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Hard-gapped holographic superconductors

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ABSTRACT

In this work we discuss the zero temperature limit of a “p-wave” holographic superconductor. The bulk description consists of a non-Abelian $SU(2)$ gauge fields minimally coupled to gravity. We numerically construct the zero temperature solution which is the gravity dual of the superconducting ground state of the “p-wave” holographic superconductors. The solution is a smooth soliton with zero horizon size and shows an emergent conformal symmetry in the IR. We found the expected superconducting behavior. Using the near horizon analysis we show that the system has a “hard gap” for the relevant gauge field fluctuations. At zero temperature the real part of the conductivity is zero for an excitation frequency less than the gap frequency. This is in contrast with what has been observed in similar scalar-gravity-gauge systems (holographic superconductors). We also discuss the low but finite temperature behavior of our solution.

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1. Introduction

In recent years string theory or, more specifically, gauge-gravity duality has seen interesting applications in the field of condensed matter physics. One of the earliest such applications is the discovery of a superconductor-like phase transition in AdS with a Reissner–Nordström black hole and a charged scalar field minimally coupled to a $U(1)$ gauge field [1,2].

Earlier works on this system at low (but non-zero) temperatures indicated the presence of a gap in the spectrum, with an exponentially decaying population of normal carriers following the Boltzmann distribution. In various recent works [3–9], authors have addressed the problem in the zero temperature limit. More precisely, they consider a charged scalar field and gauge fields in an AdS_4 background at zero temperature. The solution turns out to be a solitonic solution with a zero sized horizon. The authors find that the effective potential for small gauge field fluctuations vanishes near the black hole horizon. This implies that the normal component of the AC conductivity never vanishes, even at zero temperature, which in turn indicates that the superconductor is gapless. This is a new and interesting result different from what would have been naively expected.

In this Letter we consider non-Abelian gauge fields in the background of an extremal AdS_4 black hole. The setup is quite similar to so-called “p-wave” holographic superconductors [10] and known

to have a superconducting transition.¹ We work at the zero temperature and found a fully gravity back-reacted solitonic solution to this system similar to [3]. Then we consider the frequency dependent conductivity by considering certain vector fluctuations on this background. This is an anisotropic system which shows different conductivity at different directions. We find the expected superconducting behavior for relevant currents. Interestingly we find that the corresponding effective potential does not vanish at the horizon (it goes to zero near the boundary). Following the argument in [3], we thus arrive at the conclusion that the holographic non-Abelian superconductor does have a finite gap for the relevant gauge field fluctuations.

The plan of the Letter is as follows, in Section 2 we discuss the general setup and Einstein’s equation for a non-Abelian gauge field coupled to gravity. In Section 3 we discuss the instability of the RN black hole solution. In Section 4 we construct the zero temperature solitonic solution which is the superconducting ground state of the theory. We also discuss issues related to conductivity and show the existence of a hard gap. In Section 5 we propose a possible form of the solution at non-zero temperature using the method of matched asymptotic expansion.

2. Setup

The Einstein–YM action for a non-Abelian gauge field with a negative cosmological constant is given by [11],

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¹ For other works in non-Abelian holographic superconductor look at [11–18].

$$\mathcal{L} = \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{l^2} - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a \right), \quad (1)$$

where $F_{\mu\nu}$ is the field strength of an $SU(2)$ gauge field. A hairy black hole solution of the above action is described in [10]. In this work we like to find the zero temperature limit of this black hole. Unlike [10] we would take into account the full gravity back reaction.

The ansatz for the gauge fields is,²

$$A = A(r)\tau^3 dt + B(r)\tau^1 dx. \quad (2)$$

To tally with the anisotropy of the gauge field ansatz in the spatial direction, we choose the following ansatz for our metric,

$$ds^2 = -g(r)e^{-\chi(r)} dt^2 + \frac{dr^2}{g(r)} + r^2(c(r)^2 dx^2 + dy^2). \quad (3)$$

It is straightforward to find the non-zero components of the field strength

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + iq[A_\mu, A_\nu]^a \quad (4)$$

are

$$\begin{aligned} F_{rx}^1 &= -F_{xr}^1 = B'(r), & F_{rt}^3 &= -F_{tr}^3 = A'(r), \\ F_{xt}^2 &= -F_{tx}^2 = qAB. \end{aligned} \quad (5)$$

The energy-momentum tensor is

$$T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}} = -\frac{1}{2} g_{\mu\nu} \left(\frac{1}{4} F^2 \right) + \frac{1}{2} g^{\alpha\beta} F_{\mu\alpha}^a F_{\nu\beta}^a, \quad (6)$$

and the non-zero components of the energy-momentum tensor are,

$$\begin{aligned} T_{tt} &= \frac{1}{4} g A'^2 + \frac{g^2}{4r^2 c^2} e^{-\chi} B'^2 + \frac{1}{4r^2 c^2} (qAB)^2, \\ T_{rr} &= -\frac{e^\chi}{4gc^2} A'^2 + \frac{1}{4r^2 c^2} B'^2 + \frac{e^\chi}{4g^2 r^2 c^2} (qAB)^2, \\ T_{xx} &= \frac{1}{4} e^\chi r^2 c^2 A'^2 + \frac{g}{4} B'^2 - e^\chi \frac{(qAB)^2}{4g}, \\ T_{yy} &= \frac{1}{4} e^\chi r^2 A'^2 - \frac{g}{4c^2} B'^2 + e^\chi \frac{(qAB)^2}{4gc^2}. \end{aligned} \quad (7)$$

It is to be noted that here $T_{xx} \neq T_{yy}$ due our anisotropic ansatz.³

The Maxwell's equations of $A(r)$, $B(r)$ are

$$\begin{aligned} A_r^3 &\longrightarrow A'' + A' \left(\frac{2}{r} + \frac{\chi'}{2} + \frac{c'}{c} \right) - \frac{q^2 B^2}{r^2 gc} A = 0, \\ A_x^1 &\longrightarrow B'' + B' \left(\frac{g'}{g} - \frac{\chi'}{2} - \frac{c'}{c} \right) + \frac{e^\chi q^2 A^2}{g^2} B = 0. \end{aligned} \quad (8)$$

The potential term for $A(r)$ is different from the similar term in EYMH case due to the existence of a factor $\frac{1}{r^2}$. The diagonal Einstein equations give,

$$\begin{aligned} &-g' \left(\frac{1}{r} + \frac{c'}{2c} \right) - g \left(\frac{1}{r^2} + \frac{3c'}{rc} + \frac{c''}{c} \right) + 3 \\ &= \frac{e^\chi}{4} A'^2 + \frac{g}{4r^2 c} B'^2 + e^\chi \frac{q^2 A^2 B^2}{4r^2 gc}, \\ &-\frac{\chi'}{r} + \frac{c'}{c} \left(-\chi' + \frac{g'}{g} \right) = \frac{e^\chi q^2 A^2 B^2}{g^2 r^2 c^2}, \\ &cc'' + cc' \left(\frac{g'}{g} + \left(\frac{2}{r} - \frac{\chi'}{2} \right) \right) = -\frac{B'^2}{2r^2} + e^\chi \frac{q^2 A^2 B^2}{2g^2 r^2}. \end{aligned} \quad (9)$$

The above equations are invariant under the following scaling symmetries:

$$\begin{aligned} r &\rightarrow a_1 r, & (t, x, y) &\rightarrow (t, x, y)/a_1, \\ g &\rightarrow a_1^2 g, & A &\rightarrow a_1 A, & B &\rightarrow a_1 B, \\ e^\chi &\rightarrow a_2^2 e^\chi, & t &\rightarrow a_2 t, & A &\rightarrow A/a_2, \\ x &\rightarrow x/a_3, & B &\rightarrow a_3 B. & c &\rightarrow a_3 c. \end{aligned} \quad (10)$$

The second scaling symmetry may be used to set $\chi = 0$ at infinity and the third scaling symmetry may be used to set $c = 1$ at infinity, so that the asymptotic metric is that of AdS_4 .

The fields have the following asymptotic behavior:

$$A = \mu - \frac{\rho}{r}, \quad B = B_0 + \frac{B_1}{r}, \quad (11)$$

where μ is the chemical potential and ρ is the charge density in the boundary theory. In what follows we will only consider the solutions for the field B which vanishes near the boundary, i.e. $B_0 = 0$.

3. Reissner–Nordström black hole solution and instability

To see when one expects hairy black holes at low temperature, one can study linearized perturbations of the extremal Reissner–Nordström AdS (RN-AdS) black hole [3]. The general RN-AdS solution of Eq. (8), Eq. (9) is given by,

$$\begin{aligned} c = 1, & \quad \chi = B = 0, & g &= r^2 - \frac{1}{r} (1 + \rho^2/4) + \rho^2/4r^2, \\ A &= \rho(1 - 1/r). \end{aligned} \quad (12)$$

The temperature of the black hole (12) is

$$T = \frac{[g'(ge^{-\chi})]^{1/2}}{4\pi} \Big|_{r=r_+}. \quad (13)$$

For AdS-RN, $T = (12 - \rho^2)/16\pi$. It is known that fluctuations of B_x in this background develops a tachyonic mode [10] at low temperature and for sufficiently large q . The resulting solution is a superconducting black hole with vector hairs. Following [3], we argue that such an instability persists at the extremal limit. At the extremal limit ($T = 0$), we have $\rho = 2\sqrt{3}$. The near-horizon limit of the extremal solution is $AdS_2 \times \mathbb{R}^2$ with a metric,

$$\begin{aligned} ds^2 &= -6(r-1)^2 dt^2 + \frac{dr^2}{6(r-1)^2} + dx^2 + dy^2, \\ A &= 2\sqrt{3}(r-1). \end{aligned} \quad (14)$$

Plugging this into the wave equation of $B(r)$, and changing variables $\tilde{r} = r - 1$, we recover a wave equation for AdS_2 with a new effective mass,

$$B_{,\tilde{r}\tilde{r}} + \frac{2}{\tilde{r}} B_{,\tilde{r}} - \frac{m_{\text{eff}}^2}{\tilde{r}^2} B = 0, \quad m_{\text{eff}}^2 = -\frac{q^2}{3}. \quad (15)$$

² Due to a repulsive term coming from the non-Abelian interactions, it is expected that a isotropic ansatz will have a quartic instability and would possibly have more free energy than the anisotropic ones [10,12]. We left the detailed discussion of these issues for a future study.

³ HSH thanks Matthew Roberts and Gary Horowitz for discussions on this point.

The instability to form $SU(2)$ vector hair at low temperature then is just the instability of scalar fields below the Breitenlohner–Freedman (BF) bound for AdS_2 : $m_{BF}^2 = -1/4$. Thus the condition for instability is

$$q^2 > \frac{3}{4}. \quad (16)$$

It is reasonable to expect that such an instability takes the system to its superconducting ground state.

4. Zero temperature solution

Being a single state without any degeneracy, a superconducting ground state does not have any entropy associated with it. This fact concur with the observations in [3–5] that the zero temperature limit of the superconducting black holes in EYM theory have zero horizon size. Motivated by these facts we assume that same is true for a non-Abelian hairy black hole also and the ground state is a geometry with zero horizon size. We choose the following ansatz, near $r \rightarrow 0^+$ (similar to that of EYM case [3])

$$A \sim A_0(r), \quad B \sim B_0 - B_1(r), \quad \chi \sim \chi_0 - \chi_1(r), \\ g \sim r^2 + g_1(r), \quad c \sim c_0 + c_1(r). \quad (17)$$

All the terms with subscript 1 are sub-leading and go to zero, faster than the leading part where it is applicable, as $r \rightarrow 0$. Putting Eq. (17) in Eq. (8), we get find out the various terms in Eq. (17). For example from the equation of motion for A ,

$$r^2(r^2 A_0')' = \frac{q^2 B_0^2}{A_0} \Rightarrow A_0 \sim e^{(-\frac{\alpha}{r})}, \quad \alpha = qB_0/c_0, \quad (18)$$

where we have used the observation $A_0 \rightarrow 0$ at the horizon. Following similar procedures, we get

$$A \sim A_0 e^{-\alpha/r}, \quad B \sim B_0 \left(1 - \frac{e^{\chi_0} q^2 A_0^2}{4\alpha^2} e^{-2\alpha/r}\right), \\ c \sim c_0 \left(1 + \frac{e^{\chi_0} A_0^2}{8r^2} e^{-2\alpha/r}\right), \\ \chi \sim \chi_0 - \frac{e^{\chi_0} A_0^2 \alpha}{2r} e^{-2\alpha/r}, \quad g \sim r^2 - \frac{e^{\chi_0} A_0^2 \alpha}{4r} e^{-2\alpha/r}, \quad (19)$$

where by rescaling (10) we set the coefficients $A_0 = 1$ and $\chi_0 = 0$, $c_0 = 1$. After we solve the equations we again use the rescalings of g, c, χ to make the asymptotic geometry the same as that of AdS_4 . For a given q , we choose α in such a fashion that B vanishes near the boundary [3]. The numerical evaluation shows us an almost linear relation between q and α (Fig. 1), with a slight dip for a lower value of q . Fitting a linear relation between q and α with a range of $3 < q < 8$, we get $\alpha \approx 0.088 + 0.66q$.

Our numerics is less reliable in the region $q < 0.95$. It is not clear what exactly happens in this region. As $q \rightarrow \frac{\sqrt{3}}{2} \approx 0.866$, the geometry of our solution seems to approaches extremal RN geometry (Fig. 2). However the questions of phase transitions remain unclear. It is expected that there is a second order phase transition at $q = \frac{\sqrt{3}}{2}$ from RN black hole to our solution. However one may not rule out the possibility of a first or any other type of phase transition(s). At least in the range of our numerics it seems that our solutions have lower free energy than the extremal RN black hole in a grand canonical ensemble. As expected the solitonic solution is more massive than the RN black hole, but they carry more charge for the same boundary chemical potential and consequently have a lower free energy.

General properties of our solution is similar to EYM case [3]. B, χ approaches a constant near $r = 0$, $g = r^2$ and the gauge field

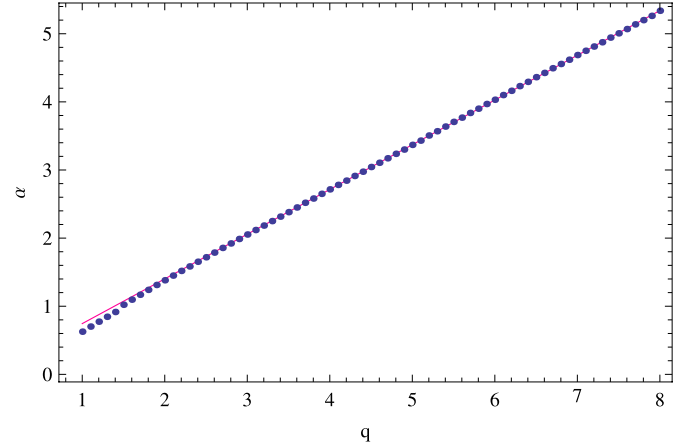


Fig. 1. α vs q plot shows a almost linear relation with a slight dip for α for a value of $q < 2.5$.

A and its derivative (electric field) vanishes. The metric approaches AdS_4 with the same value of the cosmological constant at infinity. The extremal horizon is just the Poincaré horizon of near horizon geometry AdS_4 . In terms of the dual field theory, this means that the full conformal symmetry is restored in the infrared. Due to the asymmetry of our solution in x and y direction the ratio of speed of light in IR and UV is not only non-unity but also different in different directions. We have,

$$\frac{v_x^{IR}}{v_x^{UV}} = \frac{1}{c_0} \exp\left(\frac{\chi_0}{2}\right), \quad \frac{v_y^{IR}}{v_y^{UV}} = \exp\left(\frac{\chi_0}{2}\right). \quad (20)$$

Our solution may be viewed as static, charged solitons, where the electrostatic repulsion is balancing the gravitational and non-Abelian interactions. As can be checked easily from the asymptotic behaviors the energy of the soliton is finite.⁴ One interesting feature is the existence of an “essential singularity” (i.e. an $e^{-\frac{\alpha}{r}}$ behavior) of $A(r)$ at $r = 0$. Similar singularities appear for other quantities in Eq. (19). Hence all the sufficiently high derivative of the metric and gauge fields vanishes at $r = 0$. This is in contrast with the EYM case [3], where sufficiently large curvature invariants diverge at $r = 0$ for a generic α and behavior of the fields near $r = 0$ is power law. In terms of the “tortoise” co-ordinates (27), the essential singularity is just the reflection of the fact that A is massive near the horizon and decays exponentially.

4.1. Conductivity

In order to calculate the conductivity of this system, we need to turn on a small perturbation in the vector potential. We turn

⁴ However it should be noted that $B(r)$ is not square integrable. This is not a requirement of a valid solution to the Einstein's equations and assuming that our numerics are reliable, such solutions exists. The question is that whether they play a role in the context of AdS/CFT. In a sense whether it belongs to same quantum ensemble as the extremal black hole. As our zero temperature solution is of finite energy this seems to be the case. There are well-known solutions such as the BPST instanton [19] which also have a non-square integrable gauge field configuration with a bounded action. In our case, just as in case of BPST instanton, the solution may be approached by configurations which are of finite norm with a bounded energy (just replace $B \rightarrow \exp(-\frac{\beta}{r})B(r), \beta \rightarrow 0$). At finite temperature the gauge field B is likely to be square integrable due to the cutoff provided by the horizon. Our zero temperature solution may also be thought as a zero temperature limiting case of such a solution (see Section 5).

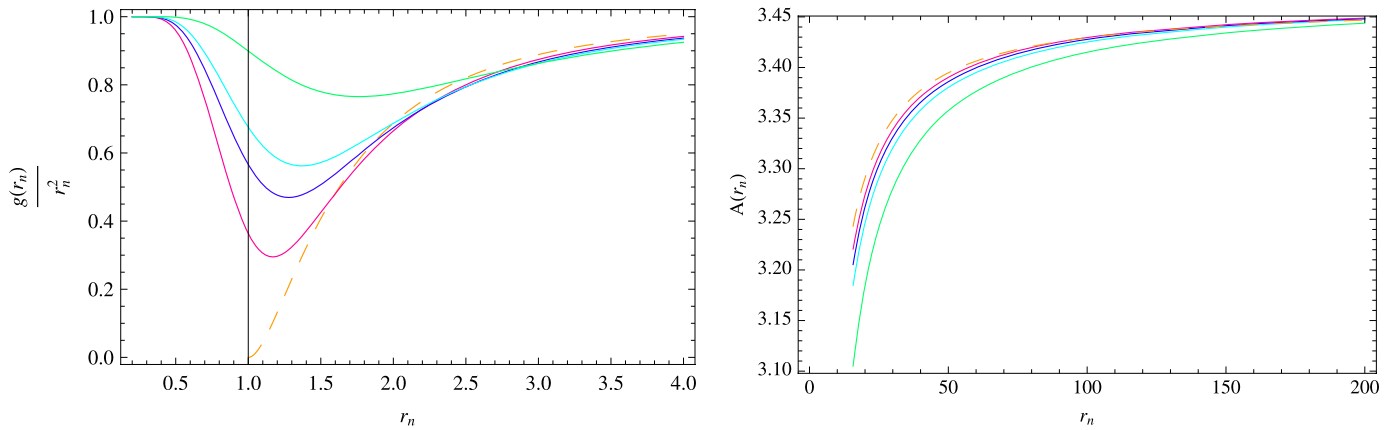


Fig. 2. The left panel shows a plot of the rescaled $\frac{g}{r^2}$ with the rescaled variable r_n . From top to bottom $q = 1.5, 1.1, 1, 0.9$. The dashed line is for the RN black hole. The right panel shows a plot of A with r_n for the same set of values of q , from bottom to top. From this plots one may see how our solutions are approaching a RN black hole as $q \rightarrow \frac{\sqrt{3}}{2}$.

on the gauge field and the associated metric perturbations of the form:

$$A_y^3 = \epsilon a(r) e^{-i\omega t} \tau^3 dy, \quad g_{ty} = \epsilon h(r) e^{-i\omega t}, \quad (21)$$

where we assume that both the gauge and the metric perturbations are of the same order $O(\epsilon)$. We won't be looking in the other component of the gauge field perturbations which decouples from the above. From Maxwell's equation we get,

$$A_y^3 \rightarrow a'' + a' \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{c'}{c} \right) + a \left(\frac{e^\chi \omega^2}{g^2} - \frac{q^2 B^2}{gr^2 c^2} \right) + \frac{e^\chi h}{g} \left(A'' + A' \left(\frac{h'}{h} + \frac{\chi'}{2} + \frac{c'}{c} \right) - \frac{q^2 B^2}{r^2 g c^2} A \right) = 0. \quad (22)$$

To linear order, Einstein's equations give,

$$T_{ry} \rightarrow -\frac{2}{r} h + h' = -a A', \quad (23)$$

which combined with the equation of $A(r)$ to give:

$$a'' + a' \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{c'}{c} \right) + a \left(\frac{e^\chi \omega^2}{g^2} - \frac{q^2 B^2}{gr^2 c^2} - e^\chi \frac{A'^2}{g} \right) = 0. \quad (24)$$

This can be written as a Schrödinger equation:

$$-a'' + V(\tilde{r})a = c^2 \omega^2 a, \quad (25)$$

where

$$V(r) = g \left(c^2 A'^2 + e^{-\chi} \frac{q^2 B^2}{r^2} \right) \quad (26)$$

and all the derivatives in Eq. (25) are in terms of the new variable \tilde{r} ("tortoise coordinate") given by:

$$\frac{d}{d\tilde{r}} \equiv e^{-\chi/2} g c \frac{d}{dr}. \quad (27)$$

Here, c approaches to unity as $r \rightarrow \infty$, so that the spacetime is asymptotically AdS_4 . It follows then from Eq. (11) that the potential $V(r)$ vanishes near the boundary. If we require $g \sim r^2$ near the horizon at $r = 0$ then the first term vanishes, while the second term is finite as $B(r=0) \equiv B_0 \neq 0$ and χ is also finite at the horizon. The value of the potential at the horizon decreases as $q \rightarrow \frac{\sqrt{3}}{2}$

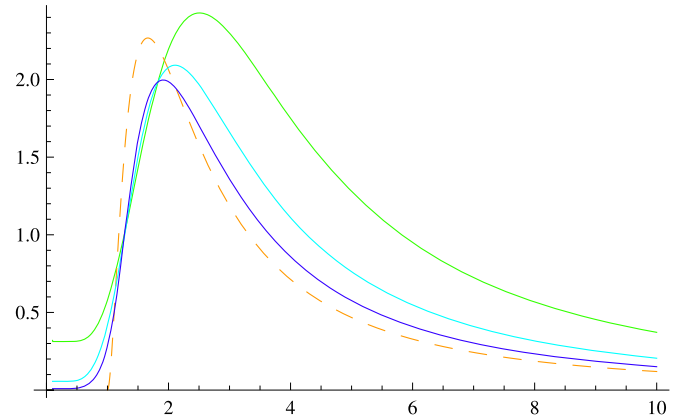


Fig. 3. Plot of the rescaled potential $V(r_n)$ for $q = 1.5, 1.1, 1$ (from the right) at $T = 0$ with rescaled co-ordinate r_n . The Poincaré horizon is at $r_n = 0$ for the zero temperature solutions. $V(r_n)$ is non-zero at the horizon. The dashed curve is for the extremal black hole with a horizon at $r = 1$.

(Fig. 3). Note that since $c \rightarrow 1$ near the boundary, the quantity ω can be interpreted as the frequency of the incoming wave.

The superconducting nature of the system is argued from the existence of a supercurrent solution. If we choose $\omega = 0$ and integrate A_x^3 from the horizon (with a regularity condition at the horizon), we are expected to get a non-trivial A_x . Existence of such a solution implies a δ function for the real part of conductivity at $\omega = 0$ [20,3]⁵

$$\text{Re}(\sigma(\omega)) \sim \delta(\omega) + \text{finite}. \quad (28)$$

The nature of the finite part of the conductivity can be inferred from the potential $V(r)$. As argued before the potential vanishes near the boundary and is finite near the horizon. The fact that the potential is non-zero at the horizon makes it possible for this system to exhibit a hard gap. From Eq. (25), the field a has the following asymptotic behaviours near the horizon ($\tilde{r} \rightarrow -\infty$) and the boundary ($\tilde{r} \rightarrow 0$):

⁵ One should be more careful about the existence of a delta function for the real part of conductivity at $\omega = 0$ and relation between superconductivity. Due to the coupling of gravity modes the potential $V(r)$ is non-trivial even in the normal phase (Fig. 3) and consequently a delta function is present in the real part of DC conductivity. Physical interpretation of such a delta function comes from the "translational invariance" of the theory. See [21] for further discussions.

$$a(\tilde{r} \rightarrow 0) \sim a_0^b + a_1^b \tilde{r}, \tag{29}$$

$$a(\tilde{r} \rightarrow \infty) = a_0 e^{i\tilde{\omega}\tilde{r}}, \tag{30}$$

where $\tilde{\omega} = \sqrt{c_0^2 \omega^2 - V_0}$, with c_h, a_h being the near-horizon values of c and a respectively. Here, we have chosen the incoming boundary condition near the horizon. The conductivity of the system is given by:

$$\sigma = -\frac{ia_1^b}{\omega a_0^b}. \tag{31}$$

It follows from Eq. (25) that:

$$a^* a'' - a a^{*''} = 0, \tag{32}$$

which implies that the quantity $\Lambda = a^* a' - a a^{*'} = 2i \text{Im}(a a^{*'})$ is a constant. Equating the values of Λ near the horizon and the boundary we get:

$$\text{Re}(\sigma) = \begin{cases} \frac{\tilde{\omega} |a_0|^2}{\omega |a_0^b|^2}, & \tilde{\omega}^2 > 0, \\ 0, & \tilde{\omega}^2 < 0. \end{cases} \tag{33}$$

Therefore, the real part of the conductivity will vanish whenever $\tilde{\omega}$ is imaginary, i.e. when $\omega < \sqrt{V_0}/c_0$, which defines the gap.

As our solution is an anisotropic superconductor (only superconducting in A_y direction), the excitations in A_x^3 channel may not be gapped [10]. The gap seems to be a property of the anisotropic non-Abelian system, as the scalar-Abelian gauge field system does not exhibit this gapped behaviour in the extremal limit [3].

5. Finite temperature analysis

Although Eq. (8) has been solved in a probe limit, the full set of equations with gravity backreaction (Eq. (9)) is not yet solved at a non-zero temperature. Hence it is an issue whether our zero temperature solution may be realized as a zero temperature limit of the finite temperature hairy black hole (and whether such a low temperature hairy black hole solution at all exists). One affirmative clue comes from the emergent conformal symmetry in the IR. In the near horizon region, i.e. $\alpha \gg r$, our solution approaches a AdS_4 geometry with a background gauge field $A_x = B_0$ and $A_t = 0$. One may consider a black hole situated deep inside this AdS space, such that the radius of the black hole $r_0 \ll \alpha$. Here, $g(r)$ near the horizon is given by, $g(r) = r^2(1 - \frac{r_0}{r})$. However this Schwarzschild like solution will get correction due to a small non-zero $A(r)$ near the horizon. The solution of $A(r)$ in the $r \ll \alpha$ is given by,

$$A(r) \approx \exp\left(-\frac{\alpha}{r}\right) \left(1 - \frac{r_0}{r}\right). \tag{34}$$

$A(r)$ is small in the region $r \ll \alpha$. In this region we can do a perturbation in $A(r)$ for other quantities. Realizing that our scaling relations Eq. (20) is just a perturbation in $A(r)$, we can match a near horizon solution with the scaling solution (i.e. Eq. (20)) in a region $r_0 \ll r \ll \alpha$. Hence we use the method of matched asymptotic expansion to argue that,

$$A_T(r) = A(r) \left(1 - \frac{r_0}{r}\right),$$

$$B_T(r) = B_0 - (B(r) - B_0) \left(1 - \frac{r_0}{r}\right)^2,$$

$$c_T(r) = c_0 + (c(r) - c_0) \left(1 - \frac{r_0}{r}\right)^2,$$

$$\chi_T(r) = \chi(r) + (\chi(r) - \chi_0) \left(1 - \frac{r_0}{r}\right),$$

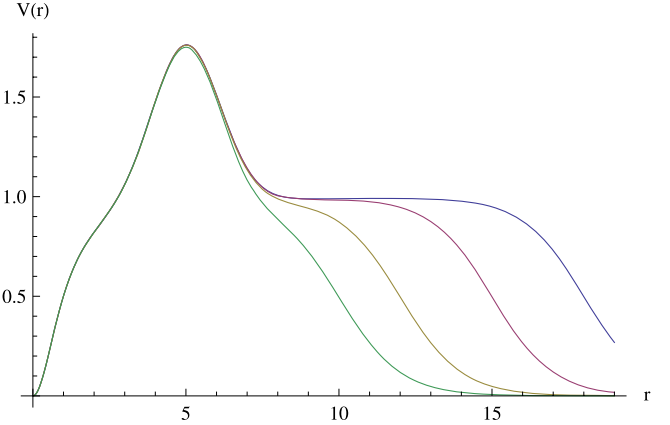


Fig. 4. Schematic plot of the potential $V(\tilde{r})$ for different T . T decreases from the left. $V(r)$ is gradually widened as T decreased. Horizon is at $\tilde{r} = \infty$.

$$g_T(r) = g(r) \left(1 - \frac{r_0}{r}\right) \tag{35}$$

is a solution to Einstein's equations if $r_0 \ll \alpha$. The quantities (A, B, c, χ) appear in the right-hand side of the above equations are our numerical solution and the quantities with a subscript T are the finite temperature versions. Whether correction to such a solution is finite or not is an open question, which may be partially settled by a convincing finite temperature numerics. However due to the possible “smooth” nature of the equations Eq. (8), i.e. a small change in the near horizon boundary condition leads to a small change near the boundary, it is reasonable to believe that corrections to Eq. (35) are well controlled. What happens to the potential $V(r)$ (see Eq. (26)) in this limit is interesting. At any non-zero r_0 , the potential $V(r)$ goes to zero at the horizon ($r = r_0$) and the height of the potential is always finite. Hence the transmission through this potential is non-zero. In our language the (i.e. Eq. (33)) $\tilde{\omega} = \omega$ is a real number. Hence the real part of the conductivity is non-zero for any value of ω . However, in the “tortoise co-ordinates” (27) the potential is gradually widened as $r_0 \rightarrow 0$ (Fig. 4). Hence the transmission through the potential gradually decreases as the horizon size is decreased. We expect a $\exp(-\frac{\alpha}{r})$ fall off for the real part of conductivity. A detailed discussion of the various gaps etc. is left for a future work.

6. Conclusion

Here we have discussed a zero temperature solitonic solution of the $SU(2)$ YM fields coupled to gravity with a negative cosmological constant. This solution may possibly be thought as the zero temperature limit of the hairy non-Abelian AdS black holes. To study the frequency (ω) response of the system we introduce the small fluctuation of certain components of gauge field on our solitonic background. We found the standard superconducting behavior which shows up as a delta function in the real part of DC conductivity. We also found the evidence of a “hard” energy gap, i.e. the real part of the AC conductivity vanishes for $\omega < \omega_c$. This has some similarity with BCS theory where also we found a energy gap at zero temperature. However our result is very different from what one gets in EYMH models discussed in [3], where the system does not show a gap and the real part of AC conductivity is always non-zero.

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