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ScienceDirect
Procedia Computer Science 93 (2016) 191 – 198
Procedia Computer Science

6th International Conference On Advances In Computing & Communications, ICACC 2016, 6-8 September 2016, Cochin, India

Threshold optimization in energy detection scheme for maximizing the spectrum utilization

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Abstract

Cognitive radio is a new paradigm in wireless communication for improving the spectrum utilization. Spectrum sensing is a fundamental component of cognitive radio networks. Among all the available spectrum sensing algorithms, energy detection scheme has gained more interest owing to its simple implementation. In this paper, we propose a threshold setting algorithm for energy detection scheme which aims at maximizing the spectrum utilization. A closed-form expression for adaptive threshold for maximal spectrum utilization is derived and analyzed. Numerical results demonstrate that the proposed scheme outperforms the conventional methods of determining the threshold.

Peer-review under responsibility of the Organizing Committee of ICACC 2016

Keywords: Cognitive radio; Spectrum sensing; Energy detection; Spectrum utilization; Adaptive threshold.

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1. Introduction

With the growing proliferation of wireless services and applications, networks are facing spectrum scarcity problem. Almost all the usable spectrum is already allocated to some licensed users for exclusive access. In 2002, the Federal Communications Commission (FCC) conducted a study on the utilization pattern of spectrum and found the allocated spectrum highly underutilized [1]. In 2003, FCC issued a notice of proposed rulemaking and order in which Cognitive Radio (CR) is identified as a potential candidate to increase spectrum utilization [2]. The term CR was first coined by Joseph Mitola in his PhD thesis [3, 4].

In cognitive radio networks, unlicensed users also known as Secondary Users (SUs) are allowed to temporarily

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Peer-review under responsibility of the Organizing Committee of ICACC 2016
doi:10.1016/j.procs.2016.07.200

access the licensed band if it is not being used by the Primary User (PU). PUs are the licensed users who are having proprietary rights for transmission over the band. To enable this, SUs have to sense the spectrum periodically to check the presence or absence of PU. If the band is found vacant, it can be used by SUs but the band is required to be vacated as soon as any PU activity is found. Therefore, spectrum sensing is categorized as a very important aspect of CRs. There are many spectrum sensing techniques available in the literature out of which energy detection has gained more interest because of its simple implementation and moreover, it does not require prior information about PU [5, 6]. The performance of any spectrum sensing algorithms is determined by the metrics called the probability of detection (P_d) and the probability of false alarm (P_{fa}). Probability of detection means detecting the presence of PU accurately. The higher the probability of detection, more the PU is protected. Probability of false alarm means detecting the PU present when actually PU is absent. Lower the probability of false alarm, more are the chances of spectrum utilization by SU.

The main challenge in energy detection scheme is the setting of an appropriate decision threshold. Most of the threshold setting algorithms are based on the Constant False Alarm Rate (CFAR) or Constant Detection Rate (CDR). In CFAR detectors, threshold is set by fixing P_{fa} which makes it difficult to achieve a desired detection probability over the wide SNR range especially in low SNR regions. Similarly, in CDR detectors threshold is set by fixing P_d and in this case it would be difficult to achieve a target false alarm probability. This has been illustrated in Fig. 1.

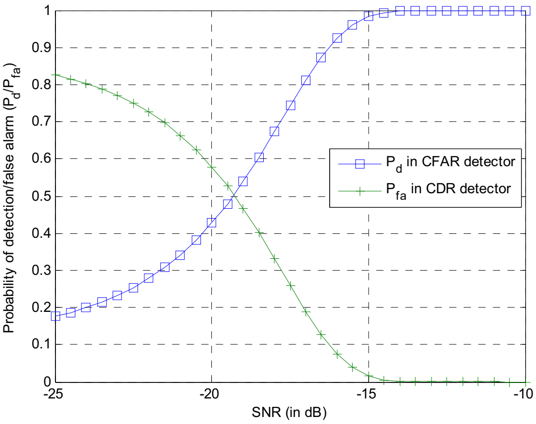

Fig. 1. Performance of CFAR (P_{fa}=0.1) and CDR (P_d=0.9) detectors.

This implies that there is a need of trade-off between the false alarm and detection probability. A number of adaptive algorithms have been proposed to minimize the miss detection and false alarm probability. In [7], a weighted combination of false alarm probability and detection probability is used to reach to optimal threshold value. By estimation noise and signal power, threshold is made adaptive to noise fluctuation. In [8], optimal threshold is derived to minimize the total sensing error probability in the presence of noise uncertainty. In [9], double threshold based energy detection method was studied to improve the performance under low SNR regions. In [10], a control parameter is introduced to vary the set threshold to obtain the desired response in accordance with the operational requirements. All of these works have assumed that PU does not appear in between two sensing epochs. Moreover, through the implementation of cognitive radio technology, the ultimate aim is to improve the spectrum utilization either by PU or SU. To the best of our knowledge, no algorithm has been developed till date which sets the threshold by maximizing the spectrum utilization. In this paper, a threshold setting algorithm is proposed for maximizing the entire spectrum utilization to study the best trade-off between the probability of detection and false alarm while taking PUs spectrum occupancy into the consideration. While formulating the expression for spectrum utilization, we have also considered the interference because of the sudden appearance/disappearance of PU between two sensing epochs. The closed-form expression for adaptive threshold is derived and simplified. Results demonstrate that the proposed scheme gives improved spectrum utilization. The rest of the paper is organized as follows: Section 2 presents the generic system model. In section 3, spectrum utilization formulation is described. Section 4 covers the proposed threshold setting algorithm. Simulation results are discussed in section 5 and section 6 concludes the paper.

2. System Model

The problem of PU detection can be formulated as a testing of two hypotheses. One is Hypothesis H_o when PU is considered as absent and received signal can be represented as:

$$H_o : y(n) = u(n) \tag{1}$$

Another is hypothesis H_1 when PU is considered as present and received signal can be represented as:

$$H_1 : y(n) = h(n)s(n) + u(n) \tag{2}$$

Where *u(n)* is Additive White Gaussian Noise (AWGN) which is assumed as independent and identically distributed (i.i.d.) random process with zero mean and variance σ_n^2. *s(n)* is PU signal and it is also assumed to be an i.i.d Gaussian random process with zero mean and variance σ_s^2. We further assume that both signals *s(n)* and noise *u(n)* are independent of each other. *h(n)* is the channel gain which has been assumed as unity. And *n=1, 2,…….N* is the index of the signal sample. Signal to Noise Ratio (SNR) is defined as the ratio of signal variance to noise variance$\left(SNR\ (\gamma) = \frac{\sigma_s^2}{\sigma_n^2}\right)$.

A conventional energy detector measures the energy (*X*) associated with the received signal and compare it with a predetermined threshold (λ) to decide among the two hypotheses H_o or H_1. The joint Probability Distribution Function (PDF) for *N* samples under hypothesis H_o can be derived as [11]:

$$p(y/H_o) = \frac{1}{\left(\sqrt{2\pi\sigma_n^2}\right)^N} exp\left(-\frac{\sum_{n=1}^{N}\left(y(n)\right)^2}{2\sigma_n^2}\right) \tag{3}$$

Under H_1, joint PDF can be written as [11]:

$$p(y/H_1) = \frac{1}{\left(\sqrt{2\pi\sigma_s^2}\right)^N} exp\left(-\frac{\sum_{n=1}^{N}\left(y(n)\right)^2}{2\sigma_s^2}\right) \tag{4}$$

Neyman-Pearson test will decide H_1 if

$$\frac{p(y/H_1)}{p(y/H_o)} > threshold\ (\lambda) \tag{5}$$

Taking logarithms on both sides and simplifying further yields test statistics (*X*) [11] as:

$$X = \frac{1}{N}\sum_{n=1}^{N}|y(n)|^2 \tag{6}$$

Test statistics *X* is a random variable for which PDF would be central chi-square distribution under hypothesis H_o and non-central chi-square distribution with non-centrality parameter under hypothesis H_1. When *N* is sufficiently large, according to the Central Limit Theorem (CLT), the PDF of *X* can be approximated by a Gaussian distribution as follows: When both *u (n)* and *s (n)* are real valued Gaussian [12]

$$under\ H_o, \qquad X \sim \mathcal{N}\left(\sigma_n^2, \frac{2}{N}\sigma_n^4\right) \tag{7}$$

$$under\ H_1, \qquad X \sim \mathcal{N}\left((1+\gamma)\sigma_n^2, \frac{2}{N}(1+\gamma)^2\sigma_n^4\right) \tag{8}$$

Where $\mathcal{N}(a,b)$ is normal distribution with mean 'a' and variance 'b'. The performance metrics P_{fa} and P_d will then be given as [7, 13]:

$$P_{fa} = Q\left(\frac{\lambda - \sigma_n^2}{\sqrt{\frac{2}{N}\sigma_n^4}}\right) \tag{9}$$

$$P_d = Q\left(\frac{\lambda - (1+\gamma)\sigma_n^2}{\sqrt{\frac{2}{N}(1+\gamma)^2\sigma_n^4}}\right) \tag{10}$$

Where $Q(.)$ is the complementary distribution function of standard Gaussian.
Probability of miss detection would be given as

$$P_{md} = 1 - P_d \tag{11}$$

3. Spectrum utilization formulation

A common frame structure used in cognitive radio networks is demonstrated in Fig. 2 where time τ is used for sensing and leftover time $T - \tau$ is used for data transmission if a band is detected as idle.

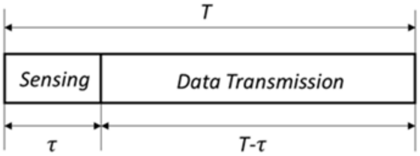

Fig. 2. Frame Structure

For our analysis, we assume that interference (or collisions between PU and SU transmission) lead to data loss and eventually to spectrum underutilization. We further assume that the traffic loads of the PUs are exponentially distributed with mean occupation and idle time per frame as α and β respectively. Then, PUs spectrum occupancy will be given as:

$$\theta = \frac{\alpha}{\alpha + \beta} \tag{12}$$

In perfect sensing, probability of wrong decisions is zero (i.e. P_{md} & P_{fa} both are equal to zero). In this case, SU will transmit only if the channel is detected as idle and will keep silent if the channel is found occupied. But PU may appear in between two sensing epochs and suffer interference on account of SUs transmission (when channel is detected as idle at initial sensing epoch). The probability that there is interference between PU and SU in such a case is given as [13]:

$$P_p = 1 - \frac{\beta}{T - \tau}\left(1 - \exp\left(-\frac{T - \tau}{\beta}\right)\right) \tag{13}$$

In imperfect sensing, there are two types of sensing errors (i) miss detection (ii) false alarm. False alarm makes the SU to keep silent even when the channel is actually vacant, thus decreasing the chances of spectrum utilization. On the other hand, miss detection leads to collisions between PU and SU transmission which results in data loss and ultimately decreases the spectrum utilization. Further, in a miss detection case, PU may become inactive before the next sensing epoch and there is possibility that there is no data loss even in miss detection. Probability of interference in miss detection is given as [13]:

$$P_{ip} = \frac{\alpha}{T - \tau}\left(1 - exp\left(-\frac{T - \tau}{\alpha}\right)\right) \tag{14}$$

In view of the above analysis, average time for which the band is utilized either by a PU or SU for successful data transmission in a single frame may be formulated as:

$$SU = (1-\theta)(T-\tau)(1-P_{fa})(1-Pp) + \theta(T-\tau)(1-P_d)(1-Pip) + \theta\tau(1-P_d) + \theta \mathrm{T} P_d \tag{15}$$

First & second term of (15) represents the data transmission of SU while third & fourth represents data transmission of PU. Normalized spectrum utilization will be given as:

$$SU_{norm} = \frac{SU}{T} \tag{16}$$

4. Proposed threshold optimization

The trade-off between P_d and P_{fa} is formulated to an equivalent form of maximizing the normalized spectrum utilization in the capacity of the PUs spectrum occupancy θ and decision threshold λ as:

$$\max\big(SU_{norm}(\lambda)\big) = \max\big(SU(\lambda)\big) \tag{17}$$

Our aim is to derive an expression for adaptive threshold to maximize the spectrum utilization. Fig. 3 shows the plot of normalized spectrum utilization expressed in (16) as a function of the decision threshold for a specified θ. It is clear from the figure that for fixed θ, there exists a threshold value for which normalized spectrum utilization is maximized.

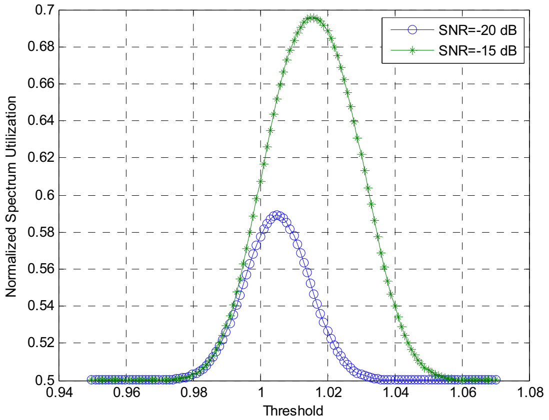

Fig. 3. Normalized spectrum utilization as a function of threshold for $N = 24000$, $\theta = 0.5$, $\tau = 2\ ms$ and $T = 100\ ms$.

Threshold that maximizes spectrum utilization can be calculated by differentiating SU with respect to threshold λ and equating it to zero as $\frac{\partial SU}{\partial \lambda} = 0$
After simplification, we will get:

$$\frac{(1-\theta)}{\theta}\frac{(1-P_p)}{P_{ip}}\frac{\partial P_{fa}(\lambda)}{\partial \lambda} = \frac{\partial P_d(\lambda)}{\partial \lambda} \tag{18}$$

Further simplification yields a quadratic equation as

$$\frac{\lambda^2(2\sigma_n^2+\sigma_s^2)}{2\sigma_n^2(\sigma_s^2+\sigma_n^2)} - \lambda - \frac{2\sigma_n^2(\sigma_s^2+\sigma_n^2)}{N\sigma_s^2} ln\left[\left(\frac{1-\theta}{\theta}\right)\left(\frac{1-P_p}{P_{ip}}\right)\frac{(\sigma_s^2+\sigma_n^2)}{\sigma_n^2}\right] = 0 \tag{19}$$

As a solution of the quadratic equation, we get

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1 + \frac{4(2\sigma_n^2 + \sigma_s^2)}{N\sigma_s^2} ln\left[\left(\frac{1-\theta}{\theta}\right)\left(\frac{1-P_p}{P_{ip}}\right)\frac{(\sigma_s^2 + \sigma_n^2)}{\sigma_n^2}\right]}}{\frac{2\sigma_n^2 + \sigma_s^2}{\sigma_n^2(\sigma_n^2 + \sigma_s^2)}} \tag{20}$$

The decision threshold should be positive and real therefore appropriate threshold is given as

$$\lambda^* = \sigma_n^2 \frac{1 + \sqrt{1 + \frac{4}{N}\left(1 + \frac{2}{SNR}\right) ln\left[\left(\frac{1-\theta}{\theta}\right)\left(\frac{1-P_p}{P_{ip}}\right)(1 + SNR)\right]}}{\frac{2 + SNR}{1 + SNR}} \tag{21}$$

5. Simulation results and analysis

System parameters for the numerical analysis are mentioned below:
- Sensing time $\tau = 2\ ms$
- Sampling frequency $f_s = 12$ MHz (twice the TV channel bandwidth i.e. 6 MHz)
- Frame duration $T = 100\ ms$
- PU spectrum occupancy θ is varied from 0 to 1
- Busy time of PU, $\alpha = \theta * T$
- Idle time of PU, $\beta = (1 - \theta)T$
- For fixed threshold $P_{fa} = 0.1$
- Number of samples, $N = \tau f_s = 24000$
- Noise variance, $\sigma_n^2 = 1$

Fig. 4 shows the curve between proposed adaptive threshold and SNR. Threshold is plotted for different PUs occupancy $\theta = 0.1, 0.5$ & 0.9. It can be seen that threshold decreases with the increase in θ. In Fig. 5 spectrum utilization is plotted as a function of θ for different SNR values of -25 dB, -20 dB and -10 dB. This curve shows better spectrum utilization for channels with high SNR. Fig. 6 shows the spectrum utilization as a function of SNR for fixed threshold, adaptive threshold derived in [7, 8] and proposed adaptive threshold for θ=0.3. It can be seen that proposed adaptive threshold has maximum spectrum utilization when compared to other methods. Fig. 7 shows the same curve, but for different PUs occupancy θ=0.3, 0.5 & 0.7. It is found that for θ=0.3, there is an improvement of almost 20% in the spectrum utilization in the complete low SNR region which is of great significance.

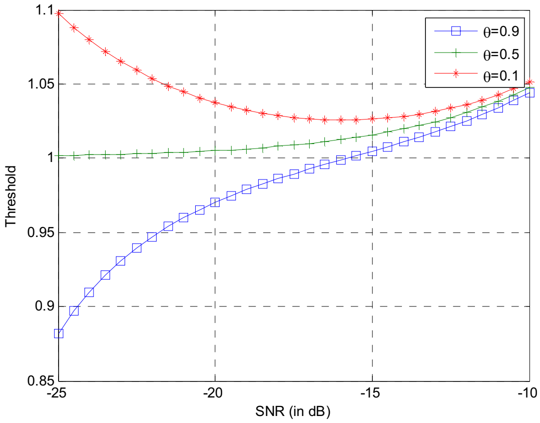

Fig. 4. Threshold as a function of SNR

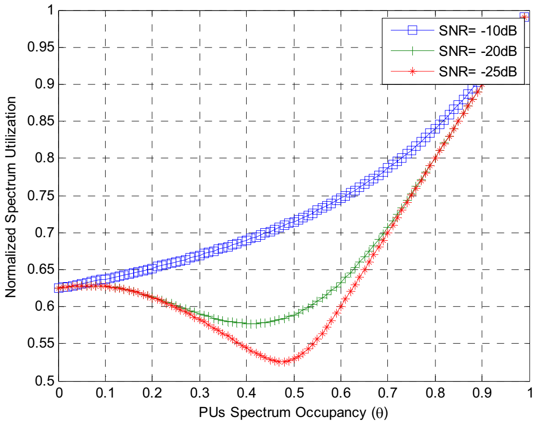

Fig. 5. Normalized Spectrum Utilization as a function of PUs spectrum occupancy θ.

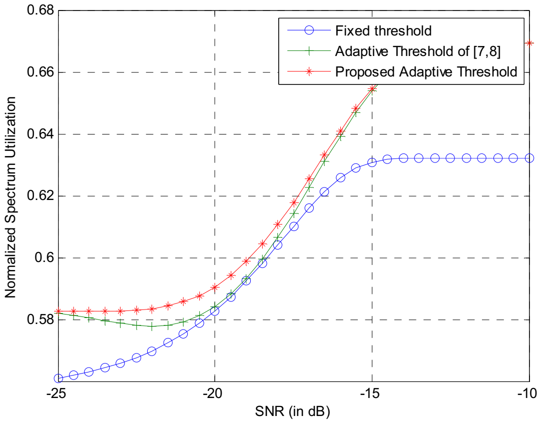

Fig. 6. Normalized Spectrum Utilization for different types of threshold when $\theta = 0.3$

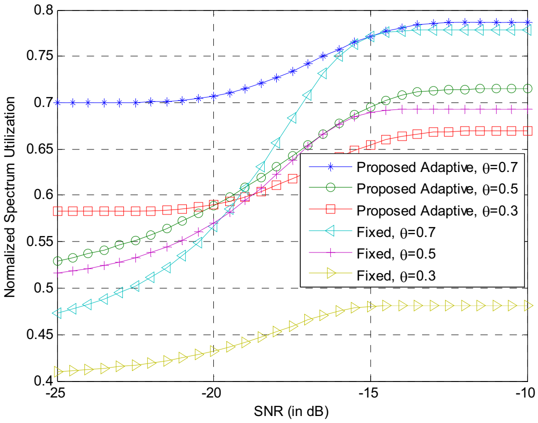

Fig. 7. Normalized Spectrum Utilization as a function of SNR for different θ.

6. Conclusion

In this paper, a threshold optimization technique is proposed to maximize the normalized spectrum utilization for specified PU spectrum occupancy. The closed form expression for adaptive threshold is derived while keeping in view the sudden appearance/disappearance of PU in between two sensing epochs. Numerical analysis shows the better spectrum utilization for the proposed scheme as compared to the other schemes available in the literature. The impact of PUs occupancy on spectrum utilization is also shown which conveys the historical PUs occupancy need to be considered when outlining CR frameworks.

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