Analysis and modelling of the cohesion strength of concrete at high strain-rates

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ABSTRACT

With the exponential increase of computational power, numerical simulations are more and more used to model the response of concrete structures subjected to dynamic loadings such as detonation near a concrete structural element or projectile-impact. Such loadings lead to intense damage modes resulting from high strain-rate tensile loadings in the concrete structure. However, the modelling of the post-peak tensile response of concrete still remains difficult due to the lack of experimental data at high strain-rates. This work aims at improving the modelling of the softening behaviour of concrete based on the following statement: despite the propagation of unstable cracks in the tested specimen cohesion strength exists in the vicinity of triggered cracks and is driving the whole softening behaviour of concrete. This statement is justified in the present work by means of experiments and Monte-Carlo calculations: firstly, concrete samples have been subjected to a dynamic tensile loading by means of spalling experiments. Several specimens have been recovered in a damaged but unbroken state and have been subsequently loaded in quasi-static tensile experiments to characterise the residual strength and damage level in the sample.

In addition, Monte-Carlo simulations have been conducted to clarify the possible influence of cohesion strength in the vicinity of cracks. Finally, the DFH (denoual–Forquin–Hild) anisotropic damage model has been adapted to take into account the cohesion strength in the damaged zone and to describe the softening behaviour of concrete. Numerical simulations of experiments conducted on dry and saturated samples at different levels of loading-rate illustrate the new capability of the model.

1. Introduction

Concrete is the main material used in civil engineering all around the world. However many aspects of its behaviour still belong to the research area. The tensile behaviour is one of them. Although it is the Achilles’ heel of concrete structures loaded by explosion or projectile-impact, few experimental works have reported dynamic tensile behaviour in terms of stress–strain curves at high strain-rates. This lack of reliable test data affects the quality of modelling and limits the accuracy of numerical predictions to calculate the response of a concrete structure subjected to an accident or an intentional attack.

At high rates of loading, the spalling technique is the most adapted tool to characterise the tensile strength of brittle materials. “Spalling fracture” is based on the reflection of a compressive wave on a free boundary (or an impedance discontinuity). The dynamic tensile loading induced in the core of the sample leads to the onset, the growth and the coalescence of multiple cracks (Antoun et al., 2003; Forquin and Erzar, 2010). This kind of fracture mode can be observed during the impact of a target by a projectile (Meyers, 1994; Li et al., 2005). Nevertheless this technique is mostly used for identifying accurately the dynamic tensile strength of materials, and a derivation of the classical split Hopkinson Pressure Bars device has been employed to identify the strain-rate sensitivity of the tensile strength of concrete (Schuler et al., 2006; Schuler and Hansson, 2006; Erzar and Forquin, 2010; Weerheijm and Van Doormaal, 2007). In parallel several works (Bazant, 2002; Bazant and Planas, 1998; Zielinski, 1982; Weerheijm, 1992; Cadoni et al., 2009) focused on the post-peak behaviour of concrete. Hillerborg et al. (1976) proposed a cohesive model to describe the propagation of cracks in concrete. In this approach, it is assumed that the propagation of crack is not the only zone where energy is consumed: indeed, presence of inclusions induces a cohesive zone behind the crack front since aggregates are responsible for bridging phenomena locally resisting to crack opening (Fig. 1). This explanation is close to the idea of fracture process zone experimentally studied by Mihashi and Nomura (1996).
and Kishen (2010) and Hu and Duan (2004). Likewise, Zhang et al. (2010) presented 3-points bending tests of high performance concrete. Specimens were notched and instrumented with gauges glued at 10 mm of the expected crack path. These tests allowed following the crack front (when tensile strain reaches its maximum) and the end of the cohesive zone (when strain drops to zero).

In previous works, Erzar and Forquin have investigated the influence of strain-rate and water content on the maximum tensile strength and cracking pattern of concrete by means of spalling tests and edge-on impact tests (Erzar and Forquin, 2010, 2011). The present work aims at quantitatively evaluating the cohesion (post-peak) strength of a common concrete and a micro-concrete and proposing a modelling of their softening behaviour. To achieve this goal, several spalling tests have been performed to damage without breaking the specimen. Subsequently, the recovered sample has been subjected to a direct tensile test to determine their residual strength. The second part of this work focuses on modelling of the dynamic tensile behaviour of concrete including cohesion strength in obscured zones. In both experimental and modelling parts, a particular attention is accorded to the role of free-water content as recent studies have reported the influence of this parameter on quasi-static and dynamic strength of concrete. Finally the proposed model is compared to experimental data provided by spalling tests.

2. Experimental study of tensile strength of concrete at high strain-rates

2.1. Concrete grades

Two types of concrete with very different mesostructures have been tested. On the one hand, the MB50 micro-concrete, providing an aggregate distribution similar to a standard concrete but at a lower scale, is an interesting candidate for dynamic testing at laboratory scale. This material has already been characterised in direct tension at low regime (10⁻⁶ to 1 s⁻¹) by Toutlemonde (1994) and under dynamic confined compression (Forquin et al., 2008; 2010). On the other hand, the R30A7 concrete is closer to a standard concrete with a compressive strength of about 30 MPa and siliceous aggregates up to 8 mm diameter (Vu et al., 2009). The composition and main mechanical properties are gathered in Table 1.

A comparison of the results obtained with both microstructures allows assessing the influence of aggregate size on the dynamic tensile response of concrete at high strain-rates.

2.2. Spalling tests performed on concrete specimens (Erzar and Forquin, 2010, 2011)

Among the techniques used to investigate the tensile behaviour of concrete at strain-rates above a ten of 1/s, the spalling technique figures prominently. Derived from the classical SHPB technique, the concrete cylinder is placed at the end of the input bar and an impedance discontinuity is ensured at the other side (cf. Fig. 2). A small detonation (Weerheijm and Van Doormaal, 2007) or a projectile-impact (Schuler et al., 2006; Klepaczko and Brara, 2001; Erzar and Forquin, 2010) induces a compressive loading that propagates through the Hopkinson bar until reaching the bar-specimen interface. Part of this wave is reflected whereas the major part of the loading is transmitted to the sample. When this pulse reaches the free opposite face, it is reflected as a tensile loading leading to damaging the material.

The instrumentation and data processing used by Erzar and Forquin are described in details in (Erzar and Forquin, 2010, 2011). Combining data gathered from the gauges glued on the specimen and laser interferometer pointed out on the rear surface of the specimen (Fig. 2) allows identifying the wave speed, the dynamic Young’s modulus, the strain-rate at failure and the dynamic tensile strength (Schuler et al., 2006; Schuler and Hansson, 2006; Erzar and Forquin, 2010, 2011). Moreover, thanks to finite-element simulations, spherical-cap ended projectiles have been optimised to obtain more homogeneous tensile stress and strain-rate fields in the specimen than with flat-end projectile (Erzar and Forquin, 2010). An ultra-high-speed camera is used to visualise the fragmentation of the concrete sample. More recently a new methodology was proposed by Pierron and Forquin (2012) to identify the Young modulus and the stress field based on the use of the Virtual Fields Method.

More than 40 spalling tests have been performed on wet and dry specimens of MB50 and R30A7 concretes. As shown in Fig. 3(a), a difference of about 3 MPa is observed in spall strength between dry and wet specimens. Fig. 3(b) presents the velocity profile of two spalling tests conducted at the same impact velocity on the micro-concrete. Firstly, it is observed that free-water has an influence on the level of velocity rebound which is directly linked to the dynamic strength of the material (Novikov et al., 1966). The tensile strength of the concrete specimen increases due to the viscosity of water. Secondly, the post-rebound curve, revealing the tensile post-peak behaviour of the material (Kanel, 2010), is largely affected too (Fig. 3(b)). Indeed, a sharp rebound and a series of oscillations are observed with wet samples due to the entrapment of waves in the spall. In contrast, supposedly due to a loss of elastic energy in phenomena of micro-cracks opening and closing, dry concrete provides a plateau after the rebound. In consequence, the experimental data highlights to what extent the free-water affects the dynamic strength as well as the post-peak behaviour of concrete at high rates of strain.

To complete the analysis, post-mortem investigations have been conducted on the tested specimens. The recovered samples were infiltrated with a hyperfluid resin, then cut and polished to reveal the inner cracks network. This post-mortem analysis shows a large number of cracks perpendicular to the specimen axis which cannot be observed with a classical observation of the external face (cf. Fig. 4).

Moreover, the fracture planes are mainly located in the cementitious matrix: cracks had circumvented the aggregates. Although

![Fig. 1. Bridging phenomenon in concrete.](image-url)
very few aggregates are broken, the dynamic strength is much higher than the quasi-static value. In the literature, several authors have linked strain-rate sensitivity of concrete to transgranular fracturing of the aggregates at high strain-rates (Zielinski, 1982). However this explanation appears no longer sufficient as very few aggregates were fractured in the present case. Finally post mortem analyses conducted on both (MB50 and R30A7) concrete samples tested at different loading-rates illustrate that a strain-rate increase induces a widest damaged zone and a more intense cracking density (Forquin and Erzar, 2010).
2.3. Residual strength of damaged specimens after spalling tests

2.3.1. Spalling damage

During the experimental campaign, several initial velocities of the projectile have been used to reach tensile strain-rates in the specimen ranging from $20 \text{s}^{-1}$ to $150 \text{s}^{-1}$. When the projectile struck the Hopkinson bar at low initial velocity (few m/s), the concrete specimens were recovered in one piece without any apparent crack. However, signals extracted from strain-gauges placed at 60 mm and 40 mm from the rear face exhibit a strain peak $\varepsilon_{\text{max}}$ followed by a residual strain $\varepsilon_{\text{res}}$ (Fig. 5). Moreover, the velocity signal recorded by the laser interferometer presents a pronounced

![Experimental configuration of (a) the “42d” and the “16d” tests conducted on MB50 concrete, (b) and (c) corresponding strain-gauges history, (d) and (e) free-surface velocity recorded by a laser interferometry technique in these two experiments.](image)
rebound suggesting a tensile damage (Fig. 5(d) and (e)). Thus, the stress peak has been reached and an onset of damage is expected.

2.3.2. Direct tensile tests on damaged specimens

The level of damage is difficult to quantify directly from the damage pattern or from the experimental data. This is why additional quasi-static tensile experiments have been performed on the spalled specimens. After the spalling tests, unbroken specimens have been glued to a specific device (cf. Fig. 6(a)) and further loaded in tension at low strain-rate (10^{-6} s^{-1}). The strain-gauges used during the dynamic load have been re-used. Moreover LVDT sensors (Linear Variable Differential Transformer) have been added for measuring the axial displacement between the plateaus of the machine.

A series of five experiments have been conducted: two of them on wet R30A7 specimens, one on dry R30A7 sample and the other ones on dry and saturated MB50 concrete. The results gathered from an experiment performed on a dry specimen are plotted in Fig. 6(b). Depending on the zone where the gauge is glued, different responses are obtained: on the one hand, a lower \( E_{\text{damaged}} = \frac{d\sigma}{dc} \) is noted in the damaged zones. On the other hand, the signal of gauges located close to the contact surface indicates a Young’s modulus similar to the initial (undamaged) value identified experimentally during the spalling test (G-100 mm in Fig. 6(b)).

A summary of data obtained from these tensile experiments is gathered in Table 2 and in Fig. 7. Table 2 provides several information concerning spalling tests conducted without breaking the specimen to pieces. First, the strain-rate of the spalling test is given. Only tests conducted at strain-rates up to 60 s^{-1} ended without breaking the specimen. Secondly, the dynamic Young’s modulus \( E_{\text{dyn}} \) is deduced from the wave speed \( C_0 \). This last data is directly measured during the test by measuring the travel time of the compressive wave from the strain gauge to the free-surface. In each experiment reported in Table 2, a velocity rebound is measured on the rear face resulting from the initiation of tensile damage in the concrete specimen. According to Novikov et al. (1966) the so-called pullback velocity \( \Delta V_{pb} \) (the difference between the maximum velocity reached during the test and the value corresponding to the rebound) is related to the dynamic strength of the material \( \sigma_{\text{dyn}} \) thanks to the following acoustic approximation \( \sigma_{\text{dyn}} = \frac{1}{2} \rho C_0 \Delta V_{pb} \), \( \rho \) being the density the tested sample. This ultimate tensile stress is also reported in Table 2. Finally, \( E_{\text{res}} \) and \( \sigma_{\text{res}} \), respectively the maximum and residual strains are identified directly from a strain-gauge placed over the damaged zone (cf. Fig. 5). The damaged Young’s modulus and the quasi-static residual strength are deduced from the direct tensile test conducted on damaged samples.

Fig. 7 correlates the level of the strain peak with the residual strain in spalling test and the damaged Young’s modulus measured in quasi-static test: the higher the strain peak reached during the spalling test, the higher the residual strain and, the lower the quasi-static strength of the damaged sample. Similarly the damaged modulus is weaker in case of higher residual strains while it is close to its dynamic (undamaged) value when the residual strain is low. Based on this damaged Young’s modulus the Fig. 7(c) and (d) present the damage level respectively as function of the peak strain and strain-rate in spalling test. Again, the level of damage is directly linked to the level of peak strain reached during the spalling test. Finally, these “post-spalling quasi-static tensile experiments” appear as a possible tool to evaluate the level of damage induced in the spalling experiments.

### Table 2

<table>
<thead>
<tr>
<th>Spalling data</th>
<th>( \dot{c} ) (s^{-1})</th>
<th>( E_{\text{dyn}} ) (GPa)</th>
<th>( \sigma_{\text{dyn}} ) (MPa)</th>
<th>( \Delta V_{\text{dyn}} ) (jstrain)</th>
<th>( \Delta V_{\text{res}} ) (jstrain)</th>
<th>Damaged Young’s modulus (GPa)</th>
<th>Quasi-static residual strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16d – dry R30A7</td>
<td>33</td>
<td>31.5</td>
<td>7.5</td>
<td>670</td>
<td>136</td>
<td>10</td>
<td>1.04</td>
</tr>
<tr>
<td>3w – wet R30A7</td>
<td>60</td>
<td>42.0</td>
<td>11.7</td>
<td>3090</td>
<td>1230</td>
<td>22</td>
<td>0.98</td>
</tr>
<tr>
<td>21w – wet R30A7</td>
<td>28</td>
<td>43.2</td>
<td>10.9</td>
<td>600</td>
<td>90</td>
<td>2.5</td>
<td>0.40</td>
</tr>
<tr>
<td>42d – dry MB50</td>
<td>58</td>
<td>30.4</td>
<td>8.6</td>
<td>1590</td>
<td>430</td>
<td>9</td>
<td>1.68</td>
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<tr>
<td>45w – wet MB50</td>
<td>41</td>
<td>39.1</td>
<td>12.9</td>
<td>954</td>
<td>192</td>
<td>22</td>
<td>0.92</td>
</tr>
</tbody>
</table>

3. Modelling of the dynamic tensile behaviour of concrete

Even if the computational power increases exponentially, some key aspects of the dynamic response of materials are still difficult to model. The dynamic fragmentation of brittle materials has been widely investigated during the last decades. Several models have been proposed to fill the gap in dynamic conditions. Holmquist et al. (1993) proposed a damage model including plasticity and irreversible compaction under high confining pressure. This formulation has been extended by Riedel et al. (1999) to model the residual strength of the material. Based on the model of Mazars (1984), the PRM (Pontiroli–Rouquand–Mazars) model (Pontiroli, 1995; Pontiroli et al., 2010) allows modelling both compressive and tensile non-linear behaviour taking into account phenomena like hysteresis or crack closure. Visco-plastic damage models have been developed and applied to concrete (Sercombe, 1997; Pedersen et al., 2008). Lu and Xu (2004) have proposed a damage model based on concepts of initiation, growth and coalescence of microcracks. This model is fitted to describe the increase of strength with loading rate. In this work, the DFH (Denoual–Forquin–Hild) anisotropic damage model has been extended to concrete. Firstly, Monte-Carlo simulations have been used to assess the macroscopic influence of cohesive strength in obscurance volumes at microscopic scale. A cohesion law is proposed to describe the
softening behaviour of concrete. It has been implemented in the commercial finite-element code Abaqus/explicit through a VUMAT user subroutine. Finally a comparison of the experimental and numerical results allows evaluating the improvement of the model applied to concrete.

3.1. DFH model: a micromechanical approach

The Denoual–Forquin–Hild model (Denoual and Hild, 2000; Forquin and Hild, 2010) is based on the description of micromechanisms activated during the dynamic fragmentation process. It depicts the random distribution of defects in the microstructure, the onset, the propagation of unstable cracks and the obscuration of critical defects in the vicinity of cracks.
3.1.1. Distribution of critical defects

Weibull first assimilated the quasi-static fracture of brittle materials to a statistical process (Weibull, 1939; 1951). In dynamic conditions, the distribution of critical defects is supposed to be the same than the one identified in quasi-static tests for example in 3-points bending experiments (Forquin and Erzar, 2010). The density of critical defects \( \dot{\lambda} \) is assumed to be a power law function of the applied stress \( \sigma \):

\[
\dot{\lambda} = \dot{\lambda}_0 \left( \frac{\sigma}{\sigma_0} \right)^m
\]

where \( \sigma \) is used to define the positive part of \( \sigma \), \( m \) is the Weibull modulus (it reflects the scatter, i.e. the lower \( m \), the higher the scatter) and \( \sigma^0 / \dot{\lambda}_0 \) constitutes the Weibull scale parameter. The Weibull parameters of dry and wet MB50 and R30A7 concretes have been set in previous works (Erzar and Forquin, 2010, 2011). These parameters are gathered in Table 3.

Moreover, for wet concrete, an increase of tensile strength is observed for low to intermediate strain-rates, which is the consequence of the presence of free-water. A simple law has been proposed (Forquin and Erzar, 2010) to take into account the influence of free water on the triggering of cracks:

\[
\sigma_0^{\text{wet}} = \sigma_0 \left( \frac{\dot{e}}{\dot{e}_0} \right)^{n_{\text{Q5}}}
\]

\( \dot{e}_0 \) being a reference strain-rate and \( n_{\text{Q5}} \), a parameter fitted to obtain a tensile strength doubled over the range \( 10^{-5} \) to \( 1 \) s\(^{-1} \), according to results of quasi-static experiments conducted by Toutlemonde (1994) on concrete and microconcrete.

3.1.2. Propagation of cracks

When the stress level reaches the critical stress of a defect an unstable crack propagates in the material at a celerity assumed to be constant and proportional to the wave speed \( c_0 \):

\[
V_{\text{crack}} = k c_0
\]

Experimentally, several works have been conducted to characterise the crack speed in brittle materials. Dynamic experiments have reported values ranging from 0.23 to 0.59 \( c_0 \) (Ravi-chandar and Knauss, 1982; 1984a,b,c,d; Sieradzki and Dienes, 1988; Sharon et al., 1995). Recently, a new testing technique named the rocking spalling test has been proposed to identify the crack speed in concrete (Forquin and Cheriguène, 2011; Forquin, 2012). According to this technique a value of \( k \) close to 0.32 has been evaluated.

Based on the concept of energy conservation, an analytical solution of \( k \) has been reported by Broek (1982) and Kanninen and Popelar (1985). Considering the energy consumed by the growth of an elliptic crack from an initial length of \( 2a_0 \) to \( 2a \) and equating this expression to the kinetic energy obtained from displacement fields yields to a crack speed \( V_{\text{crack}} = 0.38c_0(1 - a_0/a) \). This
Fig. 11. Parametric study of the cohesive term influence on the dynamic tensile behaviour of concrete.

Fig. 12. Numerical simulation of 45 h spalling test. Visualisation of the axial stress field (in Pa).
expression tends rapidly towards an asymptote of $0.38C_0$. Consequently, in the fragmentation model, it has been considered that $k = 0.38$.

### 3.1.3. The obscuration phenomenon

When a crack is initiated, the stress field in the surrounding matter is affected. The tensile stress is locally released and no critical defect localised in the vicinity of this crack can be activated. The obscuration volume $Z_0$ around the crack increases proportionally to the crack length at power $n$. It is expressed according to the relation (4) (Denoual and Hild, 2000) as illustrated in Fig. 8.

$Z_0(T - t) = S(V_{\text{crack}}(T - t))^n$,

where $T$ and $t$ denote respectively the current time and the inception time, $Z_0$ denote the cracking velocity, $S$ is a shape parameter depending on dimension $n$ of the considered domain, i.e. $S_{n=3} = 4\pi/3$ for a sphere, $S_{n=2} = \pi$ for a circle or $S_{n=1} = 2$ for a one-dimensional problem.

### 3.1.4. The damage model

The description of the three main mechanisms activated during the fragmentation of brittle materials allows defining a non-obscuration probability $P_{\text{no}}$ for a point $P$ at a time $T$ corresponding

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**Fig. 13.** Stress–strain curve of saturated MB50 microconcrete at 50, 100 and 150 s$^{-1}$.

**Fig. 14.** Comparison between numerical results and experimental data of spalling tests performed on wet microconcrete. (a) Test 45w ($\dot{\varepsilon} = 41$ s$^{-1}$) and (b) Test 15w ($\dot{\varepsilon} = 130$ s$^{-1}$).
to the probability of non-existence of a crack in the horizon of the point considered (cf. Fig. 8) (Forquin and Hild, 2010):

\[
P_{\text{no}}(P, T) = \prod_{\text{horizon of }(P,T)} P'_{\text{no}}(x, t)
= \exp \left( -\int \frac{d\lambda}{Z_0(T-t)} dz dt \right).
\] (5)

Considering a homogeneous stress field for any time \( t < T \), the obscuration probability \( P_0 \) reads (Denoual and Hild, 2000):

\[
P_0(T) = 1 - P_{\text{no}}(T) = 1 - \exp \left( -\int_0^t \frac{d\lambda}{Z_0(T-t)} Z_0(T-t) dt \right).
\] (6)

As new cracks are initiated outside obscured zones, the evolution of the cracking density \( \lambda_b \) is linked to the density of critical defects \( \lambda_t \) by:

\[
\frac{d\lambda_{\text{crack}}}{dt} = (1 - P_0) \frac{d\lambda_t}{dt}.
\] (7)

According to Lemaître and Chaboche (1985), a damage variable may be defined as the real density of discontinuities of the matter. In the fragmentation model described herein, the probability of obscuration may be utilised as damage variable assuming that there is no cohesive strength in obscured zones. The growth of the obscuration probability \( P_0 \) can be deduced from Eqs. (4) and (6) as:
Assuming a constant stress-rate, characteristic parameters can be established (Denoual and Hild, 2000). The whole fragmentation process ends when the entire volume is obscured; each crack obscures an average volume equivalent to the total volume divided by the total number of cracks:

\[ \lambda_c(t_c) Z_0(t_c) = 1. \]  

The characteristic time \( t_c \) of the fragmentation process is deduced from the preceding equations. The characteristic stress \( \sigma_c \) as well as the characteristic density of critical defects \( \lambda_c \) can be computed (Eq. (11) and (12)):

\[ t_c = \left( \frac{1}{\sigma_0} \right)^{\frac{1}{\eta_D}} \left( \frac{1}{S V_{\text{crack}}} \right)^{\frac{1}{\eta_S}}, \]  

\[ \sigma_c = \bar{\sigma} t_c = \left( \frac{1}{\sigma_0} \right)^{\frac{1}{\eta_D}} \left( \frac{\bar{\sigma}}{S V_{\text{crack}}} \right)^{\frac{1}{\eta_S}}, \]  

\[ \lambda_c = \lambda_c(t_c) = \left( \frac{1}{\sigma_0} \frac{\bar{\sigma}}{S V_{\text{crack}}} \right)^{\frac{1}{\eta_D}}. \]  

Again, considering a constant stress-rate, one may derive an analytical expression of the obscuration probability from Eqs. (6) and (8):

\[ P_0 = 1 - \exp \left( -\frac{m}{m+n} \frac{\sigma}{\sigma_c} \right) ^{m+n}. \]  

In a case of zero cohesion strength on the crack lips a damage variable may be defined as the obscured fraction of domain equal to obscuration probability \( (D_{\text{no cohesion}} = P_0) \) (Denoual and Hild, 2000; Forquin and Hild, 2010). The macroscopic stress is computed according to:

\[ \Sigma_{\text{no cohesion}} = (1 - D_{\text{no cohesion}}) \sigma. \]  

The tensile damage being anisotropic in brittle materials, one damage variable is used for each principal direction:

\[ \{ \epsilon_{11}, \epsilon_{22}, \epsilon_{33} \} = -\frac{1}{E} \begin{bmatrix} 1 & -v & -v \\ -v & 1 & -v \\ -v & -v & 1 \end{bmatrix} \begin{bmatrix} \Sigma_{11} \\ \Sigma_{22} \\ \Sigma_{33} \end{bmatrix}, \]  

where \( E \) is the Young’s modulus, \( v \) the Poisson’s ratio of the undamaged material.

### 3.2. Modelling of cohesive behaviour of concrete

#### 3.2.1. Monte Carlo simulations

The bridging phenomenon is frequent in composite materials. Concrete may be compared to composite material since large and hard inclusions are spread all over the cementitious matrix. To assess the influence of microscopic cohesive strength on the macroscopic behaviour of concrete at high strain-rates, Monte-Carlo simulations have been performed using Matlab®. The routine is composed of 5 steps. Firstly, a two-dimensional mesh of the domain considered is created. The second step consists in drawing lots a random failure stress for each element considering a Weibull law while using parameters already identified for concrete in a previous study (Forquin and Erzar, 2010). Afterwards, the stress field increases in the whole domain at a constant stress-rate. When the stress reaches the failure stress of one element, the stress is set to zero in this element and in a spherical obscuration volume centred on this spot. Its radius develops at a constant speed of 0.38C0. The simulation is stopped when the whole domain is obscured.

This basic approach has been used to evaluate the dynamic response of three scenarios (cf. Fig. 9). In the first one, named “A”, the stress in each obscuration volume is set to zero corresponding to the classical assumption made in the DFH model (Forquin and Hild, 2010). In the “B” scenario, a constant negative stress-rate is assumed for any point M in an obscuration zone starting from the maximum level of stress reached on this spot. In the last one (“C”), the loading in the obscuration zone is driven by a constant negative stress-rate starting from the failure stress of the critical defect that obscured the point M. In case of overlapping of the obscuration areas, the stress drops to the minimum value of the two zones. In this last case, sudden drops of the stress level may be observed (Fig. 9(b)).

Monte-Carlo simulations have been used to determine the stress–strain relationship in each scenario. A domain 40 \( \times \) 40 mm\(^2\) in size is meshed with squared elements (\( L_e = 0.2 \) mm). The Fig. 10 presents the results obtained with the Weibull parameters identified for the wet MB50 micro-concrete (Forquin and Erzar, 2010) at a constant stress-rate of 100 s\(^{-1}\). In order to facilitate the comparison between the 3 configurations, the same negative stress-rate has been considered: \( \sigma_{\text{coh}} = -1000 \) GPa/s.

Finally, according to Fig. 10, the use of cohesion strength in obscured areas has a strong influence on the post-peak behaviour at macroscopic scale. However the number of activated defects remains unchanged since no additional defect is triggered in the obscured domain. Indeed, in any cases (A, B, C) the stress level is decreasing in obscuration zones. Furthermore, the three scenarios providing more or less the same peak stress, it is largely noted that softening (post-peak) behaviour is mainly driven by cohesion law whereas the ultimate tensile strength (peak stress) is mainly related to obscuration probability. Based on these conclusions some modifications of the DFH model are proposed.

#### 3.2.2. Introduction of a cohesion strength

Simple approaches have been proposed in the literature to model the post-peak behaviour of concrete. For instance, several authors have used a bi-linear or multi-linear softening curve
In the present work, cohesion strength is considered in obscurcation zones for describing the bridging phenomenon in the micro-cracked matrix. Based on the previous experimental and numerical analysis an additional term is added to the classical term expressed in Eq. (14). For each principal direction a new damage variable $\bar{D}$ may be calculated and the macroscopic stress is expressed as:

\[ \sigma_{\text{macro}} = \sigma_{\text{dfh}} + \tau \bar{D} \]

where $\sigma_{\text{dfh}}$ is the classical stress term, and $\tau$ is a tension-only friction coefficient.

Fig. 17. Comparison of numerical results with experimental data performed on a dry microconcrete. Velocity profiles of (a) Test 42d ($\varepsilon = 58$ s$^{-1}$) and (b) Test 24d ($\varepsilon = 135$ s$^{-1}$), (c) and (d) damaged patterns, (e) comparison of numerical damage pattern (at $t = 125\mu s$) with post-mortem analysis in 24d concrete sample (cracks identified on sample have been superimposed in white to numerical patterns).
\[ \Sigma = (1 - \bar{D})\sigma = (1 - P_0)\sigma + (P_0)^2\sigma^{\text{COH}}. \]  
\[ \sigma^{\text{COH}}(\epsilon, \dot{\epsilon}) = \sigma_0^{\text{coh}}\exp\left(-\left(\frac{\epsilon}{\epsilon_0^{\text{coh}}}\right)^n\right)\left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0^{\text{coh}}}\right)^m, \]

where \( \sigma_0^{\text{coh}}, \epsilon_0^{\text{coh}}, n_{\text{coh}}, \dot{\epsilon}_0^{\text{coh}} \) are parameters to be fitted from experimental data. This additional cohesive term \( \sigma^{\text{COH}} \) is supposed to be strain-rate sensitive similarly to the increase of tensile strength with strain-rate for which a power law applies (exponent \( n/(m + n) \)). The relative influence of each parameter included in the cohesive term is plotted on Fig. 11. A constant strain-rate of 100 s\(^{-1}\) is assumed and the DFH parameters used for the dry microconcrete come from a previous work (Forquin and Erzar, 2010). As in Monte-Carlo simulations, it is noted that additional cohesive strength has low influence on the stress peak, especially for \( \dot{\epsilon} = 1.5 \) or 2. This model has been implemented in Abaqus/explicit through a user material subroutine VUMAT in order to perform numerical simulations with more complex loadings.

3.3. Numerical simulations of spalling tests

3.3.1. Identification of the cohesive behaviour for saturated microconcrete

A series of computations of spalling tests have been conducted with Abaqus/explicit finite element code. A pressure pulse extracted from experimental data is applied on one end of the cylinder. The compressive wave, having a characteristic duration from 70 to 80 \( \mu \)s, travels through the specimen until reaching the free-surface (Fig. 12). A tensile loading increases in the core of the specimen (Fig. 12, \( T = 75 \mu s \)). Subsequently, the tensile damage induces a drop of axial stress in the whole sample (Fig. 12, \( T = 78.4 \mu s \)).

Based on the analysis of calculations of spalling tests performed at different strain-rates (40 s\(^{-1}\) for 45w test, 135 s\(^{-1}\) for 15w test), a heuristic determination of parameters for the cohesive law has been carried out. The stress–strain curves of wet microconcrete are plotted in Fig. 13 considering distinct strain-rates. The experimental and numerical velocity profiles are compared in Fig. 14 including and excluding respectively the term of cohesion strength. The use of an additional cohesive strength allows improving significantly the consistency between numerical simulations and experimental data (Fig. 14). The strain signals picked in the simulation are consistent with gauges measurements.

At the end of the simulation, the obscured zone \( (P_0 = 1) \) spreads out a large part of concrete samples (Fig. 15(a) and (c)). The new damage variable \( \bar{D} \) is plotted in Fig. 15(b) and (d).

\[ \bar{D} = 1 - \Sigma/\sigma = P_0 - (P_0)^2\sigma^{\text{COH}}/\sigma. \]

Whereas an intense damage is noted at high strain-rate (130 s\(^{-1}\) in the 15w test, Fig. 15(d)), the damage is less pronounced and less extended at lower strain rate (40 s\(^{-1}\) in the 45w test, Fig. 15(b)).

As illustrated in Fig. 15(e), the damage model including cohesion strength is predicting a diffuse damage in addition to several major cracking planes. Moreover, the wideness of the damage zone is correctly predicted. Therefore, the damage law with the cohesive term can be used to model qualitatively (damage patterns) and quantitatively (local signals of gauges and laser interferometer) the dynamic response of the concrete.

3.3.2. Identification of cohesive behaviour for dry microconcrete

In order to identify the cohesive parameters of the dry MB50 concrete, two numerical simulations of spalling tests have been computed with the cohesive damage model, one at low impact velocity (Test 42d, \( \dot{\epsilon} = 55 \) s\(^{-1}\)) and one at a higher loading rate (Tests 24d, \( \dot{\epsilon} = 145 \) s\(^{-1}\)). Stress–strain curves of dry microconcrete are plotted in Fig. 16 considering distinct strain-rates. The cohesive parameters have been iteratively adjusted in order to improve the fitting of the free-surface velocity (Table 3).

Fig. 18. Comparison of damage level obtained with the cohesive model with experimental data of dynamically damaged specimens subjected to direct tensile test.
4. Conclusions

In the present work, the cohesion (post-peak) tensile behaviour of concrete has been investigated by means of spalling experiments, quasi-static experiments consecutively to spalling tests and Monte-Carlo calculations.

First, spalling experiments have been performed at strain-rates ranging from 30 to 60 s\(^{-1}\). Whereas one or several fracture planes were observed at higher strain-rates no fracture plane is visible in samples tested at such low strain-rates (30–60 s\(^{-1}\)). However, according to data from strain gauges, the strain level in the centre of the sample exceeds several times the maximum elastic strain defined as the strength to initial Young’s modulus ratio even at low strain-rates and a residual strain is also noted. In addition, the velocity profile measured on the rear face of the sample presents a rebound indicating tensile damage. Finally, despite an onset of damage during the spalling test, a “cohesion strength” of the concrete sample is expected. This statement is confirmed by quasi-static tensile tests performed with the damaged samples of concrete without cohesion (cf. Figs. 14, 15 and 17), mesh sensitivity appears to play a minor role in the tested range (element size ranging from \(L_e = 1\) mm to \(L_e = 3\) mm). Indeed, less than 10% of scatter is observed on the residual velocity of the spall. Moreover, the width of the damage zone is not affected by the spatial discretization.

**Fig. 19.** Mesh size influence (a) on the free-surface velocity and (b) on the damage pattern – 15w test.
dry and wet microconcrete and common concrete. The damaged samples present a lower but non-zero strength compared to the virgin specimens. A lower “damaged Young’s modulus” (i.e. stress to strain slope) derived from the data of strain gauges is also noted compared to the initial Young’s modulus. Based on these experimental observations, the following statement is proposed: despite the propagation of cracks in the spalled samples cohesion strength exists in the vicinity of triggered cracks resulting from bridging phenomena.

This statement is used in the next parts in Monte-Carlo simulations and in an analytical and numerical modelling. Previous works have shown the ability of the DHF (Denoual–Forquin–Hild) anisotropic damage model for describing the increase of spall strength and cracking density with strain-rate in concrete and microconcrete. However, none cohesion strength is taken into account in the model. In the present paper, cohesion strength is assumed in obscured zones in the vicinity of cracks. This assumption is investigated through Monte-Carlo simulations considering three cases: no cohesion strength, a homogenous decreasing stress level or a constant negative stress-rate confined in obscured zones. Finally, it is concluded that, even if additional cohesion strength in obscured volumes has a very small influence on the macroscopic ultimate strength and cracking density, it is actually driving the whole softening behaviour of the material.

Based on this analysis a closed form solution of the cohesion strength is proposed as function of strain and strain-rate. It was integrated to the DHF obscuration model and implemented in a FE code as a Vumat subroutine. The parameters are identified by means of a heuristic approach based on a series of numerical simulations of spalling tests. Finally, numerical simulations provide a good description of velocity profile of spalling tests performed with dry and wet samples. Moreover the theoretical evolution of damage with the proposed model appeared to be consistent with experimental data extracted from direct tensile tests conducted on dynamically damaged specimens. A mesh size analysis was conducted to check the mesh dependency of the modelling.

In subsequent works, additional experiments might be performed considering intermediate saturation ratio and a wider range of strain-rates. More complex loadings (such as impact loadings) should be investigated to evaluate the advantages provided by the proposed modelling.

References


