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Minimal Lee-Wick extension of the Standard Model

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ABSTRACT

We consider a minimal Lee–Wick (LW) extension to the Standard Model in which the fields providing the most important contributions to the cancellation of quadratic divergences are the lightest. Partners to the SU(2) gauge bosons, Higgs, top quark, and left-handed bottom quark are retained in the low-energy effective theory, which is valid up to approximately 10 TeV; the remaining LW partners appear above this cutoff and complete the theory in the ultraviolet. We determine the constraints on the low-energy spectrum from the electroweak parameters *S* and *T*, and find LW states within the kinematic reach of the LHC at the 95% confidence level.

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1. Introduction

The intriguing idea of Lee and Wick (LW) [1] to promote Pauli– Villars regulators to the status of physical fields was recently applied to develop a LW extension to the Standard Model (LWSM) [2]. While the original LW proposal was designed to render QED finite, the purpose of the LWSM is to use the LW opposite-sign propagators in loop diagrams to solve the hierarchy problem. This solution is analogous to the supersymmetric one in that it relies on cancellation between pairs of loops to remove quadratic divergences, but differs in that the LW particles carry the same statistics (and other quantum numbers) as their SM partners. Several recent papers investigate the formal properties and phenomenology of the LWSM [3–7].

In the present work, we consider a version of the LWSM in which only a subset of the full spectrum of LW partners lie within the reach of the LHC. Motivated by Little Higgs models [8], we study the possibility that only the LW partners of the SU(2) gauge bosons, Higgs, *t* quark, and left-handed *b* quark appear in the low-energy effective theory. These fields provide the most significant contributions to the cancellation of quadratic divergences in the Higgs sector and render the effective theory natural, provided the cutoff is ≤ 10 TeV. The remaining LW spectrum may appear above this cutoff, or the theory may be completed by other, more exotic physics. This minimal LW low-energy theory is distinguished by its simplicity, making it an ideal subject for comprehensive phenomenological investigation.

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In this Letter we present the constraints on this model's spectrum that follow from oblique electroweak parameters, in particular the Peskin–Takeuchi S and T parameters [9]:

$$S = -16\pi \frac{d}{dq^2} \Pi_{3B} \Big|_{q^2 = 0},\tag{1}$$

$$T = \frac{4\pi}{s^2 c^2 m_{Z_0^2}} (\Pi_{11} - \Pi_{33}) \bigg|_{q^2 = 0},$$
(2)

where Π are the usual self-energy functions, $s \equiv \sin \theta_W$, $c \equiv$ $\cos \theta_W$ parametrize the weak mixing angle and m_{Z_0} is the measured Z boson mass. Our approach is similar to that of other recent work, in particular Ref. [3] (ADSS), but differs in that we do not assume a complete LWSM spectrum with large sets of massdegenerate particles. Exact one-loop formulae for S and T, which have not appeared in previous literature, are necessary for a proper treatment of corrections in our model. We agree with the original LWSM work [2] and ADSS [3] that the leading oblique corrections occur at tree level in the LWSM, contrary to the claim in Ref. [5]. Part of this discrepancy is due to differing definitions in the literature of what physics is "oblique" (see the discussion in Section 2). Moreover, the identification of oblique parameters in the effective Lagrangian of Ref. [5] appears to be fundamentally different from ours and Refs. [2,3] so that the numerical results are not easily compared: we do not consider this issue further here.

Our Letter is organized as follows: In Section 2 we establish conventions for specifying the spectrum, which take into account potentially substantial mixing between SM and LW particles. We present the one-loop formulae for the *S* and *T* in Section 3. We present our numerical results in Section 4, and Section 5 summarizes our conclusions.

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Fig. 1. Diagram classes that may contribute to oblique parameters. Wavy lines represent gauge fields, dashed lines represent scalars, and solid lines represent fermions.

2. Preliminaries

We study the *S* and *T* parameters in an effective theory obtained by integrating out heavy-mass eigenstates. At tree level, this procedure is equivalent to eliminating the heavy fields from the Lagrangian using their classical equations of motion. Since the gauge sector of our model includes LW partners to only the SU(2) gauge bosons, one finds

$$\Delta S_{\text{tree}} = 4\pi \frac{v^2}{M_2^2} + O\left(\frac{v^4}{M_2^4}\right),$$
(3)

$$\Delta T_{\text{tree}} = 0, \tag{4}$$

in agreement with the results of ADSS in the limit $M_1 \rightarrow \infty$ [the U(1) LW gauge boson contributes to ΔT_{tree} at $O(v^2/M_1^2)$]. Here, M_1 and M_2 represent the unmixed LW U(1) and SU(2) gauge boson masses in the auxiliary field formulation of the LWSM, as defined in Ref. [2], and v is the Higgs vacuum expectation value. The parameter U turns out to be $O(v^4/M_2^4)$ and is similarly suppressed in loop effects, so we do not consider it further. Note that the constraints on new physics from oblique parameters are meaningful only if vertex corrections are small. The derivation of Eqs. (3)–(4) includes field redefinitions that force the couplings of the gauge fields to SM currents to match those of the SM at tree level. Thus, the definitions of *S* and *T* used here (and in ADSS) subsume the largest vertex corrections.

The one-loop contributions to the self-energies Π_{AB} in Eqs. (1)–(2) arise from the diagrams in Fig. 1. We evaluate these diagrams using mass eigenstates on the internal lines; since SM and LW fields mix, one must first define conventions to specify the spectrum. The following mixing effects are taken into account:

2.1. Neutral Higgs mixing

The SM Higgs field and its LW partner (h, \tilde{h}) have mass terms [2]

$$\delta \mathcal{L} = -\frac{1}{2} \begin{pmatrix} h & \tilde{h} \end{pmatrix} \begin{pmatrix} m_h^2 & -m_h^2 \\ -m_h^2 & -(m_{\tilde{h}}^2 - m_h^2) \end{pmatrix} \begin{pmatrix} h \\ \tilde{h} \end{pmatrix}.$$
(5)

The mass matrix in Eq. (5) is diagonalized via the symplectic transformation

$$\begin{pmatrix} h\\ \tilde{h} \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta\\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} h_0\\ \tilde{h}_0 \end{pmatrix}, \tag{6}$$

where subscript 0 here and below indicates mass eigenstates. The mixing angle $\boldsymbol{\theta}$ satisfies

$$\tanh 2\theta = \frac{-2m_h^2/m_{\tilde{h}}^2}{1 - 2m_h^2/m_{\tilde{h}}^2} = -\frac{2m_{h_0}^2m_{\tilde{h}_0}^2}{m_{h_0}^4 + m_{\tilde{h}_0}^4},\tag{7}$$

with mass eigenvalues

$$m_{h_0}^2, m_{\tilde{h}_0}^2 = \frac{m_{\tilde{h}}^2}{2} \left(1 \mp \sqrt{1 - \frac{4m_h^2}{m_{\tilde{h}}^2}} \right).$$
(8)

In addition, the LW sector has pseudoscalar \tilde{P} and charged scalar \tilde{h}^+ states with masses $m_{\tilde{h}}$. We work in unitary gauge, where all unphysical scalars are eliminated from the theory.

2.2. Gauge mixing

The SM SU(2) gauge boson and its LW partner (W, \tilde{W}) mix via [2]

$$\delta \mathcal{L} = (W^{\mu +} \quad \tilde{W}^{\mu +}) \begin{pmatrix} m_W^2 & m_W^2 \\ m_W^2 & m_W^2 - M_2^2 \end{pmatrix} \begin{pmatrix} W^-_{\mu} \\ \tilde{W}^-_{\mu} \end{pmatrix},$$
(9)

where $m_W = \frac{1}{2}g_2v$ is the unmixed SM W mass. The mass matrix is diagonalized by the symplectic transformation

$$\begin{pmatrix} W^{\pm} \\ \tilde{W}^{\pm} \end{pmatrix} = \begin{pmatrix} \cosh \varphi_c & \sinh \varphi_c \\ \sinh \varphi_c & \cosh \varphi_c \end{pmatrix} \begin{pmatrix} W^{\pm}_0 \\ \tilde{W}^{\pm}_0 \end{pmatrix},$$
(10)

where, using \tilde{W}_0^1 to indicate the charged heavy mass eigenstate,

$$\tanh 2\varphi_c = \frac{2m_W^2}{M_2^2 - 2m_W^2} = \frac{2m_{W_0}^2 m_{\tilde{W}_0}^2}{m_{W_0}^4 + m_{\tilde{W}_0}^4},\tag{11}$$

with eigenvalues satisfying

$$m_{W_0}^2, \ m_{\tilde{W}_0}^2 = \frac{M_2^2}{2} \left(1 \mp \sqrt{1 - \frac{4m_W^2}{M_2^2}} \right).$$
 (12)

In the neutral sector, mixing only occurs between the SM Z boson and the LW \tilde{W}^3 [2]:

$$\delta \mathcal{L} = \frac{1}{2} \begin{pmatrix} Z & \tilde{W}^3 \end{pmatrix} \begin{pmatrix} m_Z^2 & m_Z^2 c \\ m_Z^2 c & -(M_2^2 - m_W^2) \end{pmatrix} \begin{pmatrix} Z \\ \tilde{W}^3 \end{pmatrix},$$
(13)

where $m_Z = m_W/c$ is the unmixed SM Z mass. The photon decouples as a consequence of electromagnetic gauge invariance. Eq. (13) is diagonalized via the symplectic transformation

$$\begin{pmatrix} Z\\ \tilde{W}^3 \end{pmatrix} = \begin{pmatrix} \cosh\varphi_0 & \sinh\varphi_0\\ \sinh\varphi_0 & \cosh\varphi_0 \end{pmatrix} \begin{pmatrix} Z_0\\ \tilde{W}_0^3 \end{pmatrix},$$
(14)

where

$$\tanh 2\varphi_0 = \frac{2m_Z^2 c}{M_2^2 - m_Z^2 (1 + c^2)},\tag{15}$$

and the eigenvalues are given by

$$m_{Z_0}^2, m_{\tilde{W}_0^3}^2 = \frac{1}{2} \left[M_2^2 + m_Z^2 s^2 \mp \sqrt{\left(M_2^2 - m_Z^2 s^2\right)^2 - 4M_2^2 m_Z^2 c^2} \right].$$
(16)

2.3. Fermion mixing

Our model includes LW partners to the fields t_L , t_R , and b_L . The mass terms of the third-generation fermions read [3,7]

$$\delta \mathcal{L} = -\bar{T}_L \eta \mathcal{M}_t^{\mathsf{T}} T_R - \bar{B}_L \eta \mathcal{M}_b^{\mathsf{T}} B_R + \text{h.c.}, \qquad (17)$$

where

$$T_{L,R}^{T} = (t_{L,R}, \tilde{t}_{L,R}, \tilde{t}_{L,R}'),$$
(18)

$$B_{L,R}^{T} = (b_{L,R}, \tilde{b}_{L,R}, \tilde{b}_{L,R}'),$$
(19)

define our basis for the third-generation fields. The fields t_L , t_R , b_L , and b_R are the SM fields with their usual quantum numbers, while the tilded fields are LW. The unprimed LW fields are the partners

of the SM fields and hence have the same quantum numbers and chirality. The primed LW fields have the same quantum numbers as the unprimed LW fields of the opposite chirality, in order to permit SU(2) × U(1)-invariant LW mass terms. Thus, for example, \tilde{t}_L and \tilde{t}'_R both transform as a $(\mathbf{2}, +\frac{1}{6})$ under SU(2) × U(1), the same as t_L . The matrix $\eta = \text{diag}(1, -1, -1)$ conveniently encodes the opposite signs between SM and LW kinetic terms or mass terms. Then one finds

$$\mathcal{M}_{t}\eta = \begin{pmatrix} +m_{t} & -m_{t} & 0\\ -m_{t} & +m_{t} & -M_{t}\\ 0 & -M_{q} & 0 \end{pmatrix},$$
$$\mathcal{M}_{b}\eta = \begin{pmatrix} +m_{b} & -m_{b} & 0\\ -m_{b} & +m_{b} & -M_{b}\\ 0 & -M_{a} & 0 \end{pmatrix}.$$
(20)

We diagonalize these mass matrices via transformation matrices S_L^a and S_R^a , for a = t or b, such that \mathcal{M}_0 is diagonal with positive eigenvalues:

$$S_L^{\dagger}\eta S_L = \eta, \qquad S_R^{\dagger}\eta S_R = \eta, \qquad \mathcal{M}_0\eta = S_R^{\dagger}\mathcal{M}\eta S_L.$$
 (21)

Additional details regarding the solution to Eqs. (21) will appear elsewhere [10]; for the purposes of this calculation, we simply note that solutions were obtained numerically.

3. Loops

A consistent calculation of oblique parameters in a perturbative theory must yield results that are ultraviolet finite, since these parameters describe physical observables. Here we consider the deviation of S and T from their SM values, so one must subtract any purely SM contributions. While individual diagrams can diverge, we find that the final subtracted results are finite and cutoff independent.

First consider the *S* parameter, which receives contributions from the diagrams of Fig. 1(a), (b), and (d); the diagram in Fig. 1(c) is not relevant since its contributions to Π_{3B} is q^2 independent. From the purely Higgs-sector diagram in Fig. 1(a) we find

$$\Delta S_{1a} = \frac{1}{12\pi} \Big[I_1(m_{\tilde{h}_0}^2/m_{\tilde{h}}^2) \cosh^2\theta - I_1(m_{h_0}^2/m_{\tilde{h}}^2) \sinh^2\theta \Big], \tag{22}$$

where

$$I_1(\xi) = \frac{\xi^2 (3-\xi) \ln \xi}{(1-\xi)^3} - \frac{(5-22\xi+5\xi^2)}{6(1-\xi)^2}.$$
 (23)

Note that the contribution to the self-energy from Fig. 1(a) vanishes if the LW states are decoupled, so the result must be finite without any SM subtraction, as is indeed the case. The contribution from Fig. 1(b), however, involves a diagram with purely SM particles (the Higgs and Z bosons), with non-SM couplings. In this case, one must subtract the same diagrams evaluated with infinite LW masses. One finds

$$\Delta S_{1b} = -\frac{g_2^2 v^2}{4\pi M_2^2} \sum \frac{C_{3B}}{\xi_2} \left[I_2 \left(\frac{\xi_1}{\xi_2}\right) - \frac{1}{12} I_1 \left(\frac{\xi_1}{\xi_2}\right) - \frac{1}{12} \ln \xi_2 \right], \quad (24)$$

where C_{3B} is the coefficient of Fig. 1(b) diagram with internal scalar (*S*) and vector (*V*) particles of mass m_S and m_V , respectively, $\xi_1 \equiv m_S^2/M_2^2$, $\xi_2 \equiv m_V^2/M_2^2$, and

$$I_2(\xi) \equiv \frac{1 - \xi^2 + 2\xi \ln \xi}{2(1 - \xi)^3}.$$
(25)

Table 1 gives the values of C_{3B} , ξ_1 , and ξ_2 for each term summed in Eq. (24), as well as for the SM subtraction.

To compute the fermionic contribution to *S*, we first parametrize the gauge–fermion couplings evaluated in the mass eigenstate basis:

Table 1Coefficients C_{AB} for each of the contributing diagrams to Eqs. (24) and (29)

AB	C _{AB}	S	V	ξ1	ξ2
3 <i>B</i>	$+\cosh^2\theta(\sinh\varphi_0+\frac{1}{c}\cosh\varphi_0)^2$	h_0	Z ₀	$m_{h_0}^2/M_2^2$	$m_{Z_0}^2/M_2^2$
	$-\cosh^2\theta(\cosh\varphi_0+\frac{1}{c}\sinh\varphi_0)^2$	h_0	\tilde{W}_0^3	$m_{h_0}^2/M_2^2$	$m_{\tilde{W}_{2}^{3}}^{2}/M_{2}^{2}$
	$-\sinh^2\theta(\sinh\varphi_0+\frac{1}{c}\cosh\varphi_0)^2$	\tilde{h}_0	Z_0	$m_{\tilde{h}_{0}}^{2}/M_{2}^{2}$	$m_{Z_0}^2/M_2^2$
	$+\sinh^2\theta(\cosh\varphi_0+\frac{1}{c}\sinh\varphi_0)^2$	\tilde{h}_0	\tilde{W}_0^3	$m_{\tilde{h}_0}^{2}/M_2^2$	$m_{\tilde{W}^{3}}^{2}/M_{2}^{2}$
	$-\frac{1}{c^2}$	h	Ζ	m_h^2/M_2^2	m_Z^2/M_2^2
11	$-\cosh^2\theta(\cosh\varphi_c+\sinh\varphi_c)^2$	h_0	W_0^1	$m_{h_0}^2/M_2^2$	$m_{W_0^1}^2/M_2^2$
	$+\cosh^2\theta(\cosh\varphi_c+\sinh\varphi_c)^2$	h_0	\tilde{W}_0^1	$m_{h_0}^2/M_2^2$	$m_{\tilde{W}_{1}}^{2}/M_{2}^{2}$
	$+\sinh^2\theta(\cosh\varphi_c+\sinh\varphi_c)^2$	\tilde{h}_0	W_0^1	$m_{\tilde{h}_{0}}^{2}/M_{2}^{2}$	$m_{W^1}^2/M_2^2$
	$-\sinh^2\theta(\cosh\varphi_c+\sinh\varphi_c)^2$	\tilde{h}_0	\tilde{W}_0^1	$m_{\tilde{h}_0}^2 / M_2^2$	$m_{\tilde{W}^1}^2/M_2^2$
	+1	h	W^1	m_h^2/M_2^2	$m_{W^1}^2/M_2^2$
33	$-\cosh^2\theta(\sinh\varphi_0+\frac{1}{c}\cosh\varphi_0)^2$	h_0	Z_0	$m_{h_0}^2/M_2^2$	$m_{Z_0}^2/M_2^2$
	$+\cosh^2\theta(\cosh\varphi_0+\frac{1}{c}\sinh\varphi_0)^2$	h_0	\tilde{W}_0^3	$m_{h_0}^2/M_2^2$	$m_{\tilde{W}_{2}^{3}}^{2}/M_{2}^{2}$
	$+\sinh^2\theta(\sinh\varphi_0+\frac{1}{c}\cosh\varphi_0)^2$	\tilde{h}_0	Z_0	$m_{\tilde{h}_{0}}^{2}/M_{2}^{2}$	$m_{Z_0}^2/M_2^2$
	$-\sinh^2\theta(\cosh\varphi_0+\frac{1}{c}\sinh\varphi_0)^2$	\tilde{h}_0	\tilde{W}_0^3	$m_{\tilde{h}_0}^{2}/M_2^2$	$m_{\tilde{W}^{3}}^{2}/M_{2}^{2}$
	$+\frac{1}{c^2}$	h	Ζ	m_{h}^{2}/M_{2}^{2}	m_Z^2/M_2^2

$$\delta \mathcal{L} = -g_1 B_\mu \bar{\Psi}_0 \gamma^\mu (C_{\Psi}^{\mu} P_L + C_{\Psi}^{R} P_R) \Psi_0 - g_2 W_{\mu}^3 \bar{\Psi}_0 \gamma^\mu (D_{\Psi}^{L} P_L + D_{\Psi}^{R} P_R) \Psi_0,$$
(26)

where P_L (P_R) are the left (right)-handed chiral projection operators, and Ψ_0 represents T_0 and B_0 , the transformation of Eqs. (18) and (19), respectively, into mass eigenstates. The gauge coupling matrices $C_{\Psi}^{L,R}$ and $D_{\Psi}^{L,R}$ are computed numerically, taking into account the basis change Eq. (21). Denoting the mass of the *i*th fermion mass eigenstate m_i , and defining $\xi_i \equiv m_i^2/M^2$ for an arbitrary mass scale M, we find

$$\Delta S_{1d} = -\frac{2}{\pi} \sum_{\Psi=T,B} \sum_{i,j} \eta_{ii} \eta_{jj} \left\{ \left(C_{\Psi ij}^L D_{\Psi ji}^L + C_{\Psi ij}^R D_{\Psi ji}^R \right) \right. \\ \left. \times \left[I_1 \left(\frac{\xi_i}{\xi_j} \right) + \ln \xi_j \right] - 3 \left(C_{\Psi ij}^L D_{\Psi ji}^R + C_{\Psi ij}^R D_{\Psi ji}^L \right) \right. \\ \left. \times \sqrt{\frac{\xi_i}{\xi_j}} I_2 \left(\frac{\xi_i}{\xi_j} \right) \right\} - \frac{1}{2\pi} \left[1 - \frac{1}{3} \ln \left(\frac{m_{t,SM}^2}{m_{b,SM}^2} \right) \right].$$
(27)

The last term of Eq. (27) represents the SM subtraction, with $m_{t,SM}$ and $m_{b,SM}$ the *t* and *b* masses, respectively, obtained in the decoupling limit of the LW states. The cancellation of logarithmic divergences between various contribution to ΔS_{1d} requires

$$\sum_{i,j} \eta_{ii} \eta_{jj} \left(C_{ij}^L D_{ji}^L + C_{ij}^R D_{ji}^R \right) = 0,$$
(28)

which we find to be satisfied to any desired numerical precision. The numerical results for *S* presented in the next section represent the total $\Delta S = \Delta S_{1a} + \Delta S_{1b} + \Delta S_{1d}$ given by Eqs. (22), (24) and (27).

Our approach to evaluating T is analogous. In agreement with ADSS, we find that the contributions to T from Fig. 1(a) exactly cancel, as do those from Fig. 1(c). The coefficients C_{11} , C_{33} for the diagrams in Fig. 1(b), including SM subtractions, appear in Table 1. We find

$$\Delta T_{1b} = -\frac{m_Z^2}{4\pi s^2 m_{Z_0}^2} \sum \left(\frac{C_{11}}{\xi_1 - \xi_2} \bigg[\xi_1 \ln \xi_1 - \xi_2 \ln \xi_2 - \frac{1}{4\xi_2} (\xi_1^2 \ln \xi_1 - \xi_2^2 \ln \xi_2) \bigg] - (C_{11} \to C_{33}) \right),$$
(29)

with ξ_1 and ξ_2 defined after Eq. (24). To find the fermionic contribution to *T*, we extend the parametrization of gauge–fermion couplings of Eq. (26) to include the W^1 boson:

$$\delta \mathcal{L} = -g_2 W^1_{\mu} \bar{T}_0 \gamma^{\mu} (E_L P_L + E_R P_R) B_0 + \text{h.c.}, \tag{30}$$

where the matrices $E_{L,R}$ are also evaluated in the mass eigenstate basis. One finds

$$\Delta T_{1d} = -\frac{3}{4\pi s^2 c^2 m_{Z_0}^2} \left\{ M^2 \sum_{\Psi=T,B} \sum_{i,j} \eta_{ii} \eta_{jj} \left[-\left(D_{\Psi ij}^L D_{\Psi ji}^L \right)^L + D_{\Psi ij}^R D_{\Psi ji}^R \right) \left(\frac{\xi_i^2 \ln \xi_i - \xi_j^2 \ln \xi_j}{\xi_i - \xi_j} \right) + 4 \left(D_{\Psi ij}^L D_{\Psi ji}^R + D_{\Psi ij}^R D_{\Psi ji}^L \right) \sqrt{\xi_i \xi_j} \left(\frac{\xi_i \ln \xi_i - \xi_j \ln \xi_j}{\xi_i - \xi_j} \right) \right] + M^2 \sum_{i,j} \eta_{ii} \eta_{jj} \left[2 \left(E_{ij}^L E_{ji}^{L\dagger} + E_{ij}^R E_{ji}^{R\dagger} \right) \left(\frac{\xi_i^2 \ln \xi_i - \xi_j^2 \ln \xi_j}{\xi_i - \xi_j} \right) \right] - 8 \left(E_{ij}^L E_{ji}^{R\dagger} + E_{ij}^R E_{ji}^{L\dagger} \right) \sqrt{\xi_i \xi_j} \left(\frac{\xi_i \ln \xi_i - \xi_j \ln \xi_j}{\xi_i - \xi_j} \right) \right] - \frac{1}{4} \left[m_{t,SM}^2 + m_{b,SM}^2 - \frac{2m_{t,SM}^2 m_{b,SM}^2}{(m_{t,SM}^2 - m_{b,SM}^2)} \ln \left(\frac{m_{t,SM}^2}{m_{b,SM}^2} \right) \right] \right\}.$$
(31)

The removal of divergences from ΔT_{1d} [leading to the finiteness of Eq. (31)] requires delicate cancellations between the *t*, *b*, and *tb* diagrams not only for the LL + RR coefficients of the quadratic divergences, but also between the LL + RR and LR + RL coefficients of the logarithmic divergences. Indeed, these cancellations may be verified [10].

4. Results

To obtain the constraints on our model, we choose as input parameters M_2 , $m_{\tilde{h}}$, and a common fermion mass parameter $M_F = M_q = M_t$. With the lightest gauge boson mass eigenvalues m_{W_0} , m_{Z_0} fixed by the measured masses, specifying M_2 fixes the gauge boson spectrum of the model. We choose the Higgs mass parameter m_h that appears in the SM Lagrangian to be 115 GeV, which provides our reference mass in defining S and T; specifying $m_{\tilde{h}}$ then completely fixes the Higgs spectrum of the theory. Finally, we set the lightest fermion mass eigenvalues m_{t_0} , m_{b_0} to the physical quark masses, and decouple the LW partner \tilde{b}_R , which is not part of the minimal low-energy theory, by taking $M_b \rightarrow \infty$; specifying M_F then completely fixes the fermionic spectrum of the theory. Note that the choice $M_F = M_q = M_t$ is merely a convenience, although naturalness suggests M_q and M_t are comparable; the general case provides substantial additional freedom in accommodating current experimental bounds [10]. Finally, note that each set of input parameters $(M_2, m_{\tilde{h}}, M_F)$ corresponds to a (slightly) different value for the Lagrangian mass parameter m_t . This mass corresponds to the SM reference value in the limit of decoupled LW partners, about which deviations in S and T are measured. We shift [9] the model predictions for the oblique parameters for each input parameter set to coincide with the t reference mass 170.9 GeV assumed in the computation of the experimentally allowed region of the S-T plane.

Fig. 2 shows our results for the choice $m_{\tilde{h}} = 750$ GeV; over the phenomenologically interesting range 250 GeV $< m_{\tilde{h}} < 1$ TeV we find a remarkably weak dependence of *S*, *T* on $m_{\tilde{h}}$, and therefore opt to fix $m_{\tilde{h}}$ at an intermediate value. The grid shows model predictions as M_2 and M_F are varied from 2–10 TeV. The 95% C.L. allowed region is based [11] on an analysis by the LEP Electroweak



Fig. 2. Oblique corrections for $m_{\tilde{h}} = 750$ GeV. The grid shows model predictions as M_2 and M_F are varied from 2-10 TeV. The Higgs and *t* reference masses are 115 GeV and 170.9 GeV, respectively.



Fig. 3. Limits on the smallest 95% C.L. allowed masses for the LW eigenstates $\tilde{t}_0^{(1,2)}$ and \tilde{b}_0 following from Fig. 2, as functions of the LW gauge mass parameter M_2 .

Working Group [12], but shifted to convenient Higgs and *t* reference masses, 115 GeV and 170.9 GeV, respectively. We find points with M_2 and M_F both $\lesssim 5.2$ TeV just within the allowed region; for $M_2 = 10$ TeV, M_F can be as small as ~ 4 TeV. Fig. 3 displays the smallest allowed masses for the LW eigenstates $\tilde{t}_0^{(1,2)}$ and \tilde{b}_0 following from Fig. 2. For example, this figure indicates that one of the \tilde{t}_0 's can be as light as 4 TeV. With LW gauge and fermion states typically heavier than this, other low-energy constraints on the model, such as those from flavor-changing processes [6,7], are likely only to be relevant in the lighter LW Higgs sector. A more complete investigation of flavor and electroweak constraints on the effective theory of interest will appear in Ref. [10].

5. Conclusions

We have considered the constraints from oblique electroweak parameters in an extension to the SM that includes LW partners to the SU(2) gauge bosons, the Higgs doublet, and the $t_{L,R}$ and b_L quarks. This low-energy theory has the smallest particle content required to cancel the largest contributions to the Higgs quadratic mass divergences, rendering the effective theory natural up to ~ 10 TeV (similar to Little Higgs models). Above this scale one may uncover the remaining particle content of the LWSM, or perhaps an even more exotic ultraviolet completion. This effective theory is meritorious because its spectrum is simple and allows a more focused and complete study of phenomenological constraints and collider signatures; the electroweak analysis presented here is a necessary first step. Our conclusion that the LW partners in this effective theory can be kinematically accessible at the LHC (as specified in Figs. 2–3) suggests a broad range of interesting phenomenological issues for further study.

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References

T.D. Lee, G.C. Wick, Nucl. Phys. B 9 (1969) 209;
 T.D. Lee, G.C. Wick, Phys. Rev. D 2 (1970) 1033.

- [2] B. Grinstein, D. O'Connell, M.B. Wise, Phys. Rev. D 77 (2008) 025012, arXiv: 0704.1845 [hep-ph].
- [3] E. Alvarez, L. Da Rold, C. Schat, A. Szynkman, JHEP 0804 (2008) 026, arXiv: 0802.1061 [hep-ph].
- [4] T.G. Rizzo, JHEP 0706 (2007) 070, arXiv: 0704.3458 [hep-ph];
- T.G. Rizzo, JHEP 0801 (2008) 042, arXiv: 0712.1791 [hep-ph]; J.R. Espinosa, B. Grinstein, D. O'Connell, M.B. Wise, arXiv: 0705.1188 [hep-ph];
- B. Grinstein, D. O'Connell, M.B. Wise, Phys. Rev. D 77 (2008) 065010, arXiv: 0710.5528 [hep-ph];
- B. Grinstein, D. O'Connell, M.B. Wise, arXiv: 0805.2156 [hep-th];
- B. Grinstein, D. O'Connell, arXiv: 0801.4034 [hep-ph];
- E. Gabrielli, Phys. Rev. D 77 (2008) 055020, arXiv: 0712.2208 [hep-ph];
- F. Wu, M. Zhong, Phys. Lett. B 659 (2008) 694, arXiv: 0705.3287 [hep-ph];
- A. van Tonder, Int. J. Mod. Phys. A 22 (2007) 2563, hep-th/0610185;
- Z. Fodor, K. Holland, J. Kuti, D. Nogradi, C. Schroeder, PoS LAT2007 (2007) 056,
- arXiv: 0710.3151 [hep-lat];
- F. Knechtli, N. Irges, M. Luz, arXiv: 0711.2931 [hep-ph].
- [5] T.E.J. Underwood, R. Zwicky, arXiv: 0805.3296 [hep-ph].
- [6] T.R. Dulaney, M.B. Wise, Phys. Lett. B 658 (2008) 230, arXiv: 0708.0567 [hepph].
- [7] F. Krauss, T.E.J. Underwood, R. Zwicky, Phys. Rev. D 77 (2008) 015012, arXiv: 0709.4054 [hep-ph].
- [8] For a review, see M. Schmaltz, D. Tucker-Smith, Annu. Rev. Nucl. Part. Sci. 55 (2005) 229, hep-ph/0502182.
- M.E. Peskin, T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964;
 M.E. Peskin, T. Takeuchi, Phys. Rev. D 46 (1992) 381.
- [10] C.D. Carone, R.F. Lebed, in preparation.
- [11] G.D. Kribs, T. Plehn, M. Spannowsky, T.M.P. Tait, Phys. Rev. D 76 (2007) 075016, arXiv: 0706.3718 [hep-ph].
- [12] LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPEWWG.