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Compromise of localized graviton with a small cosmological constant in Randall–Sundrum scenario

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Abstract

A new mechanism which leads to a linearized massless graviton localized on the brane is found in the AdS/CFT setting, i.e., in a single copy of AdS_5 spacetime with a singular brane on the boundary, within the Randall–Sundrum brane-world scenario. With an help of a recent development in path-integral techniques, a one-parameter family of propagators for linearized gravity is obtained analytically, in which a parameter ξ reflects various kinds of boundary conditions that arise as a result of the half-line constraint. In the case of a Dirichlet boundary condition ($\xi = 0$) the graviton localized on the brane can be massless *via* coupling constant renormalization. Our result supports a conjecture that the usual Randall–Sundrum scenario is a regularized version of a certain underlying theory.

The most remarkable feature of the Randall–Sundrum (RS) brane-world scenario is that it leads to a massless graviton localized on the 3-brane at the linearized fluctuation level [1]. In fact, this striking feature seems to furnish a motivation for the recent application of this scenario to various branches of physics such as cosmology [2–6], the cosmological constant hierarchy [7–9], and blackhole physics [10–12]. The fact that RS spacetime is composed of the two copies of AdS_5 attached along the boundary ($y = 0$) also provides another motivation for the recent activity on the relation of this scenario to AdS/CFT [11,13–19].

When solving a linearized fluctuation equation, however, the authors of Ref. [9] chose a Dirichlet boundary condition (BC) on the brane to explain a small cosmological constant of the brane. In this BC the 3-brane acts effectively as a perfectly reflecting mirror, and the cosmological constant becomes naturally very small through thermal radiation of vacuum energy from the brane into the bulk. Then, it is very unclear why two different BCs are necessary to explain two distinct phenomena. There should exist a single physical BC which explains these two different phenomena simultaneously. In this context it is important

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to find a possible compromise of these two phenomena, which is a purpose of this Letter. As will be shown below, there exists a novel mechanism which leads to a massless physical graviton with the Dirichlet BC *via* coupling constant renormalization in the AdS/CFT setting, i.e., that of a single AdS_5 spacetime with a singular brane on the boundary. We argue here that the mixture procedure of Dirichlet BC and the coupling constant renormalization is a most probable candidate for the compromise. It also makes us conjecture that RS scenario is a regularized version of a certain underlying theory.

Recently, it was shown [20] that at nonzero temperature only half of the full spacetime in the RS scenario becomes Schwarzschild–AdS_5 due to the manifest Z_2-symmetry breaking. Therefore, the choice of a single AdS_5 in the RS scenario also guarantees that the close relationship of the RS scenario with AdS/CFT is maintained at finite temperature.

We start with the gravitational fluctuation equation [1] in the RS scenario, i.e.,

$$\widehat{H}_{\mathrm{RS}}\hat{\psi}(z) = \frac{m^2}{2}\hat{\psi}(z),$$
$$\widehat{H}_{\mathrm{RS}} = -\frac{1}{2}\partial_z^2 + \frac{15}{8(|z|+\frac{1}{k})^2} - \frac{3}{2}k\delta(z), \tag{1}$$

where $\hat{\psi}(z)$ is related to a linearized gravitational field $h(\bar{x}, y)$ as follows:

$$h(\bar{x}, y) = \psi(y)e^{ip\bar{x}},$$
$$\hat{\psi}(z) = \psi(y)e^{\frac{k|y|}{2}}, \tag{2}$$

where $z = \epsilon(y)(e^{k|y|} - 1)/k$, $p^2 = -m^2$, and $\bar{x}$ is the worldvolume coordinate. Since all components are the same, the Lorentz indices are suppressed in Eq. (2). When deriving the fluctuation equation, RS used the gauge choice

$$h_{55} = h_{\mu 5} = 0, \qquad h^{\nu}_{\mu,\nu} = 0, \qquad h^{\mu}_{\mu} = 0, \tag{3}$$

where $\mu, \nu = 0, 1, 2, 3$. However, the choice of this gauge in the bulk generates in general a non-trivial bending structure of the 3-brane which is fully discussed in Refs. [11,21,22]. Since it does not change the main conclusion, we will not explore the subtlety of this gauge choice in detail here. What we want to do is to examine the properties of the Feynman propagator explicitly for the general Hamiltonian

$$\widehat{H} = \widehat{H}_0 - v\delta(z),$$
$$\widehat{H}_0 = -\frac{1}{2}\partial_z^2 + \frac{g}{(|z|+c)^2}, \tag{4}$$

when z is non-negative. Of course, $\widehat{H}$ coincides with $\widehat{H}_{\mathrm{RS}}$ when $g = 15/8$, $c = 1/k \equiv R$, and $v = 3k/2$, where R is the radius of AdS_5.

From the purely mathematical point of view the Hamiltonian $\widehat{H}$ is a singular operator due to its point interaction. While the proper treatment of the one-dimensional δ-function potential in the Schrödinger picture was found long ago [23], it is not quite so long ago that one understood how to treat it within the path-integral formalism. Following Schulman's procedure [24,25], it is possible to express the fixed-energy amplitude $\widehat{G}[z_1, z_2 : E]$ for $\widehat{H}$ in terms of the fixed-energy amplitude $\widehat{G}_0[z_1, z_2 : E]$ for $\widehat{H}_0$ as follows[1]

$$\widehat{G}[z_1, z_2 : E] = \widehat{G}_0[z_1, z_2 : E] + \frac{\widehat{G}_0[z_1, 0 : E]\widehat{G}_0[0, z_2 : E]}{\frac{1}{v} - \widehat{G}_0[0, 0, E]}. \tag{5}$$

The remaining problem, therefore, is to compute a fixed-energy amplitude for the Hamiltonian $\widehat{H}_0$.

As mentioned before, we would like to use only half of the full RS spacetime for the computation of $\widehat{G}_0[z_1, z_2 : E]$. In this case the fixed-energy amplitude is in general dependent upon the BC at the boundary arising due to the half-line constraint, $z \geqslant 0$. In this half-line $\widehat{H}_0$ becomes simply

$$\widehat{H}_0 = -\frac{1}{2}\partial_x^2 + \frac{g}{x^2}, \tag{6}$$

where $x = z + c$. Thus our half-line constraint $z \geqslant 0$ is changed into $x \geqslant c$. If $c = 0$, the Euclidean propagator $G_{>0}[a, b : t]$ and the corresponding fixed-energy amplitude $\widehat{G}_{>0}[a, b : E]$ for Hamiltonian (6) are well-known [26]:

$$G_{>0}[a, b : t] = \frac{\sqrt{ab}}{t}e^{-\frac{a^2+b^2}{2t}} I_\gamma\left(\frac{ab}{t}\right),$$

[1] The definition of the fixed-energy amplitude $\widehat{G}[x, y : E]$ in this Letter is a Laplace transform of the usual Euclidean Feynman propagator $G[x, y : t]$.

$$\widehat{G}_{>0}[a,b:E] = 2\sqrt{ab}\, I_\gamma\left(\sqrt{\frac{E}{2}}\big((a+b)-|a-b|\big)\right) \times K_\gamma\left(\sqrt{\frac{E}{2}}\big((a+b)+|a-b|\big)\right), \tag{7}$$

where $I_\gamma(z)$ and $K_\gamma(z)$ are the usual modified Bessel functions, and $\gamma = \sqrt{1+8g}/2$.

The difficulty of the computation of the fixed-energy amplitude for $\widehat{H}_0$ is mainly due to the fact that the constraint is not half-line in terms of x but $x > c$, i.e., $\widehat{G}_0[a,b:E] = \widehat{G}_{>c}[a,b:E]$. It may be extremely difficult to compute a path-integral directly with our asymmetric constraint. In this Letter, instead of this direct approach, we adopt the following technique to solve the problem. First, we impose the usual half-line constraint $x > 0$. Then, we introduce an infinite energy barrier at $x = c$ in $\widehat{H}_0$ to forbid a penetration into the region $0 < x < c$. The infinite energy barrier can be consistently introduced within the path-integral formalism using δ- and δ'-functions by assuming an infinitely large coupling constant [27–29]. For the Dirichlet and Neumann BC cases the fixed-energy amplitudes $\widehat{G}_0^{\mathrm{D}}[a,b:E]$ and $\widehat{G}_0^{\mathrm{N}}[a,b:E]$ for $\widehat{H}_0$ with the infinite barrier are obtained from $\widehat{G}_{>0}[a,b:E]$ as follows:

$$\begin{aligned}\widehat{G}_0^{\mathrm{D}}[a,b:E] &= \widehat{G}_{>0}[a,b:E] - \frac{\widehat{G}_{>0}[a,c:E]\widehat{G}_{>0}[c,b:E]}{\widehat{G}_{>0}[c^+,c:E]},\\ \widehat{G}_0^{\mathrm{N}}[a,b:E] &= \widehat{G}_{>0}[a,b:E] - \frac{\widehat{G}_{>0,b}[a,c:E]\widehat{G}_{>0,a}[c,b:E]}{\widehat{G}_{>0,ab}[c^+,c:E]},\end{aligned} \tag{8}$$

where we used a point-splitting method to avoid an infinity arising in $\widehat{G}_{>0}[a,b:E]$ and $\widehat{G}_{>0,ab}[a,b:E]$ at $a = b$.

The quantities $\widehat{G}_0^{\mathrm{D}}[a,b:E]$ and $\widehat{G}_0^{\mathrm{N}}[a,b:E]$ are straightforwardly computed using Eq. (8). The explicit expressions are

$$\begin{aligned}\widehat{G}_0^{\mathrm{D}}[a,b:E] &= \widehat{G}_{>0}[a,b:E] - 2\sqrt{ab}\,\frac{I_\gamma(\sqrt{2E}\,c)}{K_\gamma(\sqrt{2E}\,c)} \times K_\gamma(\sqrt{2E}\,a)K_\gamma(\sqrt{2E}\,b),\\ \widehat{G}_0^{\mathrm{N}}[a,b:E] &= \widehat{G}_{>0}[a,b:E] + 2\sqrt{ab}\,\frac{f_I(E)}{f_K(E)}K_\gamma(\sqrt{2E}\,a) \times K_\gamma(\sqrt{2E}\,b),\end{aligned} \tag{9}$$

where

$$f_K(E) = \frac{\gamma-\frac{1}{2}}{\sqrt{2E}\,c}K_\gamma(\sqrt{2E}\,c) + K_{\gamma-1}(\sqrt{2E}\,c)$$

$$f_I(E) = I_{\gamma-1}(\sqrt{2E}\,c) - \frac{\gamma-\frac{1}{2}}{\sqrt{2E}\,c}I_\gamma(\sqrt{2E}\,c). \tag{10}$$

It is simple to show that $\widehat{G}_0^{\mathrm{D}}[a,b:E]$ and $\widehat{G}_0^{\mathrm{N}}[a,b:E]$ satisfy the usual Dirichlet and Neumann BCs at $x = c$.

One may impose a mixing of Dirichlet and Neumann BCs at $x = c$. In this case the fixed-energy amplitude $\widehat{G}_0[a,b:E]$ for $\widehat{H}_0$ becomes a one parameter family of propagators[2]

$$\widehat{G}_0[a,b:E] = \xi\widehat{G}_0^{\mathrm{N}}[a,b:E] + (1-\xi)\widehat{G}_0^{\mathrm{D}}[a,b:E], \tag{11}$$

where $0 \leqslant \xi \leqslant 1$. Of course, the cases $\xi = 0$ and $\xi = 1$ correspond to pure Dirichlet and pure Neumann BC cases. Another interesting case is the value $\xi = 1/2$, in which the contributions of Neumann and Dirichlet have equal weighting factors. Since $\widehat{G}_0[a,b:E]$ is expressed in terms of eigenvalues E_n and eigenfunctions ϕ_n of $\widehat{H}_0$ as follows

$$\widehat{G}_0[a,b:E] = \sum_n \frac{\phi_n(a)\phi_n^*(b)}{E-E_n}, \tag{12}$$

the $\xi = 1/2$ case should correspond to the gravitational propagator without any constraint in x. As will be shown below, this case exactly reproduces the original RS result.

[2] The boundary condition for the one-dimensional singular operator involves in general four real self-adjoint parameters [30,31]. In this Letter, however, we do not explore this purely mathematically oriented approach.

Inserting (11) into (5) one can finally obtain the ξ-dependent propagator for $\widehat{H}$ whose explicit form is

$$\begin{aligned}
&\widehat{G}[a,b:E] \\
&= 2\sqrt{ab}\Biggl[I_\gamma\bigl(\sqrt{2E}\min(a,b)\bigr)K_\gamma\bigl(\sqrt{2E}\max(a,b)\bigr) \\
&\quad + \frac{K_\gamma(\sqrt{2E}\,a)K_\gamma(\sqrt{2E}\,b)}{f_K(E)} \\
&\quad \times \Biggl[\xi\Biggl(f_I(E) + \frac{1}{cE}\Biggl[\frac{f_K(E)}{\xi v} \\
&\quad - \sqrt{\frac{2}{E}}K_\gamma(\sqrt{2E}\,c)\Biggr]^{-1}\Biggr) \\
&\quad - (1-\xi)\frac{I_\gamma(\sqrt{2E}\,c)}{K_\gamma(\sqrt{2E}\,c)} f_K(E)\Biggr]\Biggr]. \qquad (13)
\end{aligned}$$

We now consider special cases of Eq. (13). As expected, taking $\xi = 1/2$ with $g = v = 0$ makes $\widehat{G}[a,b:E]$ the exact free-particle amplitude. If one takes the RS limit $g = 15/8$, $c = 1/k = R$, $v = 3k/2$ and $E = m^2/2$ at the same ξ value, it is possible to show that Eq. (13) yields

$$\widehat{G}^{\mathrm{RS}}[a=R,b:m] = \frac{1}{m}\sqrt{\frac{b}{R}}\frac{K_2(mb)}{K_1(mR)}. \qquad (14)$$

If we takes $b = R$, the amplitude becomes simply

$$\widehat{G}^{\mathrm{RS}}[R,R:m] = R(\Delta_0 + \Delta_{\mathrm{KK}}), \qquad (15)$$

where Δ_0 and Δ_{KK} represent zero-mass localized gravity and higher Kaluza–Klein excitation

$$\begin{aligned}
\Delta_0 &= \frac{2}{m^2R^2}, \\
\Delta_{\mathrm{KK}} &= \frac{1}{mR}\frac{K_0(mR)}{K_1(mR)}, \qquad (16)
\end{aligned}$$

respectively. In this case, when the separation between masses on the brane is very large, Newton's law becomes

$$V_{\mathrm{RS}} \sim \frac{1}{r}\left[1 + \left(\frac{R}{r}\right)^2\right] \qquad (17)$$

which agrees with the RS result [1].

The first term in Eq. (17) is a usual Newton potential contributed from the zero mode Δ_0. The second term represents the correction to the potential and is generated from the Kaluza–Klein excitation Δ_{KK}. It is worthwhile noting that the correction to the potential is also computed in Ref. [21] using somewhat different method and the final result is different from Eq. (17):

$$V_{\mathrm{RS}} \sim \frac{1}{r}\left[1 + \frac{2}{3}\left(\frac{R}{r}\right)^2\right]. \qquad (18)$$

The 2/3 factor in Eq. (18) is derived by considering the source term arising from the bending structure of the 3-brane. Thus, the factor difference in potential is due to our ignorance of the bending effect. It is interesting to examine how to involve the bending effect within the path-integral formalism.

If we choose $\xi = 1$ with RS limit, $\widehat{G}[a,b:E]$ of Eq. (13) reduces to

$$\widehat{G}^{\mathrm{N}}[R,R:E] = 2R\frac{\Delta}{1-\frac{3}{2}\Delta}, \qquad (19)$$

where $\Delta = \Delta_0 + \Delta_{\mathrm{KK}}$. Numerical calculation shows that there exists a massive graviton bound on the brane in this case whose mass is

$$m_N \approx 2.48R^{-1}. \qquad (20)$$

It is well-known that the potential due to the exchange of a massive particle is exponentially suppressed at long distance. This result is reasonable because the massive particle in general cannot propagate a long distance freely.

Finally, we consider the case $\xi = 0$. In this case the result (13) of the usual Schulman procedure does not yield an any modification due to the Dirichlet BC if the coupling constant v is finite. As shown in [32,33], however, we can obtain a non-trivial modification of the fixed-energy amplitude in this case *via* coupling constant renormalization if v is an infinite bare quantity. In this Letter we will follow this procedure by treating v as an unphysical infinite quantity. This means we abandon the RS limit $v = 3k/2$ at $\xi = 0$ case. As will be shown shortly, this procedure also generates a massless gravity localized on the brane when the renormalized coupling constant becomes a particular value.

To show this more explicitly we introduce a positive infinitesimal parameter ϵ for the regularization and rewrite Eq. (5) in the form:

$$\widehat{G}^{\mathrm{D}}[a,b:E] = \widehat{G}_0^{\mathrm{D}}[a,b:E]$$

$$+\lim_{\epsilon\to 0^+}\frac{\widehat{G}_0^{\mathrm{D}}[a,c+\epsilon:E]\widehat{G}_0^{\mathrm{D}}[c+\epsilon,b:E]}{\frac{1}{v}-\widehat{G}_0^{\mathrm{D}}[c+\epsilon,c+\epsilon:E]}. \tag{21}$$

Using the expansions

$$\widehat{G}_0^{\mathrm{D}}[a,c+\epsilon:E]=2\sqrt{\frac{a}{c}}\frac{K_\gamma(\sqrt{2E}\,a)}{K_\gamma(\sqrt{2E}\,c)}\epsilon+\mathrm{O}(\epsilon^2),$$
$$\widehat{G}_0^{\mathrm{D}}[c+\epsilon,b:E]=2\sqrt{\frac{b}{c}}\frac{K_\gamma(\sqrt{2E}\,b)}{K_\gamma(\sqrt{2E}\,c)}\epsilon+\mathrm{O}(\epsilon^2),$$
$$\widehat{G}_0^{\mathrm{D}}[c+\epsilon,c+\epsilon:E]=2\epsilon+\frac{2\epsilon^2}{c}\Omega(\sqrt{2E}\,c,\gamma)+\mathrm{O}(\epsilon^3), \tag{22}$$

where

$$\Omega(z,\nu)=1+z\frac{K_\nu'(z)}{K_\nu(z)}+\frac{z^2}{2}\big(I_\nu''(z)K_\nu(z)-I_\nu(z)K_\nu''(z)\big), \tag{23}$$

it is straightforward to derive a non-trivial fixed-energy amplitude

$$\begin{aligned}\widehat{G}^{\mathrm{D}}[a,b:E] &= 2\sqrt{ab}\Bigg[I_\gamma\big[\sqrt{2E}\min(a,b)\big]K_\gamma\big[\sqrt{2E}\max(a,b)\big] \\ &\quad-\frac{K_\gamma(\sqrt{2E}\,a)K_\gamma(\sqrt{2E}\,b)}{K_\gamma^2(\sqrt{2E}\,c)} \\ &\quad\times\bigg[I_\gamma(\sqrt{2E}\,c)K_\gamma(\sqrt{2E}\,c)+\frac{1}{2[\Omega(\sqrt{2E}\,c,\gamma)-v^{\mathrm{ren}}c]}\bigg]\Bigg]\end{aligned} \tag{24}$$

where the renormalized coupling constant v^{ren} is defined in terms of the bare coupling constant as follows:

$$v^{\mathrm{ren}}=\frac{1}{2\epsilon^2}\left(\frac{1}{v}-2\epsilon\right). \tag{25}$$

One can easily show that v^{ren} has the same dimension as the bare coupling constant v. Following the philosophy of renormalization we regard v^{ren} as a finite quantity. Taking the remaining RS limit $g=15/8$, $c=1/k=R$, and $E=m^2/2$, one can show that the fixed-energy amplitude in this case is

$$\widehat{G}^{\mathrm{D}}[R,b:E]=\sqrt{Rb}\,\frac{K_2(mb)}{K_2(mR)}\frac{1}{v^{\mathrm{ren}}R-\Omega(mR,2)}. \tag{26}$$

Using

$$\Omega(mR,2)=-\frac{3}{2}-mR\frac{K_1(mR)}{K_2(mR)} \tag{27}$$

it is possible to show that the corresponding gravitational potential at long range is

$$V_D\sim\frac{1}{r}\left[1+\left(\frac{R}{r}\right)^2\right]=V_{\mathrm{RS}} \tag{28}$$

when $Rv^{\mathrm{ren}}+3/2=0$. Hence, we obtain a massless graviton localized on the brane when $v^{\mathrm{ren}}=-3/(2R)$. Eq. (28) is a surprising result. Although we obtained a massless graviton through completely different BC and completely different procedure, its gravitational potential on the 3-brane is exactly the same as that of the original RS result. This exact coincidence strongly supports the conjecture that the Dirichlet BC for Hamiltonian $\widehat{H}_0$ is a genuine physical BC in the linearized gravity theory of RS scenario. The requirement of the coupling constant renormalization supports another conjecture that RS scenario is a regularized version of a certain underlying theory. It would be interesting to find and examine the underlying theory which might be our future work.

At $v^{\mathrm{ren}}=-3/(2R)$ the graviton propagator (24) reduces to the following simple form in the RS limit

$$\widehat{G}^{\mathrm{D}}[a,b:m]=\widehat{G}_0^{\mathrm{D}}[a,b:m]+\sqrt{ab}\,\frac{K_2(ma)K_2(mb)}{K_2^2(mR)}(\Delta_0+\Delta_{\mathrm{KK}}). \tag{29}$$

The first term in Eq. (29) is responsible for the small cosmological constant through thermal radiation of vacuum energy from the brane into the bulk due to its Dirichlet nature [9]. The second term is responsible for the massless graviton localized on the brane. Of course, because of the second term the 3-brane cannot act as a perfectly reflecting mirror in the bulk. This may be a physical reason why the cosmological constant of our universe is nonzero. Therefore, it might be also interesting to estimate the value of the cosmological constant within the present scenario and compare it with real experimental data $(0.01\ \mathrm{eV})^4$.

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