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Fibonacci oscillators in the Landau diamagnetism problem

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HIGHLIGHTS

- We apply (q_1, q_2)-deformation algebra of Fibonacci oscillators to Landau diamagnetism.
- We investigate the q-deformed magnetization and magnetic susceptibility.
- The results revealed that q-deformation acts as a factor of impurity.

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ABSTRACT

We address the issue of the Landau diamagnetism problem via q-deformed algebra of Fibonacci oscillators through its generalized sequence of two real and independent deformation parameters q_1 and q_2. We obtain q-deformed thermodynamic quantities such as internal energy, number of particles, magnetization and magnetic susceptibility which recover their usual form in the degenerate limit $q_1^2 + q_2^2 = 1$.

1. Introduction

The Landau diamagnetism problem continues to play a role in several issues of many physical systems and has strong relevance today [1–5]. The diamagnetism can be used as an illustrative phenomenon that plays essential role in quantum mechanics on the surface, the perimeter, and the dissipation of statistical mechanics of non-equilibrium.

In this paper, we are interested in investigating this phenomenon in q-deformed algebra in order to understand impurities effects in, for example, magnetization and susceptibility. The magnetic susceptibility is an intrinsic characteristic of a material and its identity is related to the atomic and molecular structure. In Ref. [6] it was performed the calculation of susceptibility for electrons moving in a uniform external magnetic field, developing Landau diamagnetism, by applying the nonextensive Tsallis statistics [7–10], which is a strong candidate for solving problems where the standard thermodynamics is not applicable — see also Ref. [11] for a similar study using another method. Of course, other noncommutative deformations can be applied, for example q-deformation via Jackson derivative (JD) [12].

The study of quantum groups and quantum algebras has attracted great interest in recent years, stimulated intense research in various fields of physics [13,14], taking into account a range of applications, covering cosmology and condensed matter, e.g. black holes, fractional quantum Hall effect, high-temperature (high-T$_c$) superconductors [15], rational field

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theories, noncommutative geometry, quantum theory of super-algebras and so on [16]. Furthermore, statistical and thermodynamic properties by studying q-deformed physical systems have been intensively investigated in the literature [17–31].

Another important discussion is about the main reasons to consider *two* deformation parameters in some different physical applications. Starting from the generalization of the q-algebra [32], in Ref. [33] it was generalized the Fibonacci sequence, which is a well-known linear combination where the third number is the sum of two predecessors and so on. Here, the numbers are in that sequence of generalized Fibonacci oscillators, where new parameters (q_1, q_2) are introduced [33–35]. They provide a unification of quantum oscillators with quantum groups, keeping the degeneration property of the spectrum invariant under the symmetries of the quantum group. The quantum algebra with two deformation parameters may have a greater flexibility when it comes to application in the concrete phenomenological physical models [36,37], and may increase interest in physical applications.

The paper is organized as follows. In Section 2 we introduce the q-deformed algebra. In Section 3 we develop the (q_1, q_2)-deformed Landau diamagnetism problem and in Section 4 we make our final comments.

2. Fibonacci oscillators algebra

We consider a system of generalized oscillators now entering two parameters in statistical distribution function, whose energy spectrum may be determined by Fibonacci's generalized sequence [33–35]. This will establish a statistical system depending on the deformation parameters (q_1, q_2), allowing us to calculate the thermodynamic quantities in the limit of high temperatures.

The q-deformed quantum oscillator is now defined by the Heisenberg algebra in terms the annihilation and creation operators in c, $c^\dagger$, respectively, and the number operator N [19,35], as follows:

$$c_i c_i^\dagger - K q_1^2 c_i^\dagger c_i = q_2^{2n_i} \quad \text{e} \quad c_i c_i^\dagger - K q_2^2 c_i^\dagger c_i = q_1^{2n_i}, \tag{1}$$

$$[N, c^\dagger] = c^\dagger, \qquad [N, c] = -c, \tag{2}$$

where $K = \pm 1$, stands for bosons and fermions, respectively. In addition, the operators also obey the relations

$$c^\dagger c = [N], \qquad c c^\dagger = [1 + KN], \tag{3}$$

$$[1 + K n_{i,q_1,q_2}] = K q_1^2 [n_i] + q_2^{2n_i}, \quad \text{or} \quad [1 + K n_{i,q_1,q_2}] = K q_2^2 [n_i] + q_1^{2n_i}. \tag{4}$$

The Fibonacci *basic number* is defined as [33]

$$[n_{i,q_1,q_2}] = c_i^\dagger c_i = \frac{q_2^{2n_i} - q_1^{2n_i}}{q_2^2 - q_1^2}, \tag{5}$$

The q-Fock space spanned by the orthonormalized eigenstates $|n\rangle$ is constructed according to

$$|n\rangle = \frac{(c^\dagger)^n}{\sqrt{[n]!}}|0\rangle, \qquad c|0\rangle = 0, \tag{6}$$

The actions of c e $c^\dagger$ and N on the states $|n\rangle$ in the q-Fock space are known to be

$$c^\dagger|n\rangle = [n+1]^{1/2}|n+1\rangle, \tag{7}$$

$$c|n\rangle = [n]^{1/2}|n-1\rangle, \tag{8}$$

$$N|n\rangle = n|n\rangle. \tag{9}$$

To calculate the q-deformation statistical occupation number, we begin with the Hamiltonian of q-deformed noninteracting oscillators (bosons or fermions) [16],

$$\mathcal{H}_{q_1,q_2} = \sum_i (\epsilon_i - \mu_{q_1,q_2}) N_i, \tag{10}$$

where μ_{q_1,q_2} is the (q_1, q_2)-deformed chemical potential. It should be noted that this Hamiltonian is a two-parameter deformed Hamiltonian and depends implicitly on the deformation parameters q_1 and q_2, since the number operator is deformed via Eq. (5).

The mean value of the (q_1, q_2)-deformed occupation number can be calculated by

$$[n_i] \equiv \langle [n_i] \rangle = \frac{tr(\exp(-\beta\mathcal{H}) c_i^\dagger c_i)}{\Xi}, \tag{11}$$

$$[n_{i,q_1,q_2}] = \frac{z'\left(\exp(\beta\epsilon_i) - z'\right)}{\left(\exp(\beta\epsilon_i) - q_2^2 z'\right)\left(\exp(\beta\epsilon_i) - q_1^2 z'\right)}, \tag{12}$$

where $z_{q_1,q_2} = \exp(\beta\mu_{q_1,q_2})$ is the fugacity of the system, and we shall use the notation $z_{q_1,q_2} = z'$. When $q_1 = q_2 = 1$, we find the usual form

$$n_i = \frac{1}{z^{-1}\exp(\beta\epsilon_i) - 1}. \tag{13}$$

In the present application of the Fibonacci oscillators, we are interested in obtaining new (q_1, q_2)-deformed thermodynamic quantities such as internal energy, magnetization, and magnetic susceptibility for the high-temperature case, i.e. the limit $(z \ll 1)$.

3. Fibonacci oscillators in the Landau diamagnetism

To explain the phenomenon of diamagnetism, we have to take into account the interaction between the external magnetic field and the orbital motion of electrons. Disregarding the spin, the Hamiltonian of a particle of mass m and charge e in the presence of a magnetic field $\mathbf{H}$ is given by the expression [38]

$$\mathcal{H} = \frac{1}{2m}\left(p - \frac{e}{c}\mathbf{A}\right)^2, \tag{14}$$

where $\mathbf{A}$ is the vector potential associated with the magnetic field $\mathbf{H}$ and c is the speed of light in *CGS* units. Let us start to formalize the statistical mechanical problem by using the grand partition function with the parameters q_1 and q_2 inserted through Eq. (12), in the form

$$\ln \Xi = -K\frac{2eHL^2}{hc}\sum_n^{\infty}\frac{L}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k_z \frac{1}{(q_1^2 - q_2^2)}\Big\{\ln\Big[1 - Kz'q_1^2\exp(-\beta\epsilon)\Big](q_1^{-2} - 1) + \ln\Big[1 - Kz'q_2^2\exp(-\beta\epsilon)\Big](1 - q_2^{-2})\Big\}, \tag{15}$$

where $k_z = -\infty, \ldots, \infty$, $\epsilon = \frac{\hbar k_z^2}{2m} + \hbar\omega\left(n + \frac{1}{2}\right)$, $\omega = \frac{eH}{mc}$. However, our study is focused on the analysis of diamagnetism in the limit of high temperatures $(z' \ll 1)$. Thus, performing the sum and integrals, we find the partition function is written as follows:

$$\ln \Xi = \frac{z'K^2HC_1}{\sinh(\gamma)} + \frac{z'^2K^3HC_2Q}{2\sinh(2\gamma)}, \quad \text{where } C_1 = \frac{eL^3}{2\pi hc\lambda},\ C_2 = \frac{eL^3}{2\pi\sqrt{2}hc\lambda} \tag{16}$$

being $\lambda = \frac{\hbar}{(2\pi m\kappa_B T)^{\frac{1}{2}}}$ the thermal wavelength, $\gamma = \beta\mu_B H$ and $Q = q_1^2 + q_2^2 - 1$.

We note that Eq. (16) shows the (q_1, q_2)-deformation in the second term. In the first order does not appear q-deformation. It appears after considering at least the second order. Notice, however, the case $(q_1 = q_2 = 1)$, *as expected, does recover the underformed thermodynamic quantities up to a second order correction which are usually disregarded. On the other hand, taking computation up to second order corrections is necessary to get the effects of the q-deformation and as a consequence only in the unit circle on the* (q_1, q_2)*-space, i.e.,*

$$q_1^2 + q_2^2 = 1 \tag{17}$$

the deformation ceases. The case in Eq. (17) shows an interesting degeneration on the (q_1, q_2)-space. Deformations show up as $q_1^2 + q_2^2 < 1$ or $q_1^2 + q_2^2 > 1$. In former case appears the possibility of finding some unexpected negative thermodynamic quantities such as negative specific heat. In the following we shall consider the latter case to calculate the q-deformed thermodynamic quantities of interest in the present study.

3.1. (q_1, q_2)-deformed thermodynamic quantities

We obtain the number of particles N by setting,

$$N = z'\frac{\partial}{\partial z'}\ln \Xi = \frac{z'K^2HC_1}{\sinh(\gamma)} + \frac{z'^2K^3HC_2Q}{2\sinh(2\gamma)}. \tag{18}$$

We determine the internal energy, and we can write it in terms of N, in the form

$$U = -\frac{\partial}{\partial\beta}\ln \Xi = \frac{N\mu_B H\Big[C_1\coth(\gamma)\sinh(2\gamma) + z'KC_2\sinh(\gamma)\coth(2\gamma)Q\Big]}{C_1\sinh(2\gamma) + z'KC_2\sinh(\gamma)Q}. \tag{19}$$

In Fig. 1 we have the behavior of internal energy U as a function of the magnetic field $\mathbf{H}$ and for some values of q_1 and q_2 — see caption. We note that all the curves have different maximum peaks (depending on the values adopted for q_1 and

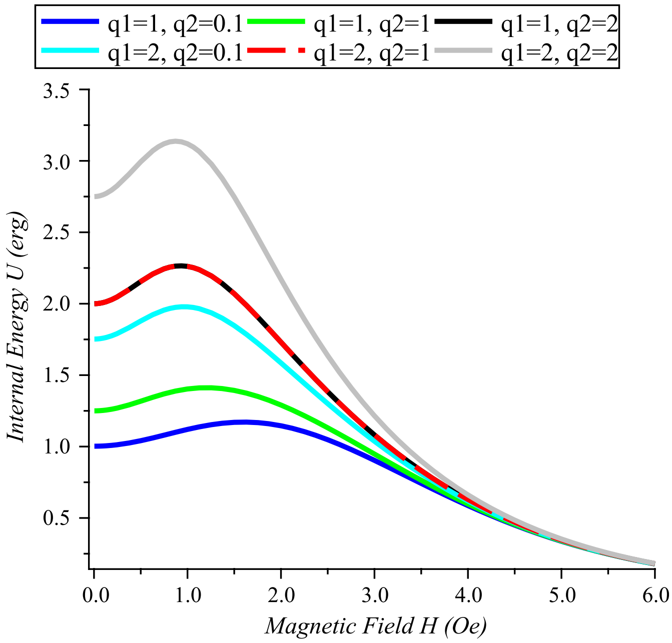

Fig. 1. (q_1, q_2)-deformed internal energy as a function of magnetic field **H** for several choices of q_1 and q_2.

q_2), for small magnetic field **H**. The curves exhibit the same behavior asymptotically. We also have proven the symmetry between the oscillators, i.e. when $q_1 = 1$ and $q_2 = 2$ (black curve) and when $q_1 = 2$ and $q_2 = 1$ (red curve), they overlap. An expected effect due to the symmetry of q_1 and q_2 defined in Q.

The grand potential ϕ is determined as

$$\phi = -\frac{1}{\beta}\ln \Xi = -\left(\frac{z'K^2HC_1}{\beta\sinh(\gamma)} + \frac{z'^2K^3HC_2Q}{2\sinh(2\gamma)}\right). \quad (20)$$

To determine the magnetization, we carried out the thermodynamic derivative by using Eq. (20), that gives

$$M = -\frac{\partial\phi}{\partial H} = \frac{z'C_1K^2\,(1-\gamma\coth(\gamma))}{\beta\sinh(\gamma)} - \frac{z'C_2K^3Q\,(1-\gamma\coth(2\gamma))}{2\beta\sinh(2\gamma)}. \quad (21)$$

We can also eliminate the chemical potential through the number of particles N and insert the Langevin functions

$$\mathcal{L}(\gamma) = \coth(\gamma) - \frac{1}{\gamma}, \qquad \mathcal{L}(2\gamma) = \coth(2\gamma) - \frac{1}{2\gamma}, \quad (22)$$

to rewrite the magnetization as

$$M = -\frac{N\mu_B\left[C_1\sinh(2\gamma)\mathcal{L}(\gamma) + z'KC_2\gamma\sinh(\gamma)\mathcal{L}(2\gamma)\right]}{C_1\sinh(2\gamma) + z'KC_2\sinh(\gamma)Q}. \quad (23)$$

The results obtained for the deformed magnetization are very interesting, because we can compare it with experimental results obtained for superconducting materials (which are perfect diamagnetic materials) as a function of temperature variation [39], in order to strength the understanding of the q-deformation as a factor of impurity. In Ref. [39] one was found that the minimum of magnetization deepens as temperature or pressure decreases.

In Fig. 2 we have the magnetization curves (M) versus magnetic field (**H**) for some values of q_1 and q_2, and we note that some observations made for internal energy such as oscillators symmetry are also valid for the magnetization, as expected. Notice that the minimum of magnetization deepens as q-deformation *increases*. This means that by increasing temperature or pressure we may diminish the effects of disorders or impurities of the system. This explains why we should reduce the deformation parameters until they assume the underformed degenerate case $q_1^2 + q_2^2 = 1$.

Now, computing the susceptibility reads,

$$\chi = \frac{\partial M}{\partial H} = \frac{N\beta\mu_B^2}{C_1\sinh(2\gamma) + z'KC_2\sinh(\gamma)Q}\Bigg[C_1\sinh(2\gamma)\Big(2\coth(\gamma)\mathcal{L}(\gamma) - 1\Big)$$
$$+ 2z'C_2\sinh(\gamma)Q\Big(2\coth(2\gamma)\mathcal{L}(2\gamma) - 1\Big)\Bigg]. \quad (24)$$

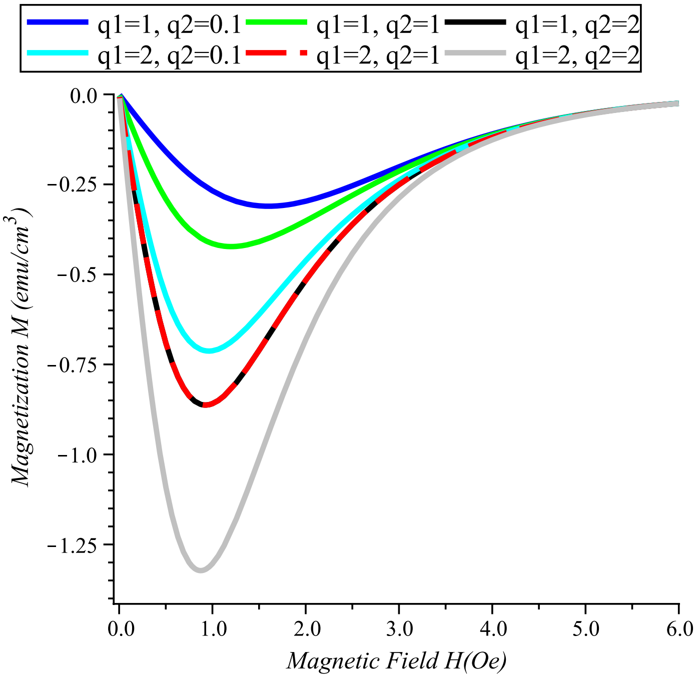

Fig. 2. (q_1, q_2)-deformed magnetization as a function of magnetic field $\mathbf{H}$ for several choices of q_1 and q_2.

In the limit of weak fields $\gamma \ll 1$ we have the leading term

$$M = -\frac{2N\mu_B \sinh(\gamma)\cosh(\gamma)(C_1 + z'C_2KQ)}{3(2C_1\cosh(\gamma) + z'KC_2Q)}, \tag{25}$$

and thus, we have the susceptibility in zero field

$$\chi_0 = -\frac{2\mu_B z'K^2(C_1 + z'C_2KQ)}{3(2C_1 + z'KC_2Q)}. \tag{26}$$

4. Conclusions

As in our previous works [12,40,41], in which we have shown that the q-parameter is associated with impurities in a sample, in particular diamagnetic materials, as in the present study, we put forward new results to strength this interpretation of the q-deformation.

In this work, we expand the application of q-calculation through two deformation parameters (q_1, q_2), known as Fibonacci oscillators. We work in the limit of high temperatures ('dilute gas' $z \ll 1$), and a (q_1, q_2)-deformed partition function. In first order the results reported in the literature [38,42], are recovered. However, the q-deformation takes place at second order for non-degenerate case $q_1^2 + q_2^2 > 1$.

We note that in the obtained results were found several interesting behaviors by just varying the values of q_1 and q_2. Of course, we performed a theoretical application, and it allows various assumptions. By comparing these results with similar experimental curves, one could understand how impurities could be entering into a material that affects, e.g., superconductivity such that its critical temperature increases, which would be of great interest to those who works with high T_c superconductors — see Ref. [43] for a recent alternative theoretical investigation on these type of superconductors whose structure can be extended via q-deformation in order to introduce impurities.

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