

# Gravitational time delay of light for various models of modified gravity

Hideki Asada

*Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan*

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## Abstract

We reexamined the gravitational time delay of light, allowing for various models of modified gravity. We clarify the dependence of the time delay (and induced frequency shift) on modified gravity models and investigate how to distinguish those models, when light propagates in static spherically symmetric spacetimes. Thus experiments by radio signal from spacecrafts at very different distances from Sun and future space-borne laser interferometric detectors could be a probe of modified gravity in the solar system.

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The nature of dark energy and dark matter has become a central issue in modern cosmology. Recent observations such as the magnitude-redshift relation of type Ia supernovae (SNIa) [1] and the cosmic microwave background (CMB) anisotropy by WMAP [2] strongly suggest a certain modification, in whatever form, in the standard cosmological model. We are forced to add a new component into the energy–momentum tensor in the Einstein equation or modify the theory of general relativity itself [3]. Indeed, there have been a lot of proposals motivated by, for instance, scalar tensor theories, string theories, higher dimensional scenarios and quantum gravity. (For recent reviews of modified gravity models inspired by the dark energy observation, e.g., [4].) Therefore, it is of great importance to observationally test these models.

The theory of general relativity has passed “classical” tests, such as the deflection of light, the perihelion shift of Mercury and the Shapiro time delay, and also a systematic test using the remarkable binary pulsar “PSR 1913 + 16” and several binary pulsars now known [5]. In the twentieth century, these tests proved that the Einstein’s theory is correct with a similar accuracy of 0.1%.

Since the time delay effect along a light path in the gravitational field was first noticed in 1964 by Shapiro [6], this

effect has successfully tested the Einstein’s theory [7]. A significant improvement was reported in 2003 from Doppler tracking of the Cassini spacecraft on its way to the Saturn, with  $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$  [8]. Here,  $\gamma$  is one of parameters in the parameterized post-Newtonian (PPN) formulation of gravity [5]. The bending and delay of photons by the curvature of spacetime produced by any mass are proportional to  $\gamma + 1$ , where  $\gamma$  is unity in general relativity but zero in the Newtonian theory, and the quantity  $\gamma - 1$  is thus considered as a measure of a deviation from general relativity. The sensitivity in the Cassini experiment approaches the level at which, theoretically, deviations  $10^{-6}$ – $10^{-7}$  are expected in some cosmological models [9,10]. Therefore, it is important to investigate the Shapiro time delay with such a high accuracy.

In addition to the above theoretical motivation, there are advances in technologies concerning the high precision measurement of time and frequency such as optical lattice clocks [11] and attoseconds ( $10^{-18}$  s) laser technologies [12]. ASTROD project with three spacecrafts aims at measuring  $\gamma$  at the level of  $10^{-9}$  [13].

The purpose of this Letter is to clarify the dependence of the time delay (and induced frequency shift) on modified gravity models and investigate how to distinguish those models by using the Shapiro time delay. An important point in this Letter is that we allow for various modified gravity theories beyond the scope of the PPN formulation. Introducing a new energy or

*E-mail address:* [asada@phys.hirosaki-u.ac.jp](mailto:asada@phys.hirosaki-u.ac.jp).

length scale (e.g. extra dimension scale) may make changes in functional forms of the gravitational field. Thus it is worthwhile to investigate how to probe such a modified functional form, by using the light propagation in the solar system. Throughout this Letter, we take the units of  $G = c = 1$ .

In this Letter, we assume that the electromagnetic fields propagate in four-dimensional spacetimes (even if the whole spacetime is higher dimensional). Thus photon paths follow null geodesics (as the geometrical optics approximation of Maxwell equation).

We shall consider a static spherically symmetric spacetime, in which light propagates, expressed as

$$ds^2 = -A(r) dt^2 + B(r) dr^2 + r^2 d\Omega^2, \quad (1)$$

where  $r$  and  $d\Omega^2$  denote the circumference radius and the metric of the unit 2-sphere, respectively. The functions  $A(r)$  and  $B(r)$  depend on gravity theories.

The time lapse along a photon path is obtained as

$$t(r, r_0) = \int_{r_0}^r \frac{dr}{b} \sqrt{\frac{B(r)}{A(r)}} \frac{1}{\sqrt{\frac{A(r_0)}{r_0^2} - \frac{A(r)}{r^2}}}, \quad (2)$$

where  $b$  and  $r_0$  denote the impact parameter and the closest point, respectively. Their relation is  $b^2 = r_0^2/A(r_0)$ .

According to a concordance between solar-system experiments and the theory of general relativity, we can assume that the spacetime is expressed as the Schwarzschild metric (rigorously speaking, its weak field approximation) with a small perturbation induced by modified gravity. For practical calculations, we keep only the leading term at a few AU in the corrections. Namely,  $A(r)$  and  $B(r)$  are approximated as

$$A(r) \approx 1 - \frac{2M}{r} + A_m r^m, \quad (3)$$

$$B(r) \approx 1 + \frac{2M}{r} + B_n r^n, \quad (4)$$

where  $M$  denotes the mass of the central body. Here,  $A_m$ ,  $B_n$ ,  $m$  and  $n$  rely on a theory which we wish to test. For simplicity, we assume  $m = n > 0$ , which corresponds to a wide class of theories of gravity.

Examples of modified gravity theories are as follows. (1)  $n = 1/2$ ,  $A_n = -2B_n = \pm 2\sqrt{M/r_c^2}$  for DGP model with  $r_c$  is the extra scale within which gravity becomes five-dimensional [14]. (2)  $n = 3/2$ ,  $A_n = (2/3)m_g^2\sqrt{2M/13}$  and  $B_n = -m_g^2\sqrt{2M/13}$  with graviton mass  $m_g$  for one of massive gravity models [15,16]. (3)  $n = 2$ ,  $A_n = -B_n = -\Lambda/3$  for the Schwarzschild–de Sitter spacetime, that is, general relativity with the cosmological constant  $\Lambda$  as a possible candidate for the dark energy, though this is not a manifest modification of gravity. The solar system experiments are not sensitive to this model with  $\Lambda \sim 10^{-52} \text{ m}^{-2}$  [17]. Here, it should be noted that the examples (1) and (2) give conformally flat spacetimes (in the weak field approximation) and their conformal factors generate the gravitational time delay (and induced frequency shift), though the null geodesic in any conformally flat spacetime is mapped into that in the Minkowski one.

The Cassini experiment has put the tightest constraint on the solar gravity, especially near the solar surface with the accuracy of  $10^{-5}$  [8]. This implies that deviations in  $A(r_\odot)$  and  $B(r_\odot)$  must be less than  $10^{-5} \times 2M_\odot/r_\odot \sim 10^{-10}$ , that is,  $|A_m r_\odot^m|, |B_n r_\odot^n| < 10^{-10}$ .

We consider the round-trip time between pulse transmission and echo reception, denoted by  $\Delta T$ . The pulse is emitted from Earth at  $r_E$ , and reflected at  $r_R$ .

Up to the linear order in  $M$ ,  $A_n$  and  $B_n$ ,  $\Delta T$  is expressed as

$$\begin{aligned} \Delta T = & 2\left(\sqrt{r_E^2 - r_0^2} + \sqrt{r_R^2 - r_0^2}\right) \\ & + 2M\left(2\ln\frac{r_E + \sqrt{r_E^2 - r_0^2}}{r_0} + 2\ln\frac{r_R + \sqrt{r_R^2 - r_0^2}}{r_0}\right. \\ & \left. + \sqrt{\frac{r_E - r_0}{r_E + r_0}} + \sqrt{\frac{r_R - r_0}{r_R + r_0}}\right) + \delta t. \end{aligned} \quad (5)$$

The extra time delay induced by a correction to general relativity is expressed as

$$\begin{aligned} \delta t = & r_0^{n+1} \left( \int_1^{R_E} + \int_1^{R_R} \right) dR \\ & \times \left( -A_n \frac{R^{n+3} - 2R^{n+1} + R}{(R^2 - 1)^{3/2}} + B_n \frac{R^{n+1}}{\sqrt{R^2 - 1}} \right), \end{aligned} \quad (6)$$

where we define non-dimensional radial coordinates as  $R \equiv r/r_0$ ,  $R_E \equiv r_E/r_0$  and  $R_R \equiv r_R/r_0$ . For a radar tracking of a spacecraft such as Cassini,  $r_E$  and  $r_R$  are of the order of 1 AU ( $\sim 10^8$  km), and  $r_0$  is several times of the solar radius ( $R_\odot \sim 10^5$  km). Eq. (6) can be rewritten by using special functions, though it seems less informative. Therefore, we take expansions of Eq. (6) in  $r_0$  because of  $r_E, r_R \gg r_0$ . For  $n \neq 1$ , we obtain

$$\begin{aligned} \delta t = & \frac{B_n - A_n}{n+1} (r_E^{n+1} + r_R^{n+1}) \\ & + \frac{B_n + A_n}{2(n-1)} (r_E^{n-1} + r_R^{n-1} - 2r_0^{n-1}) r_0^2 + O(r_0^4), \end{aligned} \quad (7)$$

whereas the second term of R.H.S. becomes  $(B_n + A_n) \ln(r_E r_R / r_0^2) r_0^2 / 2$  for  $n = 1$ .

It is convenient to use the relative change in the frequency, which is caused by the gravitational time delay [18], because the Doppler shift due to the receiver's motion has no effect owing to the cancellation at both the receipt and emission of radio signal [18]. This frequency shift is defined as  $y = -d(\Delta T)/dt$ . Indeed, the frequency shift was used by the Cassini experiment. For a case of  $b \ll r_E, r_R$ , which is valid for the Cassini experiment, the general relativistic contribution is expressed as [5]

$$y_{GR} = 4 \frac{M}{b} \frac{db}{dt}. \quad (8)$$

We pay attention to the extra contribution due to modified gravity. For  $n \neq 1$ , the extra frequency shift becomes

$$\delta y = -\frac{A_n + B_n}{n-1} \{r_E^{n-1} + r_R^{n-1} - (n+1)r_0^{n-1}\} b \frac{db}{dt}, \quad (9)$$

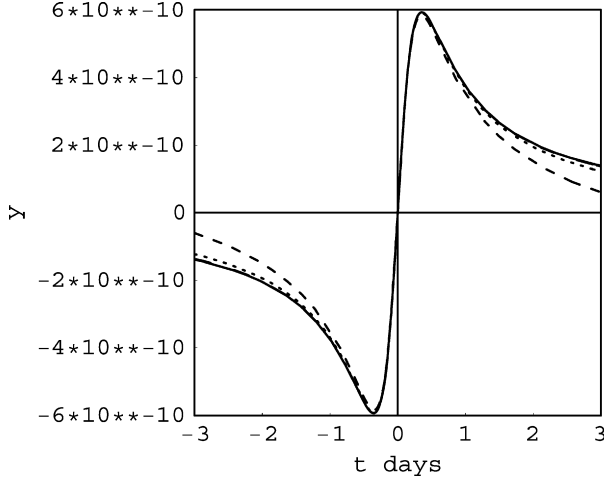


Fig. 1. Dependence of the frequency shift on the distance  $r_R$  and the index  $n$ . The long dashed, short dashed and dotted curves denote the frequency shift for  $(n, r_R) = (3/2, 10 \text{ AU})$ ,  $(n, r_R) = (2, 10 \text{ AU})$ ,  $(n, r_R) = (2, 1 \text{ AU})$ , respectively. The long dashed curve for  $n = 3/2$  and  $r_R = 10 \text{ AU}$  is overlapped with the solid curve denoting the general relativistic case. Here, we assume  $(A_n + B_n)r_\odot^n = 3 \times 10^{-11}$ .

while we obtain  $\delta y = -(A_n + B_n)[\ln(r_E r_R / r_\odot^2) - 1]b db/dt$  for  $n = 1$ . Here we used  $dr_E/dt, dr_R/dt \ll dr_0/dt$  ( $\sim db/dt$ ) near the solar conjunction ( $b \ll r_E, r_R$ ). The total frequency shift  $y$  is the sum,  $y_{GR} + \delta y$ . The impact parameter of light path changes with time, because of the motion of the emitter and receiver with respect to Sun. For simplicity, we assume that they move at constant velocity during short-time observations. The impact parameter changes as  $b(t) = \sqrt{b_0^2 + v^2 t^2}$ , where  $b_0$  denotes the minimum of the impact parameter near the solar conjunction at  $t = 0$ , and  $v$  is the velocity component perpendicular to the line of sight.

Here, we make an order-of-magnitude estimate of the frequency shift. First, we obtain  $y_{GR} \sim 10^{-9}(M/M_\odot)(r_\odot/b) \times (\dot{b}/v_E)$ , where the dot denotes the time derivative, and  $v_E$  is the orbital velocity of Earth ( $\sim 30 \text{ km/s}$ ). The Cassini experiment reported  $y$  at the level of  $10^{-14}$  by careful processing of the frequency fluctuations largely due to the solar corona and the Earth's troposphere [8]. Multi-band measurements are preferred in order to avoid the astrophysical effect of the corona and interplanetary plasma on the delay, which is proportional to the square inverse of the frequency.

For a receiver at  $r_R > r_E$ , the extra frequency shift is

$$\begin{aligned} \delta y &\sim (A_n + B_n)r_R^n \frac{b}{r_R} \frac{db}{dt} \\ &\sim 10^{-17} \left( \frac{10 \text{ AU}}{r_\odot} \right)^n \left( \frac{(A_n + B_n)r_\odot^n}{10^{-10}} \right) \left( \frac{r_R}{10 \text{ AU}} \right)^{n-1} \\ &\quad \times \left( \frac{b}{r_\odot} \right) \left( \frac{db/dt}{v_E} \right), \end{aligned} \quad (10)$$

where  $10 \text{ AU}/r_\odot \sim 2 \times 10^3$ . The larger the index of  $n$ , the longer the delay  $\delta y$ .

Fig. 1 shows that an extra distortion due to  $\delta y$  would appear especially in the tail parts of  $y - t$  curves. According to the fact that no deviation from general relativity has been re-

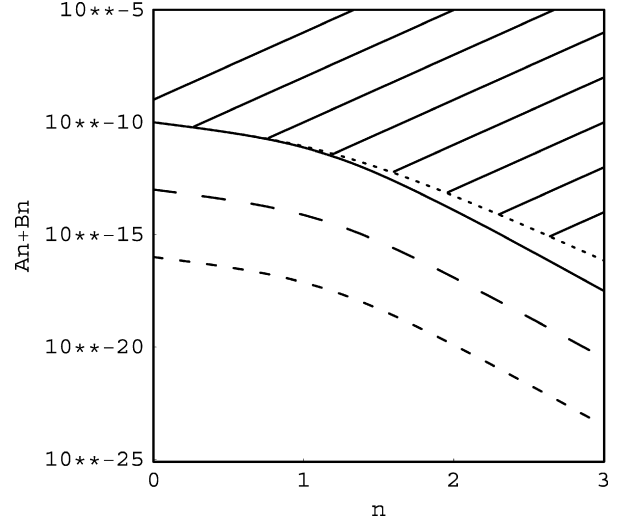


Fig. 2. Contours of  $\delta y$  on the  $n - |A_n + B_n| r_\odot^n$  plane. The solid, long-dashed and short-dashed curves correspond to  $\delta y = 10^{-14}, 10^{-17}, 10^{-20}$ , respectively, where we assume  $r_E = 1 \text{ AU}$ ,  $r_R = 40 \text{ AU}$ ,  $b \sim r_\odot$  and  $db/dt \sim v_E$ . The limit due to the current technology is  $\delta y \sim 10^{-17}$ . The shaded region above the dotted curve ( $\delta y = 10^{-14}$  for  $r_R = 8.43 \text{ AU}$ ) is excluded, because no deviation up to  $O(10^{-14})$  has been detected by the Cassini experiment [8].

ported by the Cassini experiment [8], we can put a constraint as  $\delta y < 10^{-14}$  at  $r_R = 8.43 \text{ AU}$ . On the other hand, Eq. (10) gives  $\delta y \sim 10^{-11}$  for  $n = 2$  and  $(A_n + B_n)r_\odot^n = O(10^{-10})$ , for instance, which are thus rejected. One can distinguish modified gravity models, which are characterized by various values of  $n$ ,  $A_n$ ,  $B_n$ , from observations using receivers at very different distances from Sun, as shown by Fig. 1.

Fig. 2 shows the dependence of  $\delta y$  on  $n$  and  $A_n + B_n$ . Hence, one can put a constraint on  $n$  and  $A_n + B_n$  from  $\delta y$  observed.

Eq. (8) shows that the frequency shift depends only on the impact parameter  $b$  but not the locations of the emitter and receiver. Strictly speaking,  $y_{GR}$  still has weak dependence on  $r_E$  and  $r_R$  as shown by Eq. (5). On the other hand,  $\delta y$  depends strongly on  $r_E$  and  $r_R$ . The dependence of  $y_{GR}$  and  $\delta y$  on  $r_E$  and  $r_R$  plays a crucial role in constraining (or detecting) a correction to general relativity in the solar system.

Let us imagine that time delays (or induced frequency shifts) are measured along two light trajectories, whose impact parameters are denoted as  $b_1$  and  $b_2$ , respectively. Then, we make a comparison of the two time delays. If they are in good agreement after taking account of a difference in the impact parameters, general relativity can be verified again. Otherwise, a certain modification could be required for the solar gravitational field. At this stage, however, one can say nothing about functional forms of the correction because the parameters of both  $n$  and  $A_n + B_n$ , which we wish to determine, enter the frequency shift.

In order to break this degeneracy, therefore, we consider three light paths, for which the impact parameters of the photon paths are almost the same (several times of the solar radius) for convenience sake. The locations of the receivers are denoted as  $r_{R1}, r_{R2}$  and  $r_{R3}$ , where the subscripts from 1 to 3 denote each light path. We assume that  $r_E$  is constant in time for simplicity. It is a straightforward task to take account of the eccentricity

of the Earth orbit and a difference between the impact parameters.

We make use of a difference such as  $y_2 - y_1$  and  $y_3 - y_1$ , in order to cancel out general relativistic parts. We find

$$y_2 - y_1 = \frac{A_n + B_n}{n - 1} (r_{R1}^{n-1} - r_{R2}^{n-1}) b \frac{db}{dt}. \quad (11)$$

It should be noted that  $y_2 - y_1$  is proportional to  $A_n + B_n$ . Hence, the following ratio depends only on  $n$  as

$$\frac{y_3 - y_1}{y_2 - y_1} = \frac{r_{R1}^{n-1} - r_{R3}^{n-1}}{r_{R1}^{n-1} - r_{R2}^{n-1}}. \quad (12)$$

Thereby, one can determine the index  $n$ . Next, one obtains  $A_n + B_n$  by substituting the determined  $n$  into Eq. (11).

In summary, we have clarified the dependence of the gravitational time delay on modified gravity models. For neighboring light rays, Eq. (7) gives almost the same value so that one can hardly distinguish models of gravity. This implies that we should prepare receivers at very different distances from Sun. Our result could be used for exterior planets explorers such as new horizons, which were launched in 2006 and their primary target is Pluto and its moon, Charon at distance from Sun  $\sim 40$  AU [19]. In future practical data analyses, however, it would be safer to use the original integral form as Eq. (6), because Eq. (7) is an approximate expression.

Furthermore,  $b$  becomes the same order of  $r_E, r_R$  for future space-borne laser interferometric detectors such as LISA [20], DECIGO [21] and especially ASTROD [13]. These detectors are in motion in our solar system. Namely,  $r_R$  and  $b$  change with time. Therefore, the sophisticated experiments by space-borne laser interferometric detectors, which are originally designed to detect time-dependent part of gravity, i.e. gravitational waves, could probe also a time-independent part of gravity at the relative level of  $\Delta y \sim \Delta v/v \sim \Delta L/L < 10^{-20}$ . It would be important to make a feasibility estimate for these detectors. Clearly, a stronger test can be done by not a single experiment but combining several ones. In addition, more precise measurements of the Shapiro time delay with binary pulsars may

put a constraint on the modifications discussed in this Letter, especially in the strong self-gravitating regime. Further investigations along these lines will be done in the future.

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