhavana et al. [14] investigated the soret effect on free convective unsteady MHD flow over a vertical plate with heat source. Venkateswarlu et al. [15] examined the effects of chemical reaction and heat generation on MHD boundary layer flow of a moving vertical plate with suction and dissipation. Sarada and Shanker [16] studied the effects of soret and dufour on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Prabhakar [17] examined radiation and viscous dissipation effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with hall current in the presence of chemical reaction.

The role of thermal radiation on the flow and heat transfer process is of major importance in the design of many advanced energy conversion system operating at higher temperature. Thermal radiation within this system is usually as a result of emission by hot walls and the working fluid, Seigel and Howell [18]. Effect of radiation and soret in the presence of heat source/sink on unsteady MHD flow past a semi-infinite vertical plate was studied by Srinivas and Srinivasa [19]. Makinde et al. [20] examined combined effects of buoyancy force, convective heating, Brownian motion, thermophoresis and magnetic field on stagnation point flow and heat transfer due to nanofluid flow towards a stretching/shrinking sheet. They observed that both the skin friction coefficient and the local Sherwood number decrease, while the local Nusselt number increases with increasing intensity of buoyancy force and note that dual solution exists for shrinking case. Effect of partial slip on hydromagnetic flow over a porous stretching sheet with
non-uniform heat source/sink, thermal radiation and wall mass transfer was investigated by Abdul Hakeem et al. [21]. Similarly, Rashidi et al. [22] delved into the studied of heat transfer of a steady incompressible water base nanofluid flow over a stretching sheet in the presence of transverse magnetic field with thermal radiation and buoyancy effects. Recently, Bhuvanavijaya and Mallikarjuna [23] examined the effect of variable thermal conductivity on convective heat and mass transfer over a vertical plate in a rotating system with variable porosity regime. Scrinivasa et al. [24] examined soret and dufour effects on MHD boundary layer flow over a moving vertical porous plate with suction. Radiation effects on MHD natural convection flow along a vertical cylinder embedded in a porous medium with variable surface temperature and concentration was studied by Madhireddy [25]. Heat and mass transfer through a porous media of mhd flow of nanofluids with thermal radiation, viscous dissipation and chemical reaction effects was investigated by Eshetu and Shankar [26]. Mohammed [27] investigated an unsteady mhd convective heat and mass transfer past an infinite vertical plate embedded in a porous medium with radiation and chemical reaction under the influence of dufour and soret effects. The magnetic field effect on a steady two-dimensional laminar radiative flow of an incompressible viscous water based nanofluid over a stretching/shrinking sheet with second order slip boundary condition was investigated Abdul Hakeem et al. [28] using Lie symmetry group transformations and both analytical and numerical methods of solution. They concluded that unique exact solution exists for momentum equation in stretching sheet case and dual solutions are obtained for shrinking sheet case which has upper and lower branches. Motivated by the above literatures and the numerous possible industrial applications of the problem (as seen in isotope separation), it is of paramount interest in this paper to investigate the effects of thermal radiation, soret and dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation.

To our knowledge, no studies have been reported in literature with the regard to the effects of thermal radiation, soret and dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. Keeping this in mind, our concern in this paper is to illustrates the use of Spectral Relaxation Method (SRM) to solve the governing equations representing the physical model under investigation and examine the effects of different flow parameters as encountered in the equations. SRM as proposed by Motsa [29] is a new numerical method applicable to solve non-linear systems of boundary value problems. The method is applied by defining a rule of solution expression based on bivariate lagrange interpolation. The spectral relaxation method algorithm is then applied to decompose the governing nonlinear PDEs into a sequence of linear PDEs. The resulting linear sequence of PDEs contain variable coefficients that are not amenable to exact solution. Consequently, chebyshev spectral collocation method with bivariate Lagrange interpolation is applied independently in the space and time variables.

2. Mathematical analysis

Consider an unsteady two-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects (see Rao et al. [30]). The $x'$-axis is taken along the vertical infinite plate in the upward direction and the $y'$-axis normal to the plate since the plate is considered infinite in $x'$-direction (See Fig. 1), all flow become self-similar away from the leading edge. Therefore, all the physical variables becomes function of $t'$ and $y'$ only. The effects of soret, dufour and viscous dissipation are taken into account. By applying Boussinesq’s approximation, the flow field is govern by the following equations.

\[
\frac{\partial u'}{\partial y'} = 0, 
\]

\[
\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial y'} = v' \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u'}{\rho}, 
\]

\[
\frac{\partial T}{\partial t'} + u' \frac{\partial T}{\partial y'} = \alpha' \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho c_p} \left( \frac{\partial u'}{\partial y'} \right)^2 + \frac{Dk_T \partial^2 C}{C_s c_p \partial y'^2}, 
\]

\[
\frac{\partial C}{\partial t'} + u' \frac{\partial C}{\partial y'} = D \frac{\partial^2 C}{\partial y'^2} - K_r^2(C - C_\infty) + \frac{DK_T \partial^2 T}{T_m \partial y'^2}. 
\]
subject to the conditions

\[ u' = U_0, \quad T = T_w + \psi(T_w - T_\infty)e^\eta y', \quad C = C_w + \psi(C_w - C_\infty)e^\eta y' \quad \text{at} \quad y' = 0 \]  

\[ u' \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y' = \infty \]

where \( u' \) and \( v' \) are velocity components in \( x' \) and \( y' \) direction respectively, \( r' \)-time, \( C \)-dimensional concentration, \( c_p \)-specific heat at constant pressure, \( D \)-mass diffusivity, \( g \)-acceleration due to gravity, \( k'_p \)-chemical reaction parameter, \( \alpha \)-fluid thermal diffusivity, \( \beta \)-thermal expansion coefficient, \( \beta^* \)-concentration expansion coefficient, \( \mu \)-coefficient of viscosity, \( \rho \)-fluid density, \( K_T \)-thermal diffusion ratio, \( T_m \)-mean fluid temperature, \( T_\infty \)-free stream temperature, \( C_\infty \)-free stream concentration, \( \sigma \)-electrical conductivity of the fluid, \( B_0 \)-external imposed magnetic field strength in \( y \) direction, \( q_r \)-radiative heat flux, \( C_s \)-concentration susceptivity, \( \nu \)-viscosity, \( U_0 \)-the scale of free stream velocity, \( T_w \) and \( C_w \) are the wall dimensional temperature and concentration respectively, \( T_\infty \) and \( C_\infty \) are the free stream dimensional temperature and concentration respectively, \( n' \)-the constant.

From the continuity equation (1), it is obvious that suction velocity normal to the plate can either be a constant function or function of time. We consider a case when it is function of both constant and time, hence it is express as:

\[ v' = -V_0(1 + \epsilon Ae^{n'y'}) \]

In order to simplify the radiative heat flux on the flow, we have given preference to the application of Rosseland diffusion approximation as reported in Adegbie and Fagbade [31] such that:

\[ q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y'} \]

where \( \sigma_s \) is the Stefan–Boltzman constant and \( k_{e} \) is the mean absorption coefficient. By using the Roseland approximation, the present study is limited to optically thin fluids. If temperature difference within the flow is sufficiently small, then Eq. (8) can be linearize by expanding \( T^4 \) in the Taylor series about \( T_\infty \) and neglecting higher order term; we obtain

\[ T'^4 = 4T_\infty^2 T - 3T_\infty^4 \]

In view of Eqs. (8) and (9), Eq. (3) reduces to;

\[ \frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \alpha \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma_s T_\infty^3}{3pc_pk_e} \frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u'}{\partial y'} \right)^2 + \frac{D_m k_T}{C_s c_p} \frac{\partial^2 C}{\partial y'^2}. \]

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduce;

\[ u = \frac{u'}{u_0}, \quad y = \frac{v_0^2 t'}{v}, \quad t = \frac{v_0^2 t'}{u}, \quad n = \frac{vn'}{v_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad Pr = \frac{v\rho c_p}{k} = \frac{v}{\alpha}, \quad Sc = \frac{v}{D}, \quad G_r = \frac{g\beta v (T_w - T_\infty)}{u_0 v_0^2}, \quad G_m = \frac{g\beta^* v (C_w - C_\infty)}{u_0 v_0^2}, \quad Ec = \frac{u_0^2}{c_p(T_w - T_\infty)}, \quad k_r^2 = \frac{k_r^2 v}{v_0^2}, \quad R = \frac{16\sigma_s T_\infty^3}{3k_{e}k}, \quad M = \frac{\sigma B_0^2 v}{\rho v_0^2}. \]
In view of Eqs. (11)–(15), Eqs. (2)–(10) reduce to the following dimensionless form:

\[
\frac{\partial u}{\partial t} - (1 + \epsilon A\epsilon t) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \phi - M^2 u
\]

\[
\frac{\partial \theta}{\partial t} - (1 + \epsilon A\epsilon t) \frac{\partial \theta}{\partial y} = \left(1 + \frac{R}{Pr}\right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y}\right)^2 + Du \frac{\partial^2 \phi}{\partial y^2}
\]

\[
\frac{\partial \phi}{\partial t} - (1 + \epsilon A\epsilon t) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - k_r \frac{\partial^2 \theta}{\partial y^2}
\]

where \(G_r, G_m, Pr, R, Ec, Sc, k_r, Du\) and \(Sr\) are the thermal Grashof number, modified Grashof number, Prandtl number, radiation parameter, Eckert number, Schmidt number, chemical reaction parameter, dufour number and soret number respectively.

The transformed initial and boundary conditions are:

\[
u = 1, \quad \theta = 1 + \epsilon e^{nt}, \quad \phi = 1 + \epsilon e^{nt} \quad \text{at} \quad y = 0 \quad (19)
\]

\[
u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \quad \text{at} \quad y \rightarrow \infty. \quad (20)
\]

In practical engineering application, the physical quantities of practical values are the local skin friction coefficient, local nusselt number of the flow and sherwood number. The coefficient of skin-friction, Nusselt number and Sherwood number at the plate are given as:

\[
\text{Skin friction coefficient} : C_f = \frac{t_w}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y}\right) \bigg|_{y=0}, \quad (21)
\]

\[
\text{Nusselt Number} : Nu = -x \left(\frac{\partial T}{\partial y}\right) \bigg|_{y=0}, \quad (22)
\]

\[
\text{Sherwood Number} : Sh = -x \left(\frac{\partial C}{\partial y}\right) \bigg|_{y=0}. \quad (23)
\]

Using dimensional quantities defined in Eqs. (11)–(15), The physical quantities are simplify as:

\[
NuRe_s^{-1} = -\left(\frac{\partial \theta}{\partial y}\right) \bigg|_{y=0}. \quad (24)
\]
Likewise, the rate of mass transfer coefficient is given as

\[ ShRe_x^{-1} = - \left( \frac{\partial \phi}{\partial y} \right) \bigg|_{y=0} \]  

(25)

where

\[ Re_x = \frac{V_0 x}{v} \]  

(26)

is the local Reynolds number.

3. Spectral relaxation method

Gauss-seidel type of relaxation approach is adopted while deriving spectral relaxation method (SRM) as established to linearize and decoupled the system of non-linear differential equations, Motsa [29]. The linear terms in each equations are evaluated at the current iteration level (denoted by \( r + 1 \)) and non-linear terms are assumed to be known from the previous iteration level (denoted by \( r \)), Hoffman [32]. The resulting sequence of equation is integrated using Chebyshev spectral collocation method. Implementing the SRM on the resulting system of nonlinear partial differential equations (16)–(18) give the following linear partial differential equations

\[ u_{r+1}'' + a_{0,r} u_{r+1}' + a_{1,r} - M^2 u_{r+1} = \frac{\partial u_{r+1}}{\partial t}, \]  

(27)

\[ b_{0,r} \theta_{r+1}'' + b_{3,r} \theta_{r+1}' + b_{1,r} + b_{2,r} = Pr \frac{\partial \theta_{r+1}}{\partial t}, \]  

(28)

\[ \phi'' + c_{0,r} \phi_{r+1}' + c_{1,r} \phi_{r+1} + c_{2,r} = Sc \frac{\partial \phi_{r+1}}{\partial t}. \]  

(29)
Subject to

\begin{align}
    u_{r+1}(0, t) &= 1, \quad u_{r+1}(\infty, t) = 0, \quad \theta_{r+1}(0, t) = 1 + \epsilon e^{nt}, \\
    \theta_{r+1}(\infty, t) &= 0, \quad \phi_{r+1}(0, t) = 1 + \epsilon e^{nt}, \quad \phi_{r+1}(0, t) = 0 \\
\end{align}

where the coefficient parameters \( a_{0,r}, a_{1,r}, b_{0,r}, b_{1,r}, b_{2,r}, c_{0,r}, c_{1,r}, c_{2,r} \) are defined as:

\begin{align}
    a_{0,r} &= 1 + \epsilon A e^{nt}, \quad a_{1,r} = G_r \theta_r + G_m \phi_r, \quad b_{0,r} = 1 + R, \\
    b_{1,r} &= Du \phi_{r+1}' Pr, \quad b_{2,r} = Ec (u_{r+1}')^2 Pr, \quad b_{3,r} = Pr(1 + \epsilon Ae^{nt}), \\
    c_{0,r} &= Sc(1 + \epsilon Ae^{nt}), \quad c_{1,r} = -Sc k_r^2, \quad c_{2,r} = ScSr \theta_{r+1}'.
\end{align}

The initial approximations \( u_0(y, t), \theta_0(y, t), \phi_0(y, t) \) for solving the linearized governing equations (27)–(29) are obtain on the basis of the boundary conditions (19)–(20) and present as:

\begin{align}
    u_0(y, t) &= e^{-y}, \quad \theta_0(y, t) = e^{-y} + \epsilon e^{nt}, \quad \phi_0(y, t) = e^{-y} + \epsilon e^{nt}.
\end{align}

Then, starting from the initial approximations given in (35) above, Eqs. (27)–(29) are solved iteratively. Eqs. (27)–(29) are discretize using the chebyshev spectral collocation method in the \( t \) – direction, while implicit finite difference method is apply in the \( y \) – direction. The finite difference scheme employs the centering about a mid-point between \( t^{n+\frac{1}{2}} \) to any function, say \( u(y, t) \) and its associated derivative is given as:

\begin{align}
    t^{n+\frac{1}{2}} &= \frac{t^{n+1} + t^n}{2}, \quad u(y, t^{n+\frac{1}{2}}) = u_n^{n+\frac{1}{2}} = \frac{u_{n+1}^n + u_n^n}{2}, \quad \left( \frac{\partial u}{\partial t} \right)^{n+\frac{1}{2}} = \frac{u_{n+1}^n - u_n^n}{\delta t}.
\end{align}
The spectral method is applied on Eqs. (27)–(29) to obtain:

\begin{align*}
(D^2 + a_{0,r} D - M^2)u_{r+1} + a_{1,r} &= \frac{du_{r+1}}{dt}, \quad u_{r+1}(x_0, t) = 1, \quad u_{r+1}(\infty, t) = 0 \quad (37) \\
(b_{0,r} D^2 + b_{3,r} D)\theta_{r+1} + b_{2,r} &= Pr \frac{d\theta_{r+1}}{dt}, \quad \theta_{r+1}(x_0, t) = 1 + \epsilon e^{nt}, \quad \theta_{r+1}(\infty, t) = 0 \quad (38) \\
(D^2 + c_{0,r} D - c_{1,r} t)\phi_{r+1} + c_{2,r} &= Scd \frac{d\phi_{r+1}}{dt}, \quad \phi_{r+1}(x_0, t) = 1 + \epsilon e^{nt}, \quad \phi_{r+1}(\infty, t) = 0 \quad (39)
\end{align*}

where

\begin{align*}
u_{r+1} &= \begin{bmatrix} u_{r+1}(x_0, t) \\ u_{r+1}(x_1, t) \\ \vdots \\ u_{r+1}(x_{N_x-1}, t) \\ u_{r+1}(x_{N_x}, t) \end{bmatrix}, \quad a_{0,r} = \begin{bmatrix} a_{1,r}(x_0, t) \\ \vdots \\ a_{1,1}(x_1, t) \\ \vdots \\ a_{1,r}(x_{N_x}, t) \end{bmatrix} \\
\theta_{r+1} &= \begin{bmatrix} \theta_{r+1}(x_0, t) \\ \theta_{r+1}(x_1, t) \\ \vdots \\ \theta_{r+1}(x_{N_x-1}, t) \\ \theta_{r+1}(x_{N_x}, t) \end{bmatrix}, \quad b_{1,r} = \begin{bmatrix} b_{1,r}(x_1, t) \\ \vdots \\ b_{1,1}(x_1, t) \\ \vdots \\ b_{1,r}(x_{N_x}, t) \end{bmatrix}
\end{align*}
\[ \phi_{r+1} = \begin{bmatrix} \phi_{r+1}(x_0, t) \\ \phi_{r+1}(x_1, t) \\ \vdots \\ \phi_{r+1}(x_{N_x-1}, t) \\ \phi_{r+1}(x_{N_x}, t) \end{bmatrix}, \quad c_{0,r} = \begin{bmatrix} c_{1,r}(x_0, t) \\ c_{2,r}(x_1, t) \\ \vdots \\ \vdots \\ c_{1,r}(x_{N_x}, t) \end{bmatrix}. \] (42)

Thereafter we apply finite difference scheme on (37)–(39), in the \( t \) direction with centering about the mid-point \( t_{n+\frac{1}{2}} \) to obtain the following systems:

\[ A_1 U_{r+1}^{n+1} = B_1 U_{r+1}^n + K_1, \] (43)
\[ A_2 \theta_{r+1}^{n+1} = B_2 \theta_{r+1}^n + K_2, \] (44)
\[ A_3 \phi_{r+1}^{n+1} = B_3 \phi_{r+1}^n + K_3. \] (45)

Subject to the following initial and boundary conditions:

\[ u_{r+1}(x_0, t^n) = 1, \quad u_{r+1}(x_{N_x}, t^n) = 0 \]
\[ \theta_{r+1}(x_0, t^n) = \phi_{r+1}(x_0, t^n) = 1 + e^{\epsilon t^n}, \]
\[ \theta_{r+1}(x_{N_x}, t^n) = \phi_{r+1}(x_{N_x}, t^n) = 0, \]
\[ u_{r+1}(y, 0) = e^{-y}, \quad \theta_{r+1}(y, 0) = e^{-y} + e^{\epsilon t^n}, \]
\[ \phi_{r+1}(y, 0) = e^{-y} + \epsilon e^{\epsilon t^n} \]
where

\[
A_1 = \left( \frac{1}{2} (D^2 + a_{0,r} D - M^2) - \frac{1}{\delta t} \right) u_{r+1}^{n+1},
\]
\[
A_2 = \left( \frac{1}{2} (b_{0,r} D^2 + b_{3,r} D) - \frac{P r}{\delta t} \right) \theta_{r+1}^{n+1},
\]
\[
A_3 = \left( \frac{1}{2} \left( D^2 + c_{0,r} D + c_{1,r} I - \frac{S c}{\delta t} \right) \right) \phi_{r+1}^{n+1},
\]
\[
B_1 = - \left[ \frac{1}{\delta t} + \frac{1}{2} (D^2 + a_{0,r} D - M^2) \right] u_{r+1}^{n},
\]
\[
B_2 = - \left[ \frac{P r}{\delta t} + \frac{1}{2} (b_{0,r} D^2 + b_{3,r} D) \right] \theta_{r+1}^{n}, \quad B_3 = - \left[ \frac{S c}{\delta t} + \frac{1}{2} (D^2 + c_{0,r} D + c_{1,r} I) \right] \phi_{r+1}^{n},
\]
\[
K_1 = -a_{1,r}^{n+\frac{1}{2}}, \quad K_2 = - \left( b_{2,r}^{n+\frac{1}{2}} + b_{1,r}^{n+\frac{1}{2}} \right), \quad K_3 = -c_{2,r}^{n+\frac{1}{2}}
\]

where \( I \) is an identity vector of size \((N_x + 1) \times (N_x + 1)\). \( U, \theta \) and \( \phi \) are the vectors of the functions \( u, \theta, \phi \) when evaluated at the grid points. The boundary conditions above are imposed on the first and last rows as follows.

\[
u_{0,r}^{n+1}(x_j, t^n) = u_{0,r+1}(x_j, t^n), \quad \theta_{0,r}^{n+1}(x_j, t^n) = \theta_{0,r+1}(x_j, t^n), \quad \phi_{0,r}^{n+1}(x_j, t^n) = \phi_{0,r+1}(x_j, t^n), \quad j = 0, 1, 2, 3, \ldots, N_x.
\]

Hence, starting from the initial approximations \( u_0(t, y), \theta_0(t, y), \phi_0(t, y) \) given by Eq. (35), the Eqs. (43)–(45) can be solved iteratively until a solution that converges to a given level of accuracy is obtained.

Fig. 6. Velocity, temperature and concentration profiles for different values of \( Du \).
Fig. 7. Velocity, temperature and concentration profiles for different values of $E_c$.

Table 1
Comparison of computational values for Skin-friction coefficient $C_f$ and Nusselt number $N_u = R e^{-1}$ for different values of thermal radiation parameter when $M = 0.5, Sr = Du = 0$, $Ec = 0.001, Sc = 0.6, G_m = G_r = 2$, $A = K_r = 0.5, Pr = 0.71, n = 0.1$ and $t = 1.0$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>Present study</th>
<th>Rao et al. [30]</th>
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<tbody>
<tr>
<td>$C_f$</td>
<td>$-\theta'(0)$</td>
<td>$C_f$</td>
</tr>
<tr>
<td>0.0</td>
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4. Results and discussion

An efficient numerical method called spectral relaxation method has been used to solve the transformed Eqs. (16)–(18) subject to the boundary conditions (19) and (20). The paper examined the effect of governing parameters on the transient velocity profile, temperature profile as well as the concentration profile. For purpose of discussing our results, the SRM approach has been applied for various values of flow controlling parameters: $G_r = 2.0, Pr = 0.71, Ec = 0.001, Sc = 0.6, R = k_r = A = 0.5, n = 0.1, Du = 0.2, Sr = 0.5, t = 1.0$ to obtain a clear insight into the physics of the problem. Therefore all the graphs and tables correspond to the values above unless otherwise stated.

In the absence of soret and dufor parameter, the results obtained in this paper are more generalized form of Rao et al. [30] and hence can be taken as a limiting case by taking $Du = Sr = 0$. Remarkably, the present results as shown in Table 1 are in excellent agreement with the result of Rao et al. [30] in the absence of soret
Fig. 8. Velocity, temperature and concentration profiles for different values of $Pr$.

<table>
<thead>
<tr>
<th>Sr</th>
<th>Du</th>
<th>$u'(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
</tr>
</thead>
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Table 3
Computed values of the local skin friction, local Nusselt number and local Sherwood number for various values of the thermal radiation parameter when $Pr = 0.71, Gr = Gm = 2.0, R = 0.5, t = 1.0, M = 0.1$.

<table>
<thead>
<tr>
<th>R</th>
<th>$u'(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
</tr>
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<td>1.0</td>
<td>2.0512</td>
<td>0.5892</td>
<td>0.6994</td>
</tr>
</tbody>
</table>
and dufor parameter. This establishes the validity of the present solution and justifies the use of the present numerical method. Furthermore, we present the computational values for skin-friction coefficient, local heat transfer rate (Nusselt number) and Sherwood number at different values of the soret number $Sr$, dufor number $Du$ and thermal radiation intensity $R$ (see Tables 2–3). Table 2 illustrates the local skin friction, local Nusselt number and local sherwood number at various values of the soret number ($Sr$) and dufour number ($Du$). It can be seen that there is an increase in the generation of skin friction coefficient, whereas Nusselt number reduces with an increase in both soret and dufor numbers respectively. While an increase in soret number drastically reduces the magnitude of Sherwood number along the flow. Table 3 gives account on the effect of the thermal radiation parameter on the local skin friction, local Nusselt number and the local sherwood number with a variation in the magnetic parameter $M$. It is noticed that an increase in thermal radiation parameter with an optimized value of magnetic parameter enhances skin friction coefficient but reduces the local heat transfer rate.

Fig. 2 shows the influence of thermal buoyancy force parameter $Gr$ on the velocity, temperature and concentration profiles. From the figure, velocity profile increases with an increase in the value of the thermal buoyancy force. Thermal buoyancy force parameter $Gr$ is a dimensionless number which approximates the ratio of the buoyancy to viscous force acting on a fluid in boundary layer. At higher thermal buoyancy force parameter $Gr$, the boundary layer is laminar. As expected, it is observed that there is an upward increase in the velocity profile due to the enhancement of thermal buoyancy force. Also, as $Gr$ increases, the maximum value attained by the velocity of the fluid increases rapidly near the porous plate and then decay smoothly to the free stream velocity. The influence of thermal buoyancy force parameter $Gr$ on the temperature distribution and concentration profile are negligible as seen in Fig. 2. Fig. 3 illustrates the influence of the modified Grashof number $Gm$ on the velocity, temperature and concentration profiles. The fluid velocity increases and reached the peak due to an increase in the species buoyancy force. The velocity distribution attained a distinctive maximum value in the vicinity of the plate and then decreases steadily to approach the free stream value. Whereas, there is little or no change in the temperature and concentration distributions within
the boundary layer as modified Grashof number increases. Fig. 4 shows the velocity, temperature and concentration profiles for different values of the radiation parameter. We observed that thermal radiation enhances convective flow such that as thermal radiation intensity $R$ increases, flow velocity and temperature distribution increase within the thermal buoyancy layer very close to the plate. Furthermore, it is noted that an increase in the thermal radiation parameter produces a significant increase in the thermal condition of the fluid and its thermal boundary layer. It is noticed that an increase in thermal radiation intensity does not significantly influence the fluid concentration, this is evident in governing equation. Similarly, we investigate the influence of the Dufour and Soret effects separately in order to clearly observe their respective effects on the velocity, temperature and concentration profiles on the flow. The variation of lateral velocity and concentration profiles for different value of the Soret number $Sr$ are shown in Fig. 5. It is clearly seen that the velocity distribution in the boundary layer increases with an increase in the Soret number from which we conclude that the fluid velocity rises due to greater thermal diffusion. In addition, concentration profile increases with an increase in the value of the Soret number. Fig. 6 represents the variation of different value of Dufour number $Du$ on the dimensionless velocity, temperature and concentration distributions. From the figure, it is observed that as the Dufour number increases the velocity of the flow increases. It was noticed that diffusion thermal effect greatly affect the fluid temperature. As value of the Dufour parameter increases, the fluid temperature distribution also increases whereas the effect of Dufour number is very marginal on concentration profile. It was observed that the behavior of $Du$ and $Sr$ on temperature and the concentration distribution is opposite. The effects of the viscous dissipation parameter i.e. the Eckert number $Ec$ on the velocity, temperature and concentration profiles are shown in Fig. 7. The Eckert number $Ec$ expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

The effect of the Prandtl number on the velocity, temperature and concentration profiles is shown in Fig. 8. It is observed that the velocity profile decreases with the increase in Prandtl number $Pr$; this is due to the fact that fluids...
with higher Prandtl number possess greater viscosities and this will serve to reduce velocities, thereby lowering the skin friction. In the same vein, an increase in the Prandtl number corresponds to a decrease in the temperature and the thermal boundary layer thickness. This is because for small values of the Prandtl number $Pr(<1)$, the fluid is highly conductive. Fig. 9 illustrates the influence of the magnetic parameter $M$ on the flow profiles. It is found that the velocity profile of the flow decreases with an increasing magnetic parameter. The presence of transverse magnetic field produces a resistive force on the fluid flow, this force is called the Lorentz force. Its lower the motion of electrically conducting fluid since the magnetic field exact a retarding force on the free convective flow. This is clearly seen in velocity profile as shown in Fig. 8(a). The effects of the chemical reaction parameter $k_r$ on the velocity, temperature and concentration profiles are shown in Fig. 10. We noticed that an increase in the chemical reaction parameter leads to decrease in the velocity and concentration profiles. The effect of the Schmidt number on the velocity, temperature and concentration profiles is shown in Fig. 11. It is noticed that as the Schmidt number increases, the velocity and concentration profiles decrease. This is as a result of concentration buoyancy effects that often retard the fluid velocity. Reduction in the velocity and concentration distributions as a result of an increase in chemical reaction parameter correspond to a simultaneous reduction in the velocity and concentration boundary layers.

5. Conclusion

This study investigated an approximate analysis on the problem of an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation under the influence of thermal radiation, soret and dufour. Rosseland diffusion approximation has been used to describe and model radiative heat loss on the flow. In the analysis, a spectral relaxation method is employed to solve the resulting coupled non-linear partial differential equations. The spectral collocation method expressed in term of Lagrange interpolation polynomials and
adapted to decouple nonlinear systems of partial differential equations using relaxation techniques has been used. The following conclusions are drawn in the study.

- An increase in the thermal radiation parameter leads to an increase in both the velocity and temperature profiles. It is very obvious from our results that thermal radiation intensifies the convective flow.
- Velocity profile as well as the concentration profile increases with an increase in the soret number. Whereas soret parameter has little or no effect on temperature distribution.
- Both the velocity and temperature profiles increase with an increase in the dufour number.
- An increase in the magnetic parameter decreases the velocity profile.
- Velocity Profile as well as concentration profile decreases with an increase in the chemical reaction parameter and Schmidt number respectively.
- The influence of thermal radiation, soret and dufour effect as well as chemical reaction parameter on the flow profiles is significant and hence finds application in engineering problems such as isotope separation, chemical catalytic reactors and processes etc.

References