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# Two loop renormalization of the magnetic coupling in hot QCD

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## Abstract

Well above the critical temperature hot QCD is described by 3d electrostatic QCD with gauge coupling  $g_E$  and Debye mass  $m_E$ . We integrate out the Debye scales to two loop accuracy and find for the gauge coupling in the resulting magnetostatic action  $g_M^2 = g_E^2 \left(1 - \frac{1}{48} \frac{g_E^2 N}{\pi m_E} - \frac{17}{4608} \left(\frac{g_E^2 N}{\pi m_E}\right)^2 + O\left(\left(\frac{g_E^2 N}{\pi m_E}\right)^3\right)\right)$ .  
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## 1. Introduction

Notable progress [1] in the standard model at high  $T$  is due to the systematic separation of perturbative scales like the temperature, the Debye mass  $gT$  and the non-perturbative scale  $g^2 T$ . It is the latter that is three-dimensional and can be treated numerically on the lattice and has given us a wealth of information on the plasma state of the standard model and QCD itself. In this Letter we will be concerned with QCD, but we will admit for  $N$  instead of three colours.

For small gauge coupling  $g$  one can integrate out the integer Kaluza–Klein modes  $2\pi nT$  and obtain a static effective action,  $S_E$ , with a running coupling  $g_E(T)$ . This action is three-dimensional, and its degrees of freedom are three-dimensional Yang–Mills and a massive adjoint Higgs with as mass the Debye scale  $gT$ . The scale  $g^2 T$  appears as the coupling in the three-dimensional Yang–Mills action. The

Debye scale can be integrated out when  $g \ll 1$ . We are then left with the magnetic action  $S_M$ . It is the three-dimensional Yang–Mills theory which describes physics at the magnetic scales  $g^2 T$ . Its coupling  $g_M^2$  is a function of the parameters in  $S_E$  and can be calculated perturbatively. This has up to now been done to one loop order [4]. In this Letter we report on the computation of the two loop effects.

## 2. Motivation

Our motivation stems from the need for accuracy. More precisely, integrating out effects of the K–K modes one obtains from the original action of QCD the superrenormalizable action  $S_E$ :

$$\begin{aligned} \mathcal{L}_E = & \text{Tr}(\vec{D}(A)A_0)^2 + m_E^2 \text{Tr} A_0^2 + \lambda_E (\text{Tr}(A_0^2))^2 \\ & + \bar{\lambda}_E \left( \text{Tr}(A_0)^4 - \frac{1}{2} (\text{Tr} A_0^2)^2 \right) \\ & + \frac{1}{2} \text{Tr} F_{ij}^2 + \delta \mathcal{L}_E. \end{aligned} \quad (1)$$

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This action density describes the physics of the QCD plasma down to temperatures  $T \sim 2T_c$ , including non-perturbative effects from the magnetic density  $\text{Tr} F_{ij}^2$ . These have been calculated by lattice methods [2]. The terms neglected,  $\delta\mathcal{L}_E$ , introduce an error of  $O(g^4)$  [3]. So the parameters in this action have all evaluated up to this order.

The magnetic action takes the form

$$\mathcal{L}_M = \frac{1}{2} \text{Tr} F_{ij}^2 + \delta\mathcal{L}_M \quad (2)$$

with a magnetostatic gauge coupling  $g_M^2$ .

Now the neglected terms introduce an error [3] of  $O(g^3)$ , and hence the magnetic coupling has to be computed to  $O(g^2)$  accuracy. The calculation is reported on in the next section.

### 3. Renormalization of the magnetic gauge coupling

The basic idea behind the effective actions Eqs. (1) and (2) is that one can compute with both in the region of momenta  $p \sim g^2 T$ . To know what the parameters of the latter are in terms of those of the former requires computing two-point functions, three point functions, etc. in both theories and match them. In the matching the diagrams of the pure 3d Yang–Mills theory drop out.

Here we will follow a well-known shortcut [5] by introducing a background field  $B_i$  in  $\mathcal{L}_E$ :

$$\vec{A} = \frac{1}{g_E} \vec{B} + \vec{Q}, \quad A_0 = g_E Q_0. \quad (3)$$

We calculate the fluctuations around the background in a path integral:

$$\exp\left(-\frac{1}{g_M^2} S_M(B)\right) = \int DQ_0 DQ_i \exp\left(-S_E - \frac{1}{\xi} \text{Tr}(D_i Q_i)^2\right). \quad (4)$$

We added a general background gauge term. The resulting action  $S_M(B)$  is gauge invariant to all loop orders and the renormalization of the coupling is identified from the background field two point function at a momentum  $p = O(g^2 T)$ . This momentum is the infrared cut-off in computing the r.h.s. of Eq. (4). With

dimensional regularization one finds in  $d$  dimensions, dropping the pure Yang–Mills diagrams as mentioned before:

$$\begin{aligned} & \exp\left(-\frac{1}{g_M^2} S_M(B)\right) \\ &= \exp\left(-\frac{1}{g_E^2} S_M(B)\right) \\ & \quad \times (1 + (F_1^{\text{tr}} + F_2^{\text{tr}} + \dots) S_M(B)). \end{aligned} \quad (5)$$

Here the  $F_i^{\text{tr}}$  are the transverse parts of the background two-point functions  $F_i$  as shown for the two loop case in Fig. 1.

Let the sum of all Feynman diagrams for the two point function of the background field with  $i$  loops be  $F_i$ . Then we can write

$$F_i = F_i^{\text{tr}}(\delta_{lm} p^2 - plpm) + F_i^L plpm. \quad (6)$$

The longitudinal part is zero because of gauge invariance. It is borne out by explicit calculation. Transversality is true for all  $d$  and values of the parameters.

The  $F_i^{\text{tr}}$  are still depending on the parameters  $\epsilon = \frac{3-d}{2}$ ,  $g_E$ ,  $m_E$ ,  $\xi$ , the regularization scale  $\bar{\mu}$  and the momentum  $p$ . We are interested in the limits  $d = 3$  and  $\tilde{p} = 0$  where  $\tilde{p} = \frac{p}{m_E}$ . In that limit we expect because of the superrenormalizability the UV poles and the  $\bar{\mu}$  dependence to cancel. Also the  $\xi$  dependence should disappear. And so should the IR effects in the guise of inverse powers of the momentum  $p$ . And indeed they do by explicit calculation, as the table in the next section shows.

This leaves us with the relation, using Eq. (5)

$$\frac{1}{g_M^2} = \frac{1}{g_E^2} - F_1^{\text{tr}} - F_2^{\text{tr}}, \quad (7)$$

with

$$g_E^2 F_1^{\text{tr}} = -\frac{1}{48} \frac{g_E^2 N}{\pi m_E}, \quad (8)$$

$$g_E^2 F_2^{\text{tr}} = -\frac{19}{4608} \left(\frac{g_E^2 N}{\pi m_E}\right)^2. \quad (9)$$

This is the main result.

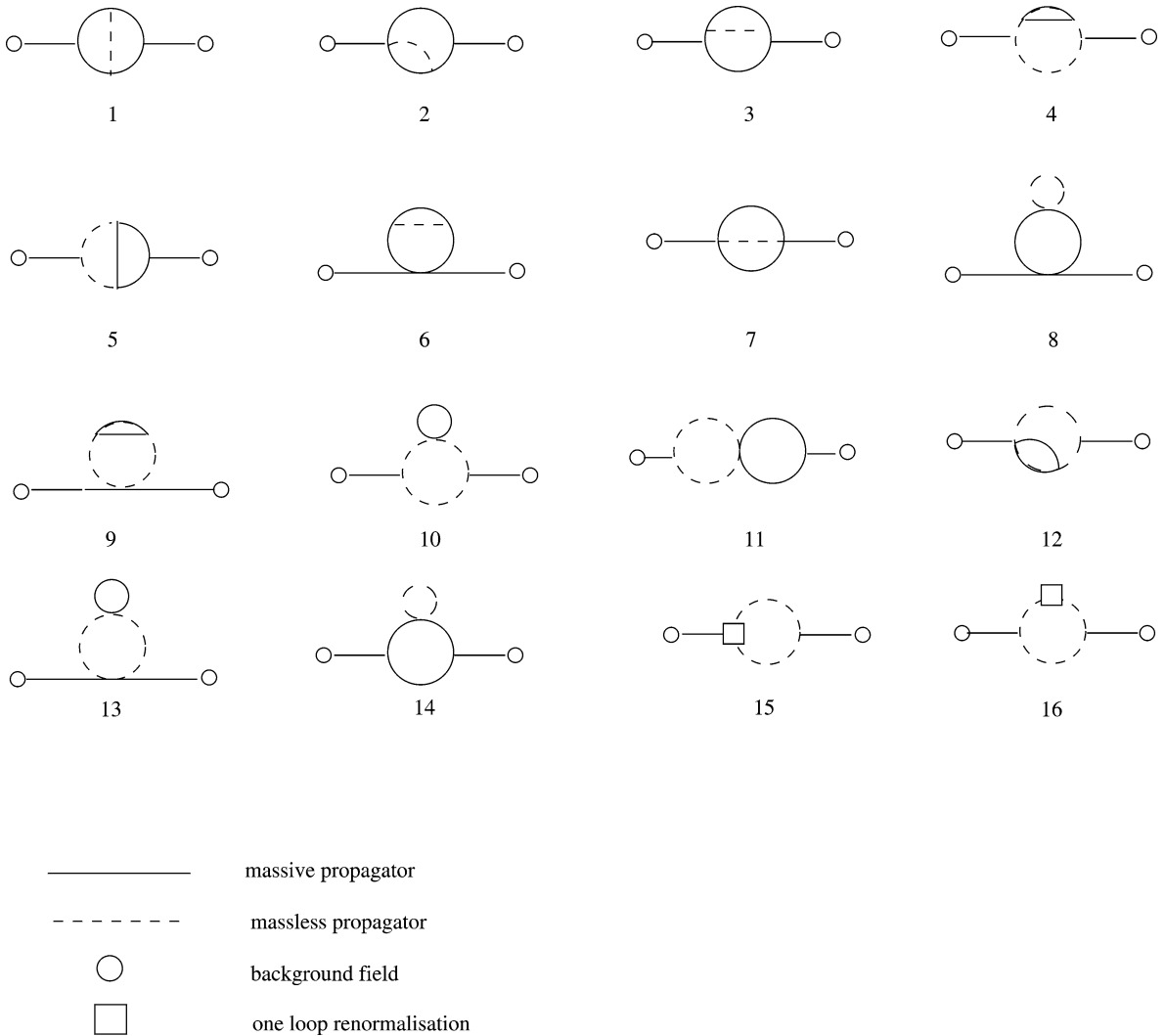


Fig. 1. Two-loop Feynman graphs for the background two-point function.

#### 4. Details of the calculation

The two loop diagrams involving at least one massive propagator are shown in Fig. 1. Also shown (graphs 15 and 16) are the insertions, discussed in Ref. [5]. They are vital for the gauge parameter independence of our result. FORM [7] was used for algebraic manipulations and secularization of integrals with reducible numerators. The program TARCER [6] served to scalarize those integrals with irreducible denominators. At that point the result  $F_2$  is expressed

in terms of eight scalar integrals. For  $d = 3$ , the values of these scalar integrals are computed in Ref. [8]. For the expansion of these integrals up to  $\epsilon$ , methods as in Ref. [8] were used and results were checked with Ref. [9].

First we checked the transversality, i.e.,  $F_2^L = 0$ . The reader can find the result for  $F_2^{\text{tr}}$  in Table 1. Individual graphs have UV and IR divergencies that do cancel when summed. The finite part is indeed gauge parameter independent, though the physically irrelevant  $O(p^2)$  terms are not.

Table 1

Graph No.	$p^2 F_2^{\text{tr}}$
1	$-\frac{\tilde{p}^2(29+24\xi)}{9216\pi^2} + \frac{1+\xi+\epsilon(2+\xi-4(1+\xi)\log(2))+4\epsilon(1+\xi)\log(\frac{\tilde{\mu}}{m})}{128\epsilon\pi^2}$
2	$\frac{\tilde{p}^2(20+6\xi)}{1536\pi^2} - \frac{3(2+\xi-2\epsilon(-1+\xi(-1+\log(4))+\log(16))+4\epsilon(2+\xi)\log(\frac{\tilde{\mu}}{m}))}{128\epsilon\pi^2}$
3	$\frac{\tilde{p}^2(-43+12\xi)}{4608\pi^2} + \frac{4+\epsilon(1+\xi-16\log(2))+16\epsilon\log(\frac{\tilde{\mu}}{m})}{64\epsilon\pi^2}$
4	$\frac{-2-\xi}{32\tilde{p}\pi} - \frac{\tilde{p}(15+\xi)}{1536\pi} + \frac{\tilde{p}^2(13+2\xi)}{3072\pi^2} - \frac{3(1+\xi)^2+4\epsilon(-8+\xi)\xi\log(\frac{\tilde{\mu}}{m})+4\epsilon(3+2\xi(7+\xi))\log(\frac{\tilde{\mu}}{2m})}{768\epsilon\pi^2\xi}$
5	$\frac{\tilde{p}(15+\xi(4+\xi))}{1536\pi} - \frac{\tilde{p}^2(23+12\xi)}{4608\pi^2} + \frac{2\epsilon+\xi+4\epsilon\xi\log(\frac{\tilde{\mu}}{2m})}{128\epsilon\pi^2}$
6	$\frac{-3}{(64e\pi^2)} + \frac{22-24(1+\log(\frac{\tilde{\mu}}{2m}))}{128\pi^2}$
7	$\frac{6(2+\xi)+\epsilon(12(1+\xi)-\tilde{p}^2(2+\xi))+24\epsilon(2+\xi)\log(\frac{\tilde{\mu}}{2m})}{512\epsilon\pi^2}$
8	0
9	$\frac{1+2\xi}{256\epsilon\pi^2\xi} + \frac{-2\xi-4(1+2\xi)-4(-1-2\xi)(1+\log(\frac{\tilde{\mu}}{2m}))}{256\pi^2\xi}$
10	$\frac{16+8\xi}{256\tilde{p}\pi}$
11	0
12	0
13	0
14	0
15	$-\frac{\tilde{p}(1+\xi)(4+\xi)}{1536\pi}$
16	$\frac{\tilde{p}(2+\xi)}{768\pi}$
Sum	$\frac{-19\tilde{p}^2}{4608\pi^2}$

## 5. Conclusions

Our main result, Eq. (9), shows that the smallness of the corrections to the magnetic coupling does persist in two loop order. In fact, at  $2T_c$  the coupling for 3 colours equals [2]  $g_E^2 = 2.7$  and the 2 loop correction is about a third of the one loop correction (itself about 3 percent).

Our result is of importance in analyzing the purely magnetic quantities, as the spatial Wilson loop, and the magnetic mass. In particular it is crucial in connecting the lattice results from the magnetic action to those obtained from the electric action, and ultimately to those of four-dimensional simulations. This will be done in a future publication [10].

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