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Dihedral flavor symmetry from dimensional deconstruction

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To my late father

Abstract

Extra dimension deconstructed on a closed chain has naturally the symmetry of a regular polygon, the dihedral symmetry D_N. We assume that the fields are irreducible representations of the binary dihedral group Q_{2N}, which is the covering group of D_N. It is found that although the orbifold boundary conditions break the dihedral invariance explicitly, the Q_{2N} symmetry appears as an intact, internal global flavor symmetry at low energies. A concrete predictive model based on Q_{6N} with an odd N is given.

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1. Introduction

The Yukawa sector of the standard model (SM) contains a large number of redundant parameters. The presence of the redundant parameters is not related to a symmetry in the SM. That is, they will appear in higher orders in perturbation theory even if they are set equal to zero at the tree level. These redundant parameters may become physical parameters when going beyond the SM, and, moreover, they can induce flavor changing neutral currents (FCNCs) and CP violating phenomena that are absent or strongly suppressed in the SM. One of the most well-known examples is the case of the minimal supersymmetric model (MSSM). Since the SM cannot control the redundant parameters, the size of the new FCNCs and CP violating phases may be unacceptably large unless there is some symmetry, or one fine tunes their values.[1]

A natural guidance to constrain the Yukawa sector and to reduce the redundancy of this sector is a flavor symmetry. It has been recently realized that non-Abelian discrete flavor symmetries, especially dihedral symmetries, cannot only reduce the redundancy,

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[1] For recent reviews, see, for instance, [1] and references therein.

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but also partly explain the large mixing of neutrinos.[2] When supersymmetrized, it has been found that the same flavor symmetries can suppress FCNCs that are caused by soft supersymmetry breaking terms [17,18] (see also [19–23]).

In this Letter we address the question of the origin of dihedral flavor symmetries. We will find that dimensional deconstruction [24,25] is a possible origin of dihedral flavor symmetries.

2. Dihedral invariance in an extra dimensional space

Consider an extra dimension which is compactified on a closed one-dimensional lattice with N sites. We assume that the lattice has the form of a regular polygon with N edges as it is illustrated in Fig. 1.

The regular polygon is invariant under the symmetry operations of the dihedral group D_N. The D_N operations are $2N$ discrete rotations, where N of $2N$ rotations are combined with a parity transformation. Clearly, a discrete polygon rotation of $n \times \theta_N, n \in \{1, \dots, N\}$ corresponds to a discrete translation of the lattice sites of $n \times a$, where a is the lattice spacing and

$$\theta_N \equiv 2\pi/N. \tag{1}$$

The coordinate of the extra dimension is denoted by y, and the N sites are located at $y = y_0, y_1, \dots, y_{N-1}$. ($y_{N+i}$ is identified with y_i.) Under a D_N transformation, the set of coordinates $(y_0, y_1, \dots, y_{N-1})$ changes to $(y'_0, y'_1, \dots, y'_{N-1})$, which we express in terms of a $N \times N$ real matrix. The matrix for the fundamental rotation (i.e., a rotation of θ_N) is given by

$$R_N = \begin{pmatrix} 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ & & \cdots & & \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}, \tag{2}$$

and that for the parity transformation is

$$P_D = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 1 & 0 \\ & & \cdots & & \\ 0 & 1 & 0 & \cdots & 0 \end{pmatrix}. \tag{3}$$

Then the $2N$ group elements of D_N are

$$\mathcal{G}_{D_N} = \{R_N, (R_N)^2, \dots, (R_N)^N = \mathbf{1}, R_N P_D, \\ (R_N)^2 P_D, \dots, (R_N)^N P_D = P_D\}. \tag{4}$$

Using the properties, $P_D^2 = \mathbf{1}$ and $P_D R_N P_D = (R_N)^{-1}$, one can convince oneself that $\mathcal{G}_{D_N}$ is indeed a group.

There exist two-dimensional representations for $\tilde{R}_N$ and $\tilde{P}_D$ [2,12]:

$$\tilde{R}_N = \begin{pmatrix} \cos\theta_N & \sin\theta_N \\ -\sin\theta_N & \cos\theta_N \end{pmatrix}, \\ \tilde{P}_D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{5}$$

which are useful representations in finding irreducible representations (irreps) of D_N (θ_N is given in (1)). It follows that D_N is a subgroup of $SO(3)$, which one sees if one embeds $\tilde{R}_N$ and $\tilde{P}_D$ into 3×3 matrices

$$\tilde{R}_N \to \begin{pmatrix} \cos\theta_N & \sin\theta_N & 0 \\ -\sin\theta_N & \cos\theta_N & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ \tilde{P}_D \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{6}$$

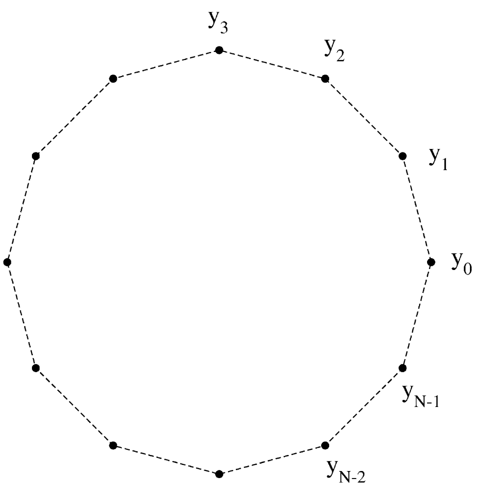

Fig. 1. A regular polygon with $N = 12$ edges, which are located at $y = y_0, y_1, \dots, y_{N-1}$.

[2] Models based on dihedral flavor symmetries, ranging from $D3(\simeq S_3)$ to Q_6 and D_7, have been recently discussed in [2–16].

Therefore, D_N has only real representations.

$SU(2)$ is the universal covering group of $SO(3)$, and has pseudo real and real irreps. Q_{2N} is a finite subgroup of $SU(2)$. It can be interpreted as the covering group of D_N in the sense that the defining matrices $\tilde{R}_{2N}$ and $\tilde{P}_Q$ for Q_{2N} satisfy

$$(\tilde{R}_{2N})^2 = \tilde{R}_N, \qquad (\tilde{P}_Q)^4 = (\tilde{P}_D)^2 = \mathbf{1}, \tag{7}$$

where

$$\tilde{R}_{2N} = \begin{pmatrix} \cos\frac{\theta_N}{2} & \sin\frac{\theta_N}{2} \\ -\sin\frac{\theta_N}{2} & \cos\frac{\theta_N}{2} \end{pmatrix},$$
$$\tilde{P}_Q = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{8}$$

The set of $4N$ elements of Q_{2N} is given by

$$\mathcal{G}_{Q_{2N}} = \{\tilde{R}_{2N}, (\tilde{R}_{2N})^2, \ldots, (\tilde{R}_{2N})^{2N} = \mathbf{1}, \tilde{R}_{2N}\tilde{P}_Q, (\tilde{R}_{2N})^2\tilde{P}_Q, \ldots, (\tilde{R}_{2N})^{2N}\tilde{P}_Q = \tilde{P}_Q\}. \tag{9}$$

There exist only one- and two-dimensional irreps for D_N and Q_{2N}. For Q_{2N}, there are $N-1$ different two-dimensional irreps, which we denote by

$$\mathbf{2}_\ell, \quad \ell = 1, \ldots, N-1. \tag{10}$$

$\mathbf{2}_\ell$ with odd ℓ is a pseudo real representation, while $\mathbf{2}_\ell$ with even ℓ is a real representation, where $\mathbf{2}_\ell$ with even ℓ is exactly $\mathbf{2}_{\ell/2}$ of D_N. Under the fundamental rotation (i.e., a rotation of θ_N which is defined in (1)), $\mathbf{2}_\ell$ transforms with the matrix

$$\tilde{R}_{2N}(\mathbf{2}_\ell) = (\tilde{R}_{2N})^\ell = \begin{pmatrix} \cos(\ell\frac{\theta_N}{2}) & \sin(\ell\frac{\theta_N}{2}) \\ -\sin(\ell\frac{\theta_N}{2}) & \cos(\ell\frac{\theta_N}{2}) \end{pmatrix}. \tag{11}$$

It is straightforward to calculate the Clebsch–Gordan coefficients for tensor products of irreps [12]. There exist four different one-dimensional irreps of Q_{2N}. Because of the relation (7), each of them has a definite Z_4 charge. Further, under the fundamental rotation, they either remain unchanged or change their sign. Therefore, one-dimensional irreps can be characterized according to $Z_2 \times Z_4$ charge:

$$\mathbf{1}_{+,0},\ \mathbf{1}_{-,0},\ \mathbf{1}_{+,2},\ \mathbf{1}_{-,2} \quad \text{for } N = 2, 4, 6, \ldots, \tag{12}$$
$$\mathbf{1}_{+,0},\ \mathbf{1}_{-,1},\ \mathbf{1}_{+,2},\ \mathbf{1}_{-,3} \quad \text{for } N = 3, 5, 7, \ldots, \tag{13}$$

where the $\mathbf{1}_{+,0}$ is the true singlet of Q_{2N}, and only $\mathbf{1}_{-,1}$ and $\mathbf{1}_{-,3}$ are complex irreps. Note that all the real representations of Q_{2N} are exactly those of D_N, which is one of the reasons why we would like to call Q_{2N} as the covering group of D_N.

3. Field theory with the dihedral invariance

Let us now discuss how to construct field theory models with a dihedral invariance. We denote the five-dimensional coordinate by

$$z^M = (x^\mu, y) \quad \text{with } \mu = 0, \ldots, 3. \tag{14}$$

The coordinates y_i of the lattice sites transform to y'_i with $N \times N$ matrices of D_N, which are given in (2) and (3). Then it is natural to assume[3] that the fields defined on the lattice are irreps of Q_{2N} which is the covering group of D_N. That is,[4]

$$\phi(x, y) \to \phi'(x, y) = \tilde{Q}_{2N}\phi(x, \tilde{D}_N^{-1}y), \quad \tilde{Q}_{2N} \in Q_{2N} \text{ and } \tilde{D}_N \in D_N. \tag{15}$$

In Table 1 explicit expressions of the matrices corresponding to the fundamental rotation and the parity transformation are given, where we assume that the gauge fields belong to the true singlet $\mathbf{1}_{+,0}$.

Given the details of the Q_{2N} irreps, it is then straightforward to construct an invariant action [24–26]. Supersymmetrization can also be straightforwardly done [26].

4. Orbifold boundary conditions and Q_{2N} flavor symmetry

In the case of a continuous extra dimension, orbifold boundary conditions are used to suppress unnecessary light fields and also to obtain four-dimensional chiral fields. We shall discuss next how an internal Q_{2N} flavor symmetry can appear even if orbifold boundary conditions break the dihedral invariance (15). Let $\phi(x, y)$ be a generic field which satisfies the periodic boundary condition, $\phi(x, y) = \phi(x, y + Na)$. Then the field $\phi(x, y)$ can be decomposed into

[3] D_N may be understood as a twisted product of Z_N and Z_2. Witten [27] has considered this Z_N (the symmetry of the boundary of a deconstructed disc) to solve the triplet–doublet splitting problem in GUTs.

[4] Non-Abelian discrete family symmetries appearing in extra dimension models of [5,28], for instance, are not directly related to a symmetry of the extra dimension.

Table 1
Explicit expressions of the matrices corresponding to the fundamental rotation (i.e., a rotation of θ_N given in (1)) and the parity transformation. $\tilde{R}_{2N}$, $\tilde{P}_Q$ and $\tilde{P}_D$ are given in (8) and (5), respectively, where $\ell \in \mathbf{N}$ and $\leqslant (N-1)/2$, r = real, c = complex, pr = pseudo real. All the real irreps of Q_{2N} are those of D_N. Complex one-dimensional irreps exist only for $N = 3, 5, 7, \ldots$, while the real one-dimensional irreps $\mathbf{1}_{-,0}$ and $\mathbf{1}_{-,2}$ exist only for $N = 2, 4, 6, \ldots$

Irreps	$\mathbf{1}_{+,0}$	$\mathbf{1}_{+,2}$	$\mathbf{1}_{-,0}$	$\mathbf{1}_{-,1}$	$\mathbf{1}_{-,2}$	$\mathbf{1}_{-,3}$	$\mathbf{2}_{2\ell-1}$	$\mathbf{2}_{2\ell}$
Rotation	1	1	-1	-1	-1	-1	$(\tilde{R}_{2N})^{2\ell-1}$	$(\tilde{R}_{2N})^{2\ell}$
Parity	1	-1	1	i	-1	$-i$	$\tilde{P}_Q$	$\tilde{P}_D$
Reality	r	r	r	c	r	c	pr	r

the cosine and sine modes

$$\phi(x, y) = \frac{\phi(x)}{\sqrt{N}} + \sum_{i=1}^{i_{\max}} \phi_{+,i}(x)\cos(k_i y) + \sum_{i=1}^{i'_{\max}} \phi_{-,i}(x)\sin(k_i y), \tag{16}$$

where

$$\phi(x) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \phi(x, y_n), \tag{17}$$

$$k_i = \frac{2\pi i}{aN}, \quad i \in \mathbf{N},$$

$$i_{\max} = \begin{cases} i'_{\max} + 1 = N/2 - 1 \\ i'_{\max} = (N-1)/2 \end{cases} \quad \text{for} \quad \begin{cases} \text{even } N, \\ \text{odd } N. \end{cases} \tag{18}$$

$\phi(x)$ is the zero mode. As in the continuous case, we can drop the cosine or sine modes by imposing an appropriate boundary condition: under the parity transformation (3), i.e.,

$$y_0 \to y'_0 = y_0, \qquad y_1 \to y'_1 = y_{N-1}, \qquad \ldots,$$
$$y_i \to y'_i = y_{N-i}, \qquad \ldots, \tag{19}$$

the zero mode $\phi(x)$ and the cosines modes are even, while the sine modes are odd.

Since the D_N transformation mixes the cosine and sine modes, the orbifold boundary conditions break the dihedral invariance explicitly. However, the Q_{2N} invariant construction of an action discussed in the previous section ensures that the Q_{2N} invariance remains intact as a global, internal symmetry. This is because there is no derivative with respect to y is used in the construction. So, the theory with orbifold boundary conditions is invariant under the internal transformation

$$\phi(x, y) \to \phi'(x, y) = \tilde{Q}_{2N}\phi(x, y), \quad \tilde{Q}_{2N} \in Q_{2N}, \tag{20}$$

which should be compared with (15). The internal symmetry is nothing but a global flavor symmetry based on Q_{2N}.

5. An example

In what follows, we would like to discuss a concrete model. One of the successful ansätze for the quark mass matrices is of a nearest neighbor interaction (NNI) type [29–31]

$$M = \begin{pmatrix} 0 & C & 0 \\ \pm C & 0 & B \\ 0 & B' & A \end{pmatrix}. \tag{21}$$

In [12] it has been proposed to derive the mass matrix (21) solely from a dihedral symmetry, and concluded that two conditions should be met: (i) There should be real as well as pseudo real nonsinglet representations, and (ii) there should be the up and down type Higgs $SU(2)_L$ doublets (type II Higgs). The smallest finite group that allows both real and pseudo real nonsinglet representations is Q_6 as we have seen. Further, the Higgs sector of the MSSM fits the desired Higgs structure. Therefore, we assume supersymmetry in four dimensions. The $D_3(S_3)$ model of [6] with a Z_2 symmetry in the leptonic sector is one of the most predictive models for the leptonic sector. However, the Z_2 symmetry in the quark sector is broken, so that the Z_2 symmetry should be seen as an approximate symmetry in that model. It was found, however, that this leptonic sector can be reproduced in a supersymmetric Q_6 model without introducing an additional discrete symmetry into the leptonic sector [12]. In Table 2 we write the Q_6 assignment of the quark and lepton chiral supermultiplets.[5]

[5] The same model exists for Q_{2N} if N is odd and a multiple of 3.

Table 2

Q_6 assignment of the matter supermultiplets. Q, Q_3, L, L_3 and H^u, H_3^u, H^d, H_3^d stand for $SU(2)_L$ doublets supermultiplets for quarks, leptons and Higgs bosons, respectively. Similarly, $SU(2)_L$ singlet supermultiplets for quarks, charged leptons and neutrinos are denoted by U^c, U_3^c, D^c, D_3^c, E^c, E_3^c and N^c, N_3^c. This is an alternative assignment to the one given in the footnote of [12]. The present assignment can more suppress the proton decay [32]. The assignment for the mirror supermultiplets can be simply read off from Table 2

	Q	$U^c, D^c, L, E^c, N^c, H^u, H^d$	Q_3	$U_3^c, D_3^c, H_3^u, H_3^d$	L_3, E_3^c	N_3^c
Q_6	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{1}_{+,2}$	$\mathbf{1}_{-,1}$	$\mathbf{1}_{+,0}$	$\mathbf{1}_{-,3}$

We impose the following orbifold boundary conditions: all the mirror chiral supermultiplets are odd under the parity transformation (19). Similarly, the $N=1$ chiral supermultiplets, which are the $N=2$ superpartners of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge supermultiplets, are also odd. It is then clear that the zero modes of the gauge, matter and Higgs supermultiplets coincide with those of the supersymmetric Q_6 model of [12], and hence it is the low energy effective theory. The low energy Yukawa superpotential W_Y is given by

$$W_Y = W_Q + W_L, \tag{22}$$

where[6]

$$\begin{aligned} W_Q = {} & Y_a^u Q_3 U_3^c H_3^u + Y_b^u Q^T \sigma_1 U_3^c H^u \\ & - Y_{b'}^u Q_3 U^{cT} i\sigma_2 H^u + Y_c^u Q^T \sigma_1 U^c H_3^u \\ & + Y_a^d Q_3 D_3^c H_3^d + Y_b^d Q^T \sigma_1 D_3^c H^d \\ & - Y_{b'}^d Q_3 D^{cT} i\sigma_2 H^d + Y_c^d Q^T \sigma_1 D^c H_3^d, \end{aligned} \tag{23}$$

$$\begin{aligned} W_L = {} & Y_c^e f^{IJK} L_I E_J^c H_K^d + Y_{b'}^e L_3 \big(H_1^d E_1^c + H_2^d E_2^c\big) \\ & + Y_b^e \big(L_1 H_1^d + L_2 H_2^d\big) E_3^c + Y_a^\nu L_3 N_3^c H_3^u \\ & + Y_c^\nu f^{IJK} L_I N_J^c H_K^u \\ & + Y_{b'}^\nu L_3 \big(H_1^u N_1^c + H_2^u N_2^c\big), \end{aligned} \tag{24}$$

and $f^{122} = f^{212} = f^{222} = -f^{111} = 1$. In [12] it has been found that by introducing a certain set of gauge singlet Higgs supermultiplets it is possible to construct a Higgs sector in such a way that CP phases can be spontaneously induced. Therefore, all the parameters appearing in the Lagrangian including the soft supersymmetry breaking (SSB) sector are real. Consequently, no CP violating processes induced by SSB terms are possible in this model, satisfying the most stringent experimental constraint coming from the EDM of the neutron and the electron [35]. Since the Higgs sector is also Q_6 invariant, it is straightforward to derive it from dimensional deconstruction. Consequently, the quark sector contains only 8 real parameters with one independent phase to describe the quark masses and their mixing, and the leptonic sector contains only 6 real parameters with one independent phase to describe 12 independent physical parameters. Predictions in the $|V_{ub}|$–$\sin 2\phi_1$ planes are shown in Fig. 2, while Fig. 3 shows the predictions in the $\sin 2\phi_1$–ϕ_3 planes.

As we can see from Figs. 2 and 3, with accurate measurements of the Cabibbo–Kobayashi–Maskawa matrix elements, the predictions could be tested.

The predictions in the leptonic sector are summarized as follows[7]:

(1) Inverted neutrino mass spectrum, i.e., $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$.
(2) $m_{\nu_2}^2/\Delta m_{23}^2 = \frac{(1+2t_{12}^2+t_{12}^4-rt_{12}^4)^2}{4t_{12}^2(1+t_{12}^2)(1+t_{12}^2-rt_{12}^2)\cos^2\phi_\nu} - \tan^2\phi_\nu$ $(r = \Delta m_{21}^2/\Delta m_{23}^2, t_{12} = \tan\theta_{12})$, where ϕ_ν is an independent phase.
(3) $\sin\theta_{13} \simeq m_e/\sqrt{2}m_\mu \simeq 3.4 \times 10^{-3}$ and $\tan\theta_{23} \simeq 1-(m_e/\sqrt{2}m_\mu)^2 = 1 - O(10^{-5})$.
(4) The prediction of $\langle m_{ee}\rangle$ is shown in Fig. 4.

We emphasize that the smallness of $\sin\theta_{13}$ and the almost maximal mixing of the atmospheric neutrinos are consequences of the Q_6 flavor symmetry. The value of $\sin\theta_{13}$ in the present model may be too small to be measured in a laboratory experiment [41], but the

[6] The Higgs sector of the model of [12] possesses a permutation symmetry $H_1^{u(d)} \leftrightarrow H_2^{u(d)}$, which ensures the stability of the VEV $\langle H_1^{u(d)}\rangle = \langle H_2^{u(d)}\rangle$. The resulting mass quark matrices are equivalent to (21). The leptonic sector given in [6] can be obtained by the interchange $1 \leftrightarrow 2$.

[7] Large mixing of neutrinos may be obtained in dimensional deconstruction models in a different mechanism. See, for instance, [5,33,34].

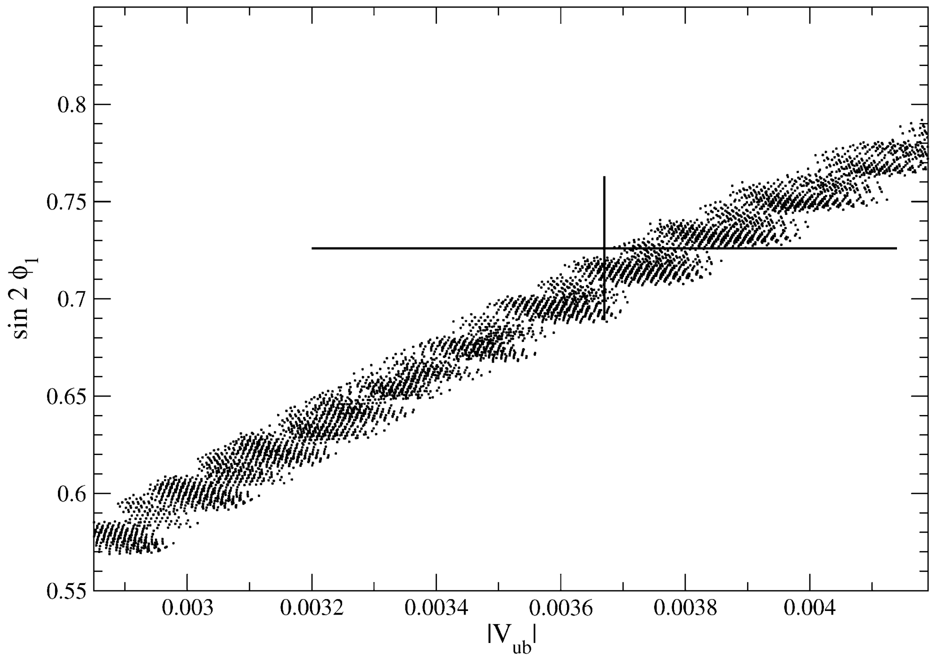

Fig. 2. Predictions in the $|V_{ub}|$–$\sin 2\phi_1$ plane. The uncertainties result from those in the quark masses and in $|V_{us}|$ and $|V_{cb}|$, where we have used $|V_{us}| = 0.2240 \pm 0.0036$ and $|V_{cb}| = (41.5 \pm 0.8) \times 10^{-3}$ [36]. The vertical and horizontal lines correspond to the experimental values, $\sin 2\beta(\phi_1) = 0.726 \pm 0.037$ and $|V_{ub}| = (36.7 \pm 4.7) \times 10^{-4}$ [37,38].

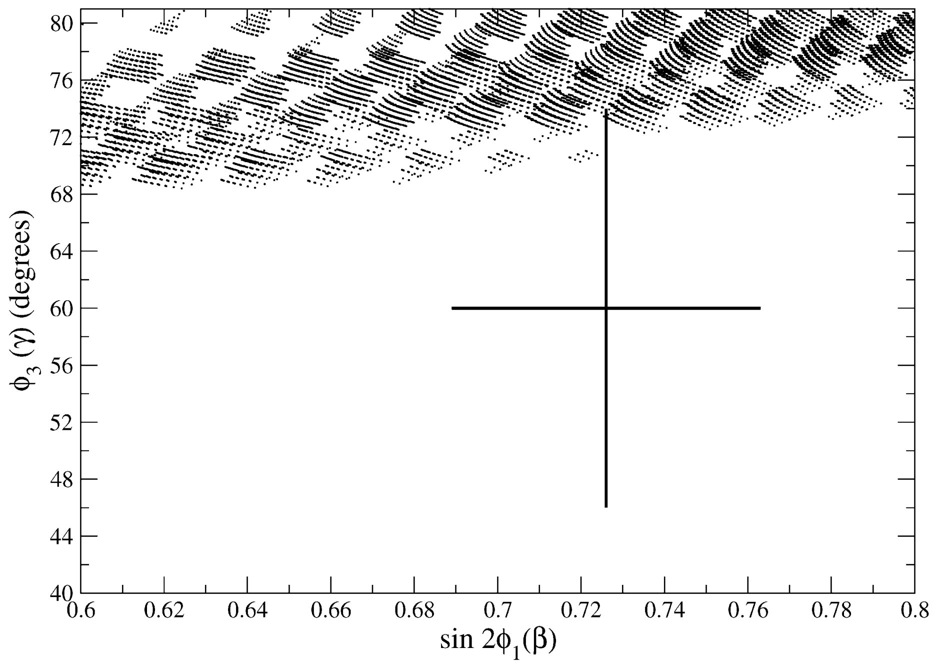

Fig. 3. Predictions in the $\sin 2\phi_1$–ϕ_3 plane. The vertical and horizontal lines correspond to the experimental values, $\sin 2\phi_1(\beta) = 0.726 \pm 0.037$ and $\phi_3 = (60^\circ \pm 14^\circ)$ [37,38].

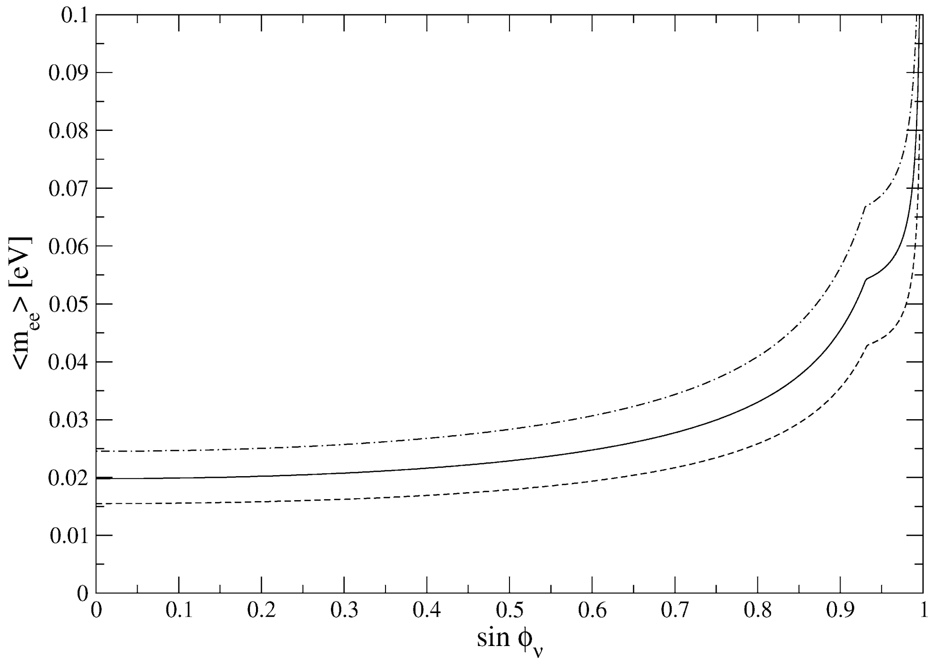

Fig. 4. The effective Majorana mass $\langle m_{ee} \rangle$ as a function of $\sin\phi_\nu$ with $\sin^2\theta_{12} = 0.3$ and $\Delta m^2_{21} = 6.9 \times 10^{-5}$ eV^2 [39]. The dashed, solid and dot-dashed lines stand for $\Delta m^2_{23} = 1.4$, 2.3 and 3.0×10^{-3} eV^2, respectively.

tiny deviation from zero ($\sin^2\theta_{13} \simeq m_e^2/2m_\mu^2 \simeq 10^{-5}$) are important in supernova neutrino oscillations [40].

6. Conclusion

In this Letter we have looked for a possible origin of dihedral symmetries. It has been recently realized that a flavor symmetry based on a dihedral group can be used to soften the flavor problem of the SM and the MSSM. We have considered an extra dimension compactified on a closed chain, which is assumed to have the form of a regular polygon. Since the symmetry group of the regular polygon is the dihedral group D_N, we assumed that the fields are irreps of the covering group of D_N, which is the binary dihedral group Q_{2N}. The construction of an action with the dihedral invariance is straightforward, and moreover we found that the Q_{2N} symmetry remains as an intact, internal flavor symmetry even if the original dihedral invariance is broken by orbifold boundary conditions. We hope that with our finding we can come closer to a deep understanding of the origin of a flavor symmetry based on a non-Abelian finite group.

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