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# Fractal nature of highway traffic data

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**Abstract**

In this paper, we applied a fractal approach to analyze the traffic data collected from the Beijing Yuquanying. The power spectrum, the empirical probability distribution function, the statistical moment scaling function and the autocorrelation function are used as indicators to investigate the presence of the fractal. The results from the fractal identification methods indicate that these data exhibit fractal behavior. A fractal framework seemed well suited for description of the data observed here, but its suitability for general traffic systems was not clear.


*Keywords:* Fractal; Power spectrum; Probability distribution function; Statistical moment scaling function; Autocorrelation function

## 1. Introduction

Traffic flow often exhibits irregular and complex behavior [1–6]. It changes abruptly when entering or leaving a congestion zone. Its current state and future evolution depend greatly on myriad properties of interacting, often highly variable physical and human elements. Proper representation of all dynamics in a model is complex: while certain relationships can be developed through analysis, incorporation of immeasurable quantities, such as laws and social codes, creates further complications.

The presence of chaotic phenomena in traffic models has been reported in recent studies. Prigogine and Herman [7] modeled traffic integrating statistical mechanics with individual choice behavior and showed it to exhibit a high degree of complexity. Disbro and Frame [8] demonstrate how the theoretically derived, Gazis, Herman and Rothery (GHR) traffic model [9] is highly chaotic, even when applied to small (eight-car) systems. Van Zuylen et al. discussed the implications of human behavior, chaos and unpredictability for urban and transportation planning and forecasting. Safanov et al. [10] showed that chaotic behavior in traffic can due to the delays in human reaction. Shang et al. [4] indicated that chaotic characteristics exist in the traffic system and techniques based on phase space dynamics can be used to predict the traffic speed. And Weidlich [11] demonstrated how random-utility-based models of relatively

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simple social behaviors produced chaotic behavior, which offers a detailed application of chaos to traffic flow data.

The outcomes of such studies, on the one hand, nearly unanimously indicated the presence of a chaotic nature in traffic and, hence, the possibility of transformation of traffic data from one scale to another. On the other hand, most of the studies have also revealed the insufficiency of the chaos approaches to characterize the traffic process. These papers were motivated by a hypothesis that transportation systems are often chaotic. The difficulty with this hypothesis is that chaos theory presumes system determinism. Since transportation systems involve humans, weather, and other possibly random agents, such an assumption is not easy to justify. Thus, chaos theory may not apply well. Also, while chaos may exist at a small level, it may be neither discernible nor of (apparent) practical significance. This raises the issue of the usefulness of chaos theory to practitioners.

Unlike the above models/studies, we employ the fractal approaches directly to investigate traffic flow. The theoretical basis of fractal approaches for traffic flow is the assumption that the variability of the traffic process could be directly modeled as a stochastic (or random) turbulent cascade process [12–16]. A cascade process is generally described as eddies breaking up into smaller sub-eddies, each of which receives a part of the flux of its parent body. It is important to note, however, that though there have been several attempts to justify their applicability to traffic flow by an analogy with the energy cascade process in fully developed turbulence, such stochastic cascade approaches are purely phenomenological. Therefore, whether or not the traffic process is a stochastic cascade remains an unanswered question. Nevertheless, the following are possible reasons for the validity of stochastic fractal approaches for traffic flow: (1) the belief that the traffic process and the breaking-up of eddies are stochastic; and (2) stochastic cascade processes generically yield fractals.

The purpose of the present study is to investigate the presence of fractal behaviors in the traffic time series so as to investigate the possibility of a fractal approach. The traffic time series observed on the Beijing Yuquanying highway are analyzed, and the study deals with fractal analyses only in time. The existence of fractal is investigated by employing four methods: (1) the power spectrum method; (2) the empirical probability distribution function method; (3) the statistical moment scaling function method; and (4) the autocorrelation function method, which is employed to investigate the existence of long-range dependence and self-similarity, specified in terms of a Hurst parameter, in traffic flow.

The organization of this paper is as follows. First, a brief review of the methods employed in the present study to identify the presence of fractals in a time series is given in Section 2. Details of the data used, analyses carried out and results achieved are presented in Section 3. Finally, important conclusions drawn from this study are provided in Section 4.

## 2. Proposed methodology

Research over the past two decades has led to the development of a wide variety of methods for identifying fractals in a time series [3,12,17–21]. However, before applying any fractal identification method, general information about the fractal behavior may be obtained by using standard statistical descriptions, such as the power spectrum and the empirical probability distribution function. These two approaches are, therefore, employed in the present study. In addition, the statistical moment scaling function can provide important information regarding the existence of a fractal and its type, whether mono-fractal or multi-fractal, and, therefore, this is also employed. Finally, the autocorrelation function method is employed to investigate the existence of long-range dependence and self-similarity in traffic flow. In this section, brief accounts of the four methods are provided.

### 2.1. Power spectrum

The power spectrum is a standard tool in fractal investigations in nonlinear time series [17,20,22,23]. When all or part of the spectrum obeys the power law form

$$E(f) \propto f^{-\delta} \tag{1}$$

where $f$ is the frequency and $\delta$ is an exponent, called the spectral exponent, the data are scaling in that range, that is, the scaling regime. The absence of characteristic timescales indicates that multi-fractal behavior may be assumed to hold. Also, the power spectrum is particularly useful for studying the oscillations of a process. In general, for a random process, the power spectrum oscillates randomly about a constant value, indicating that no frequency explains any more of the variance of the sequence than any other frequency. For a periodic or quasi-periodic sequence, only peaks at certain frequencies exist; measurement noise adds a continuous floor to the spectrum. Thus, in the spectrum, signal and noise are readily distinguished. Chaotic signals may also have sharp spectral lines, but even in the absence of noise there will be a continuous part of the spectrum.

### 2.2. Empirical probability distribution function

The empirical probability distribution function (PDF) of a time series might describe the fractal nature of the intensity thresholds of the time series fluctuations at a given scale, generally the scale corresponding to the measurement resolution. If, for high-intensity threshold values $x$, the tail of the probability distribution of the time series $X$ follows a power law of the form

$$\Pr(X > x) \propto x^{-D} \tag{2}$$

where $D$ is the probability exponent, then the series is characterized by hyperbolic intermittency [17]. This is a general, but not necessary, feature of multi-fractal processes [22]. In fact, a value of $D < 2$ indicates that mono-fractal models may be sufficient to characterize the traffic behavior, whereas multiplicative models might be necessary if $D > 2$[17,24]. From (2) it can be deduced that moments of order greater than or equal to $D$ are divergent.

### 2.3. Statistical moment scaling method

To investigate the scaling behavior of the time series, the variation of empirical moments with scale is examined. This analysis is a common technique for studying scaling properties of data [25] and has been applied to hydrography by, for example, Over and Gupta [26]. Among the existing multi-fractal identification methods, the statistical moment scaling method [26–29] is the most widely used one for nonlinear time series. In this method, the time series is divided into non-overlapping intervals of a certain time resolution. The ratio of the maximum scale of the field to this interval is termed the 'scale ratio' $k$. Thus $k$ is inversely proportional to the size of the scale examined. For different scale ratios $k$, the average intensity $\varepsilon(k, i)$, in each interval $i$ is computed and raised to the power $q$, and subsequently summed to obtain the statistical moment $M(k, q)$:

$$M(k, q) = \sum_i \varepsilon(k, i)^q. \tag{3}$$

Since values of $\varepsilon(k, i) = 0$ do not contribute to the moment, and these terms are undefined when raised to $q = 0$, they were excluded from the computation. For a scaling field, the moment $M(k, q)$ relates to the scale ratio $k$, as

$$M(k, q) = k^{\theta(q)} \tag{4}$$

where $\theta(q)$ may be regarded as a characteristic function of the fractal behavior. If $\theta(q)$ *versus* $q$ is a straight line the data set is mono-fractal. On the other hand, if $\theta(q)$ *versus* $q$ is a convex (concave) function, the data set is multi-fractal [21,27].

### 2.4. Autocorrelation function

The autocorrelation function is useful in determining the degree of dependence present in the values of a time series, separated by an interval called the lag time $\tau$. In general, the autocorrelation function of a random process fluctuates about zero, indicating that the process at any instance of time has no memory of the past at all. The autocorrelation function of a periodic process is also periodic, indicating the strong relation between values that repeat over and over again. For a chaotic process, the autocorrelation function decays exponentially with increasing lag, because the points are not independent of each other and self-similarity is present [30].

A phenomenon that is self-similar looks the same or behaves the same when viewed at different degrees of "magnification" or different scales on a dimension. This dimension may be space (length, width) or time.

Let $X = (X_t; t = 1, 2, \ldots)$ be a covariance stationary stochastic process; that is a process with constant mean $\mu = E(X_t)$, finite variance $\sigma^2 = E[(X_t - \mu)^2] > 0$, and an autocorrelation function

$$R(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{E[(X_t - \mu)^2]}, \quad \tau = 0, 1, 2, \ldots. \tag{5}$$

Let us assume that $X$ has an autocorrelation function of the form

$$R(\tau) \sim \tau^{-\beta} L_1(\tau), \quad \tau \to \infty \tag{6}$$

where $0 < \beta < 1$ and $L_1$ is slowly varying at infinity, that is, $\lim_{t\to\infty} L_1(tx)/L_1(t) = 1$ for all $x > 0$. For each $m = 1, 2, \ldots$, let

$$X^{(m)} = (X_k^{(m)}; k = 1, 2, \ldots)$$

denote a new time series obtained by averaging the original series $X$ over non-overlapping blocks of size $m$. That is, for each $m = 1, 2, \ldots, X^{(m)}$ is given by

$$X_k^{(m)} = \frac{X_{km-m+1} + \cdots + X_{km}}{m}, \quad k = 1, 2, \ldots. \tag{7}$$

The process $X$ is called (exactly second-order) self-similar [30] with a self-similarity parameter (or the Hurst parameter) $H = 1 - \beta/2$ if the corresponding aggregated processes $X^{(m)}$ have the same correlation structure, i.e.

$$R^{(m)}(\tau) = R(\tau), \quad m = 1, 2, \ldots, \tau = 1, 2, \ldots. \tag{8}$$

$X$ is called (asymptotically second-order) self-similar [30] with self-similarity parameter (or the Hurst parameter) $H = 1 - \beta/2$ if

$$R^{(m)}(1) \to 2^{1-\beta} - 1, \quad m \to \infty$$
$$R^{(m)}(\tau) \to \frac{1}{2}\delta^2(\tau^{2-\beta}), \quad \tau = 2, 3, \ldots, m \to \infty$$

where $\delta^2(g)$ denotes the second central difference operator applied to a function $g$.

Self-similarity is the property we associate with fractals — the object appears the same regardless of the scale at which it is viewed. It is manifested in the absence of a natural length of a "burst"; at every timescale ranging from a few minutes to hours and days, bursts consisting of bursty subperiods separated by less bursty subperiods.

Several methods are commonly used for measuring the self-similarity of traffic flow. We shall mention two important ones here, $R/S$ plots and variance–time plots [30].

First we deal with $R/S$ plots. Let $X_k : k = 1, 2, \ldots, n$, be a set of $n$ observations which have an expectation value (sample mean) $E[X(n)]$; the scaled, adjusted range is given by

$$\frac{R(n)}{S(n)} = \frac{\max(0, W_1, \ldots, W_n) - \min(0, W_1, \ldots, W_n)}{S(n)}$$

where $S(n)$ is the standard deviation and for each $k : 1, 2, \ldots, n$, $W_k$ is given by

$$W_k = (X_1 + X_2 + \cdots + X_k) - kE[X(n)], \quad k = 1, 2, \ldots, n.$$

The Hurst parameter is given by the equation $R(n)/S(n) \sim cn^H$. By taking *logs* we get

$$\log_{10}(R(n)/S(n)) \sim \log_{10} c + H \log_{10} n.$$

Therefore the gradient of a plot of $\log_{10}(R/S)$ against $\log_{10} n$ is the Hurst parameter.

A second method for calculation is a variance–time plot. Again, let $X_k$ be a series of observations for $k = 1, 2, \ldots, n$. If we take a sample of $m$ points, $\mathrm{Var}[X^{(m)}] \sim m^{-\beta}$. Therefore, the gradient of a plot of $\log_{10}[\mathrm{Var}(X^{(m)})]$ against $\log_{10} m$ will be $-\beta$ and the Hurst parameter can be found using the equation $H = 1 - \beta/2$.

The Hurst exponent is also directly related to the spectral exponent $\delta$, which is defined in Section 2.1. The relationship between the Hurst exponent $H$ and the spectral exponent $\delta$ is equation $\delta = 2H - 1$ (see [30,31]).

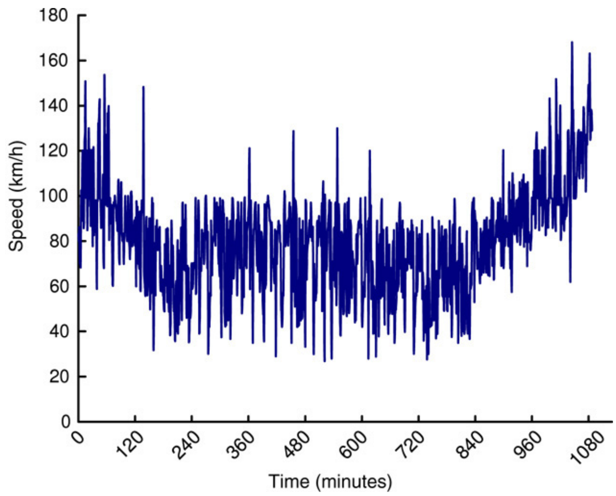


Fig. 1. Time series plot of the speed data observed at the Beijing Yuquanying (from 5:30 AM to 11:30 PM on 1/12/2004).

## 3. Results and analyses

### *3.1. Traffic data*

We use the data observed on the Beijing Yuquanying highway over a period of about 40 months, from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004 (24 h for each day). During this period the traffic data set was chosen not to be biased due to road constructions or bad weather conditions. The data were downloaded from the Highway Performance Measurement Project (HPMP) run by Beijing STONG Intelligent Transportation System Co. Ltd, Beijing. The raw data for speed, volume and occupancy are collected every twenty seconds for each lane of the instrumented highway locations. The double loops are placed on the highway every half a mile, for both the main lanes and the on–off ramps. Raw data are screened for errors, and then aggregated into one-minute data for average speed. We analyze mainly speed data during free flow conditions (which amounts to about 85% of all data) when stationarity can be assumed.

Fig. 1 shows the time series plot of the speed data observed at the Beijing Yuquanying over a period of 18 h (from 5:30 AM to 11:30 PM on 1/12/2004). A visual inspection of the speed series only indicates significant variations in speed, but this is not enough to say anything regarding the existence of fractals. Hence, additional tools are necessary to identify the possible presence of fractals.

### *3.2. Power spectrum*

We now attempt to empirically estimate the scaling ranges of the traffic speed time series, *observed at the Beijing Yuquanying* from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004 (24 h for each day). This is conveniently done by calculating the power spectrum $E(f)$ (in one dimension, the ensemble average of the square of the Fourier amplitudes as a function of the frequency $f$) and estimating the spectral exponent $\delta$ in the scaling relation $E(f) \propto f^{-\delta}$. Although the spectrum is defined as an ensemble average quantity, if the series is long enough, it may be possible to get some idea of the scaling range and exponent on individual realizations.

The power spectrum of a time series, through the spectral exponent $\delta$, can be used to obtain important information regarding the cascade which results in a process, in addition to identifying the presence or absence of characteristic timescales, i.e. a fractal nature. For instance, a necessary, but not sufficient, condition for the process to be a direct result of an unbounded cascade is that $\delta < 1$.

Fig. 2 shows the power spectrum $E(f)$ of the speed time series in the range 2 min to 1500 h, observed at the Beijing Yuquanying. The spectrum has been averaged over logarithmically spaced frequency intervals. The spectrum appears to consist of a power law interval in the range 2 min to 18 h. But the straight-line approximation may be extended up to 25 h with a reasonable fit (the red, straight regression line) with $\delta = 0.68$. Because the scaling turned out to hold for this larger interval it has been used in the moment analysis (see Fig. 4). The regression line for frequencies

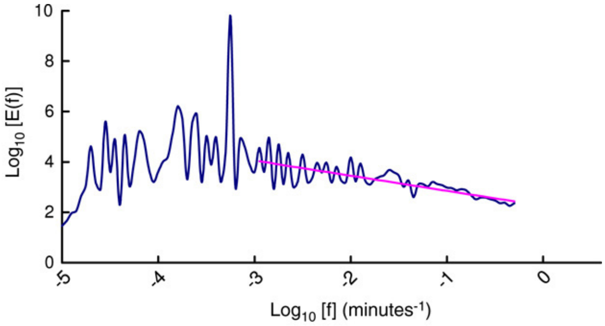


Fig. 2. Power spectrum for the speed data observed at Beijing Yuquanying (from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

larger than 18 h appears to be almost flat ($\delta \approx 0$), but is interrupted by a large peak in the spectrum corresponding to the strong periodic changes in the daily cycle. The presence of a scaling regime, shown by the red line in Fig. 2, may be considered as an indication of fractal behavior. The value of $\delta$, calculated from the slope of the red line, is approximately 0.68 and, therefore, an unbounded cascade model may be sufficient to represent the traffic flow. It should be noted, however, that the identification of the scaling regime(s) and the estimation of the spectral exponent(s) depend on individual judgment and, therefore, discrepancies and uncertainties are unavoidable. As a result, extreme caution is needed while using the power spectrum to identify the existence of fractals in a time series. Similarly, the power spectrum shown in Fig. 2 is not sufficient for identifying whether the traffic flow is chaotic or stochastic, as chaotic processes may also show a continuous part of the spectrum even in the absence of noise.

### 3.3. Probability distribution function

The empirical PDF obtained for the traffic speed time series observed at the Beijing Yuquanying (from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004 (24 h for each day)) is shown in Fig. 3. The figure shows that the traffic data exhibit a hyperbolic tail behavior with a probability exponent $D$, estimated from the fitted regression line, of about 6. Since the $D$ value obtained for the traffic time series is greater than 2.0, a mono-fractal model might be insufficient to model the traffic behavior, as it cannot accommodate a value of $D > 2$. This suggests that a multiplicative model, which can accommodate any value of $D$, might be necessary to model the traffic behavior observed at the Beijing Yuquanying [17,24].

Apparently, the study of the PDF is indicative of a different kind of fractality (such as referring to the Levy scale [31]). Mandelbrot [32] and Taqqu et al. [31] showed that for self-similar superposition processes, the probability exponent $D$ is related to the self-similarity parameter $H$ (see Section 3.5).

An alternative approach to estimating the probability exponent $D$, used in [30], is the Hill estimator [33]. The Hill estimator does not give a single estimate of $D$, but can be used to gauge the general range of $D$ values that are reasonable. We used the Hill estimator to check that the estimates of $D$ obtained using the fitted regression line method in log–log plots were within range.

### 3.4. Statistical moment scaling method

The validity of Eq. (4) may be directly tested on the traffic speed time series from the Beijing Yuquanying. Each series' intensity is averaged over successively doubled time intervals corresponding to successively halved values of the scale ratio $k$, and for each $k$ the $q$th statistical moment is calculated according to Eq. (3). By plotting $M(k, q)$ as a function of $k$ in a log–log diagram, the scaling behavior of the time series as expressed by Eq. (4) may be investigated. If Eq. (4) is valid the curve will exhibit an approximately linear behavior with a slope that is an estimate of $\theta(q)$. By performing the procedure for different values of $q$, the whole $\theta(q)$ function may be estimated. Owing to uncertainties in the estimation of high-order moments [25], the investigation is limited to orders in the range $0.5 \leq q \leq 6.5$. Fig. 4 shows $M(k, q)$ as a function of $k$ in a log–log diagram for the traffic speed time series (from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004) observed at the Beijing Yuquanying, for $0.5 \leq q \leq 6.5$. The curves in Fig. 4 exhibit a well-defined straight-line behavior over the whole scaling region for $0.5 \leq q \leq 5.5$. However, the moments of higher

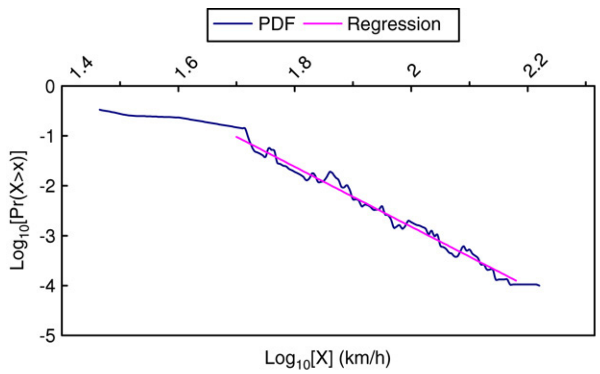


Fig. 3. Empirical probability distribution function, $\Pr(X > x)$, for the speed time series from the Beijing Yuquanying (from 5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004).

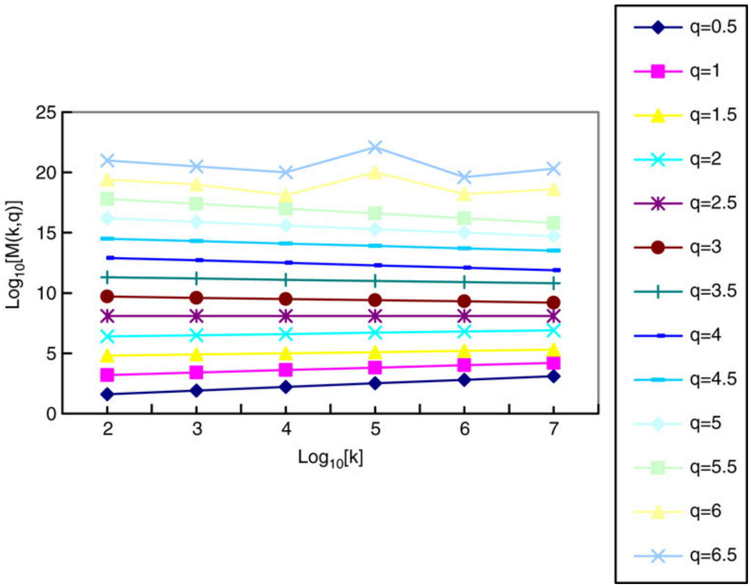


Fig. 4. The $q$th statistical moment, $M(k, q)$, as a function of the scale ratio, $k$, in the scaling regime for the speed time series from the Beijing Yuquanying.

order do not follow a straight line for the time series. In addition, it can be noted that the higher the order of the moment, the less linear the character of $M(k, q)$ is.

The relationship between $\theta(q)$ and $q$ is shown in Fig. 5. The $\theta(q)$ function was estimated from the slope of the fitted regression line for the traffic time series. The $\theta(q)$ function is shown for moment orders up to 5.5 for the time series. As seen in the figure, the $\theta(q)$ *versus* $q$ function is a concave curvature, rather than a straight line, indicating that the traffic data are multi-fractal. The $\theta(q)$ function is slightly curved for $0.5 \leq q \leq 5.5$ but becomes more linear as $q$ increases.

### *3.5. Autocorrelation function*

The degree of self-similarity of traffic flow, defined via the Hurst parameter $H$, can be used to measure the "burstiness" of the traffic flow. As $H$ increases the degree of self-similarity is increasing ($0.5 < H < 1$). For $H = 0.5$ and $H > 1$, there is almost no self-similarity.

If a self-similar process has observable bursts on all timescales, it is said to exhibit long-range dependence [30]; values at any instant are typically correlated with value at future instants.

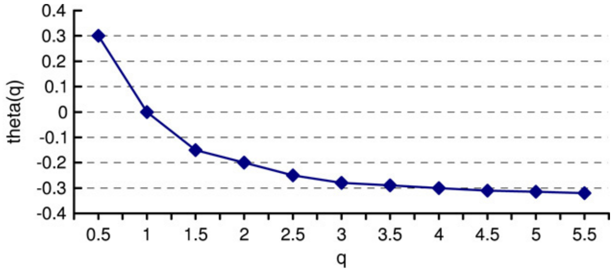


Fig. 5. Estimated $\theta(q)$ function for the speed time series from the Beijing Yuquanying.

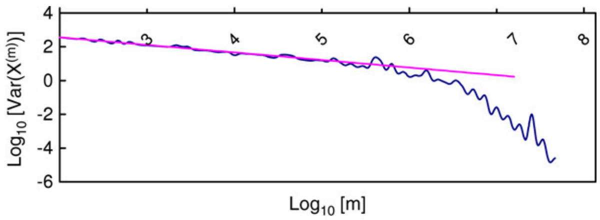


Fig. 6. The variance–time plot for the speed time series from the Beijing Yuquanying.

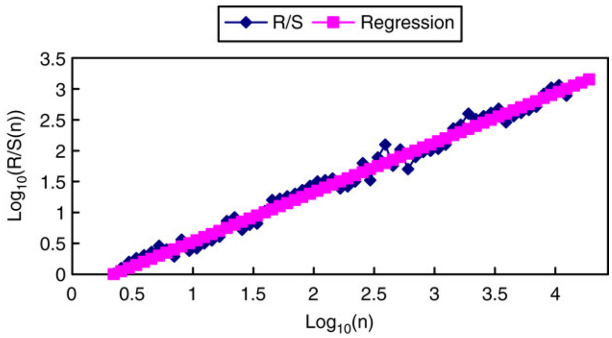


Fig. 7. The $R/S$ pox plot. The plot tightly clusters around a straight line whose asymptotic slope clearly lies between the slopes 0.5 and 1.0.

The Hurst parameter $H$ can be derived from $\beta$ in the equation $H = 1 - \beta/2$. Note that $H$ is always in the range $0 < H < 1$. The Hurst parameter is an important measure of long-range dependence. If $H = 1/2$ then a data set has independent data — an example of such a data set would be Brownian motion where points in the time series depend only on their nearest neighbor. If $H > 1/2$ then the data set has long-range dependence. That is (in loose terms) data which are spatially separated in the series are likely to be correlated. If $H < 1/2$ then the data set has negative long-range dependence, that is to say, in loose terms data which are spatially separated in the series are likely to have a negative correlation.

We give here the variance–time plots (Fig. 6) for the speed time series from the Beijing Yuquanying (5:30 AM on 1/16/2001 to 11:30 PM on 6/17/2004). The so-called variance plots are obtained by plotting $\log(\mathrm{Var}(X(m)))$ against $\log(m)$ and by fitting a simple least squares line through the resulting points in the plane, ignoring the small and big values for $m$ (because in such cases sample size becomes too small for obtaining reliable estimates of the variance). Values of the estimate $\hat{\beta}$ of the asymptotic slope between $-1$ and 0 suggest self-similarity, and an estimate for the degree of self-similarity is given by $\hat{H} = 1 - \hat{\beta}/2$. The curve shows an asymptotic slope that is distinctly different from $-1$ and is easily estimated to be about $-0.32$, resulting in an estimate $\hat{H}$ of the Hurst parameter $H$ of about $\hat{H} = 0.84$.

Estimating the Hurst parameter directly from the corresponding pox plots of $R/S$ (Fig. 7) leads to a practically identical with variance–time plot estimate; the value of the asymptotic slope of the $R/S$ plot is clearly between $1/2$ and 1, with a simple least squares fit resulting in the same estimate: $\hat{H} = 0.84$.

Finally, the Hurst parameter $H$ can be computed from the spectral exponent $\delta$, which is defined in Section 2.1, in the equation $H = \frac{1+\delta}{2}$. In Section 3.2, we have a spectral exponent estimate of $\delta = 0.68$, yielding an identical estimate ($\hat{H} = 0.84$) to the methods above. Therefore, not only the relationship $\delta = 2H - 1$ between the parameter $H$ and the exponent $\delta$ but also the accuracy of our estimates is validated, as we might expect.

## 4. Conclusion

Applications of the ideas gained from fractal theory to characterize nonlinear time series have been among the most exciting areas of research in recent times. This paper presented a detailed analysis of fractal properties of the highway speed time series. The data came from millions of measurements collected on China's highways. Our analysis aimed to examine whether this data set possesses any fractal structure or whether it has rather a stochastic character. The existence of a fractal nature was investigated using the power spectrum, the empirical probability distribution function, statistical moment scaling methods, and the autocorrelation function, which is employed to investigate the existence of self-similarity and long-range dependence, specified in terms of a Hurst parameter, in the traffic flow.

The presence of a scaling regime in the power spectrum of the traffic time series indicated the possibility of fractal behavior. The value of the spectral exponent $\delta$ was approximately 0.68 and, being less than unity, suggests that an unbounded cascade model might be sufficient. The results of the empirical probability distribution function analysis, with probability exponent $D > 2.0$, indicated insufficiency of a mono-fractal model and the need for a multiplicative model to characterize the traffic behavior. The observation of a concave curvature, rather than a straight line, of the statistical moment scaling function indicated that the traffic data analyzed exhibited multi-fractal properties. We found strong evidence, the Hurst parameter $H = 0.84$, of a long-range dependence, which was not accounted for in any of the commonly used stochastic models for highway traffic. This can serve as one ground for using this model in transportation systems.

The above results from the present investigation provided positive evidence regarding the existence of fractal behavior in the traffic process observed at the Beijing Yuquanying. A possible implication of this might be that traffic flow characterization could be viewed from a new perspective: the chaotic fractal perspective. It should be noted, however, that the methods employed in the present study possess inherent limitations. For example, insufficient experiments and the presence of noise may influence the outcomes of the fractal identification methods. The results of this paper should be considered as preliminary and their generality, at this point in time, is not guaranteed. Therefore, these results should be substantiated further using other fractal and chaos identification methods to provide strong proof regarding the existence of a fractal and chaotic nature in traffic flow. Investigations in these directions are under way, details of which will be reported elsewhere.

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## References


[1] A.S. Nair, J.-C. Liu, L. Rilett, S. Gupta, Non-Linear Analysis of Traffic Flow. http://translink.tamu.edu/docs/Research/LinearAnalysisTrafficFlow/chaos1.PDF, 2001. Accessed May 27, 2003.

[2] D. Park, L.R. Rilett, Forecasting freeway link travel times with a multilayer feedforward neural network (Advanced Computer Technologies in Transportation Engineering, Blackwell Publishers, Malden, MA and Oxford, UK), International Journal of Computer-Aided Civil and Infrastructure Engineering 14 (1999) 357–367 (special issue).

[3] H.-O. Peitgen, H. Jürgens, D. Saupe, Chaos and Fractals: New Frontiers of Science, Springer-Verlag, 1992.

[4] P. Shang, X. Li, S. Kamae, Chaotic analysis of traffic time series, Chaos, Solitons and Fractals 25 (1) (2005) 121–128.

[5] A.D. May, Traffic Flow Fundamentals, Prentice Hall, Englewood Cliffs, 1990.

[6] H.J. van Zuylen, M.S. van Geenhuizen, P. Nijkamp, (Un)predictability in traffic and transport decision making, in: Transportation Research Record, TRB, vol. 1685, National Research Council, Washington DC, 1999, pp. 21–28.

[7] I. Prigogine, R. Herman, Kinetic Theory of Vehicular Traffic, Elsevier, New York, 1971.

[8] J.E. Disbro, M. Frame, Traffic Flow Theory and Chaotic Behavior, in: Transportation Research Record, TRB, vol. 1225, National Research Council, Washington DC, 1989, pp. 109–115.

[9] D.C. Gazis, R. Herman, R.W. Rothery, Nonlinear follow-the-leader models for traffic flow, Operations Research 9 (1961) 545–567.

[10] L.A. Safanov, E. Tomer, V.V. Strygin, Y. Ashkenazy, S. Havlin, Delay-induced chaos with multi-fractal attractor in a traffic flow model, Europhysics Letters 57 (2002) 151–157.
[11] W. Weidlich, Sociodynamics: A Systematic Approach to Mathematical Modeling in the Social Sciences, Harwood Academic, Amsterdam, 2000.
[12] R.C. Hilborn, Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers, 2nd ed., Oxford University Press, Oxford, 2001.
[13] H. Kantz, T. Schreiber, Nonlinear Time Series Analysis, Cambridge University Press, Cambridge, 1996.
[14] K. Falconer, Fractal Geometry, Wiley, New York, 1990.
[15] F. Takens, Detecting strange attractors in turbulence, in: Dynamical Systems and Turbulence, in: D.A. Rand, L.S. Young (Eds.), Lecture Notes in Mathematics, vol. 898, Springer-Verlag, Berlin, 1981, pp. 366–381.
[16] Y. Tessier, S. Lovejoy, D. Schertzer, Universal multifractals: Theory and observations for rain and clouds, Journal of Applied Meteorology 32 (1993) 223–250.
[17] S. Lovejoy, B. Mandelbrot, Fractal properties of rain and a fractal model, Tellus 37A (1985) 209–232.
[18] R. Macfadyen, Self-Similarity: Complication or Distraction? From the COST-257 project. http://www.info3.informatik.uni-wuerzburg.de/cost/html/temp.htm.
[19] M. Menabde, D. Harris, A. Seed, G. Austin, D. Stow, Multiscaling properties of rainfall and bounded random cascades, Water Resources Research 33 (12) (1997) 2823–2830.
[20] J. Olsson, J. Niemczynowicz, R. Berndtsson, Fractal analysis of high-resolution rainfall time series, Journal of Geophysical Research 98 (D12) (1993) 23 265–23 274.
[21] C. Svensson, J. Olsson, R. Berndtsson, Multifractal properties of daily rainfall in two different climates, Water Resources Research 32 (8) (1996) 2463–2472.
[22] K. Fraedrich, C. Larnder, Scaling regimes of composite rainfall time series, Tellus 45A (1993) 289–298.
[23] P. Ladoy, S. Lovejoy, D. Schertzer, Extreme variability of climatological data: scaling and intermittency, in: D. Schertzer, S. Lovejoy (Eds.), Non-linear Variability in Geophysics, Kluwer Academic, Norwell, MA, 1991, pp. 241–250.
[24] Y. Tessier, S. Lovejoy, P. Hubert, D. Schertzer, S. Pecknold, Multifractal analysis and modeling of rainfall and river flows and scaling, causal transfer functions, Journal of Geophysical Research 101 (D21) (1996) 26 427–26 440.
[25] P.P.G. Kumar, E. Foufoula-Georgiou, A probability-weighted moment test to assess simple scaling, Stochastic Hydrology and Hydraulics 8 (1994) 173–183.
[26] T.M. Over, V.K. Gupta, Statistical analysis of mesoscale rainfall: Dependence of a random cascade generator on large-scaling forcing, Journal of Applied Meteorology 33 (1994) 1526–1542.
[27] U. Frisch, G. Parisi, On the singularity structure of fully developed turbulence, in: M. Ghil, R. Benzi, G. Parisi (Eds.), Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics, North-Holland, New York, 1985, pp. 84–88.
[28] D. Schertzer, S. Lovejoy, Physical modeling and analysis of rain and clouds by anisotropic scaling multiplicative processes, Journal of Geophysical Research 92 (D8) (1987) 9693–9714.
[29] I. Rodriguez-Iturbe, F.B. De Power, M.B. Sharifi, K.P. Georgakakos, Chaos in rainfall, Water Resources Research 25 (7) (1989) 1667–1675.
[30] W. Willinger, M.S. Taqqu, W. Sherman, D. Wilson, Self-similarity through high variability: Statistical analysis of Ethernet LAN traffic at the source level, IEEE/ACM Transactions on Networking 5 (1) (1997) 71–86.
[31] M.S. Taqqu, J.B. Levy, Using renewal processes to generate long-range dependence and high variability, in: E. Eberlein, M.S. Taqqu (Eds.), Dependence in Probability and Statistics, in: Progress in Prob. and Stat., vol. 11, Birkhauser, Boston, 1986, pp. 73–89.
[32] B.B. Mandelbrot, Long-run linearity, locally Gaussian processes, H-spectra and infinite variances, International Economic Review 10 (1969) 82–113.
[33] B.M. Hill, A simple general approach to inference about the tail of a distribution, The Annals of Statistics 3 (1975) 1163–1174.