Multi Stage Noise Shaping Sigma-delta Modulator (MASH) for Capacitive MEMS Accelerometers

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Abstract

Electromechanical multi stage noise shaping sigma-delta modulator (MASH) has the advantages of inherent stability, high dynamic range, and high overload input level compared with the single loop sigma-delta-modulator approach. In this paper, a fourth order electromechanical MASH is studied by Simulink modeling and hardware implementation using surface mount PCB technology. The accelerometer used in the study is fabricated using a Silicon on Insulator (SOI) wafer with a device layer thickness of 50μm, using a dicing free and dry release process. The experimental results confirm the concept of the MASH structure and show its potential as a closed loop interface concept for a high performance capacitive MEMS accelerometer.

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1. Introduction

High order electromechanical sigma-delta modulators (ΣΔM) with a single loop architecture have successfully been applied to capacitive MEMS accelerometers [1, 2]. However, a successful implementation of an electromechanical multi stage noise shaping sigma-delta modulator (MASH) has not yet been reported. The MASH architecture has the advantages of inherent stability, high dynamic range, and high overload input level compared with the single loop ΣΔM approach [3]. Nevertheless, the MASH is sensitive to component and parameter tolerances. In this paper, a fourth order electromechanical MASH with an oversampling ratio (OSR) of 64 and a bandwidth of 1 KHz is studied by Simulink modeling and hardware implementation using surface mount PCB technology. The accelerometer used in the study is fabricated using a Silicon on Insulator (SOI) wafer with a device layer thickness of 50μm, using a dicing free and dry release process [4].

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2. System Modeling and Analytical Investigation

A system level view of the MASH modulator is depicted as a Simulink model in figure 1. The first loop consists of a capacitive MEMS accelerometer (M) embedded in a digitally controlled force-feedback loop forming a second order electromechanical sigma-delta modulator. The compensator (C) is required to maintain loop stability with the under damped accelerometer. The second loop is a purely electronic, second order sigma-delta modulator, where the quantization noise from the first loop is scaled by the interface gains (KR, KS, and K2) then digitized by the second loop and cancelled by the digital filters D1 and D2. The sensor system design parameters are listed in table 1.

![Simulink model of an electromechanical forth order MASH ΣΔM architecture. The first loop comprises the micromachined accelerometer sensing element, whereas the second loop is purely electronic. The output bitstreams of the two loops are combined by digital filters.](image)

In order to design the digital filters, the system has to be linearized so that classical control theory can be applied. A linear model can be constructed using the usual assumption for the two quantizers to be modelled as gain constants (Kq1, Kq2) and additive white noise signals (Qn1, Qn2). For small mass deflection, the pickoff and feedback circuits can also be modelled as simple gain constants (Kpo, Kbst, and Kfb), respectively. The quantization noise transfer functions from the first stage (Q1NTF) and the second stage (Q2NTF) derived from the linear model are given by:

\[
Q_{1NTF} = NTF_1 D_1 = \left\{STF_2 D_2 \left[ NTF_1 \left( KS_1 K_2 - \frac{KR_1 K_2}{K_q^1} \right) + \frac{KR_1 K_2}{K_q^1} \right] \right\} \quad (1)
\]

\[
Q_{2NTF} = -\frac{D_2}{1 + G_1 AB K_{q2} + G_2 BK_{q2}} \quad (2)
\]

Where, \(NTF_1\) is the noise transfer function of the electromechanical 2\(^{nd}\) order \(\Sigma\Delta\) modulator in the first stage, and \(STF_2\) is the signal transfer function of the electronic \(\Sigma\Delta\) modulator in the second stage, which are given by:

\[
NTF_1 = \frac{1}{1 + MK_{\rho o} K_{bst} C K_q^1 K_f b} \quad (3) \quad STF_2 = \frac{A B K_{q2}}{1 + G_1 AB K_{q2} + G_2 BK_{q2}} \quad (4)
\]
To cancel the quantization noise \( Q_{n1} \) introduced in the first stage from the final output, the \( Q_1NTF \) in equation 1 must equal zero; therefore, the digital filter \( D_2 \) is given by:

\[
D_2 = D_1 \frac{NTF_1}{STF_2 \left( NTF_1 \left( K_{S1}K_2 - \frac{KR_1K_2}{K_{q1}} \right) + \frac{KR_1K_2}{K_{q1}} \right)} \tag{5}
\]

The digital filter \( D_1 \) is usually designed to introduce a delay in the path of the first stage to compensate for the mistiming between the loops. In an ideal case, the quantization noise \( Q_{n1} \) that was introduced by the first stage will be cancelled; only the quantization noise \( Q_{n2} \) from the second stage will appear at the modulator output. The value of \( Q_{n2} \) is shaped by the second stage loop filters and the digital filter \( D_2 \). However, equation 5 shows that if we know the \( NTF_1 \) of the first stage and \( STF_2 \) of the second stage, we can easily determine the digital filter \( D_2 \). However, in practice, it is not possible to precisely model these two transfer functions because \( NTF_1 \) is a function of the sensing element and \( STF_2 \) is a function of analogue electronics, both of which are subject to manufacturing tolerances and imperfections. This leads to a mismatch between the digital filter \( D_2 \) and the analogue components. This mismatch causes a leakage of the quantization noise \( Q_{n1} \) in the final output and degrades the modulator performance. Equation 5 can be written in a generic form to design the digital filters for higher order modulators as follows:

\[
D_n = D_{n-1} \frac{NTF_{n-1}}{STF_n \left( NTF_{n-1} \left( K_{S_{n-1}}K_n - \frac{KR_{n-1}K_n}{K_{q_{n-1}}} \right) + \frac{KR_{n-1}K_n}{K_{q_{n-1}}} \right)} \tag{6}
\]

where \( n > 2 \) is an integer representing the loop number. The simulation results in figures 2a and 2b show the noise shaping of the first loop, with a noise floor around -95dB, and the MASH noise shaping, with a noise floor around -130dB, respectively.

Figure 2: Simulation noise shaping analysis, (a) 2nd order sigma-delta accelerometer spectrum indicating a noise floor of -95dB, (b) 4th order MASH accelerometer spectrum indicating a noise floor of -130dB.

Table 1: MEMS accelerometer and system parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ‘m’</td>
<td>1.56μ Kg</td>
<td>Sense cap. ‘Cs’</td>
<td>1.56p F</td>
<td>Sampling freq. ‘Fs’</td>
<td>125k Hz</td>
</tr>
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<td>Damper ‘b’</td>
<td>920μ N/m/s</td>
<td>Feedback cap. ‘Cf’</td>
<td>3.40p F</td>
<td>Feedback voltage ‘Vb’</td>
<td>12 V</td>
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<tr>
<td>Spring ‘k’</td>
<td>88.25 N/m</td>
<td>Pickoff gain ‘Kpo’</td>
<td>1.2e6</td>
<td>KR, KS, K2</td>
<td>0.4, 1, 0.7</td>
</tr>
<tr>
<td>Nominal gap ‘d0’</td>
<td>7.25μ m</td>
<td>Boost gain ‘Kbst’</td>
<td>50</td>
<td>G1, G2</td>
<td>1, 2</td>
</tr>
</tbody>
</table>
3. Hardware Implementation and Experimental Results

The hardware implementation of the fourth order MASH follows the functionality of the Simulink model. A fully differential architecture was considered to minimize common mode effects. In the first loop, the sense capacitance change due to acceleration is detected by the pickoff circuit and converted into a proportional voltage; this is then digitized by the quantizer with a sampling frequency of 125 kHz. Figure 3 shows the PCB circuit with the accelerometer.

![Figure 3: Fourth order MASH PCB and capacitive MEMS accelerometer.](image)

The digital outputs of both loops are transmitted via a USB link to a PC in order to perform a real time digital filtering in Matlab and to obtain the PSD of the filtered bitstreams. The noise shaping of the real measurements is close to the expectations from the simulation analysis, and proves the concept of the MASH architecture for MEMS accelerometer. Figures 4a and 4b show the noise floor of the first stage, which is around -90dB, and the noise floor of the MASH structure, which is around -110dB.

![Figure 4: Experimental noise shaping results, (a) 2nd order sigma-delta accelerometer spectrum indicating a noise floor of -90dB, (b) 4th order MASH accelerometer spectrum indicating a noise floor of -110dB.](image)

4. Conclusions

Clearly, the MASH structure improves the performance of the modulator by 20dB. The performance matches or exceeds single loop architectures. The difference in performance between the simulation and the experimental measurements is due to other noise sources, such as cross-coupling, and will be improved further by shielding.

References


