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Generalized Soft Wall model

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ABSTRACT

We develop an exactly solvable generalization of the Soft Wall holographic model for the vector mesons. The generalization preserves the ultraviolet and infrared asymptotics of the Soft Wall model and contains an additional free parameter. This new parameter provides an arbitrary intercept in the Regge like spectrum of radial excitations and leads to a substantial modification of asymptotic expansion of the vector correlator at large momentum. The matching to the Operator Product Expansion from QCD allows to estimate the value of the new parameter which is shown to be in a good agreement with the phenomenology. In addition, the mass splitting between the vector and axial mesons arises naturally via the opposite sign of the introduced contribution to the intercept.

1. Introduction

One of spectacular manifestations of confinement in QCD is the expected Regge behavior of meson spectrum in the light quark sector, both in the spin and in the radial directions. Traditionally the related phenomenology was discussed in terms of the effective strings or dual amplitudes. The story has taken an interesting turn with the appearance of the Soft Wall (SW) holographic model for the strongly coupled QCD [1] which was inspired by the ideas of the gauge/gravity correspondence in the string theory [2–4]. Since the SW model provided a seminal theoretical setup for various applications, many authors proposed different modifications of the SW model with the aim of improving it or extending its applicability. We mention a few of them. The relation of the SW model and the light-front QCD was studied in Ref. [5]. The revealed mapping of one approach onto the other leads to interesting consequences concerning the hadron formfactors and the dependence of hadron mass spectrum on the orbital quantum number. The authors of Ref. [6] showed how to incorporate the chiral symmetry breaking in the SW model in a way consistent with QCD and with the known phenomenology. The two versions of the SW model with a positive and negative dilaton profile for both mesons and baryons of arbitrary spin were scrutinized in Ref. [7]. The SW model with the ultraviolet cutoff was analyzed in Ref. [8]. An incomplete list of the most recent developments is given in Ref. [9].

In the original SW model, the intercept of linear in mass squared trajectories of radially excited states is fixed. Naive attempts to change it spoil the ultraviolet asymptotics of the model, as a result the analytical behavior of correlation functions becomes strongly inconsistent with QCD. Namely, expanding the two-point correlators at large Euclidean momentum generically there appear an infinite number of terms which are absent in the standard Operator Product Expansion (OPE). In the present Letter, we show how to introduce an arbitrary intercept in a self-consistent way. The resulting model preserves the ultraviolet and infrared asymptotics of the original SW model and the number of unwanted terms in the OPE is reduced from infinity to one. We do not introduce any further modifications. Such a SW model with arbitrary intercept is referred to as generalized SW model. The arbitrary intercept entails some important theoretical and phenomenological consequences which will be analyzed.

The outline of this Letter is as follows. The scheme of the vector SW model is briefly reminded in Section 2. In Section 3, we derive our generalization of this model. The two-point correlator and its high-momentum expansion are calculated in Section 4. Section 5 contains numerical fits and estimations. Section 6 provides further phenomenological discussions. We conclude in Section 7.

2. Soft Wall model

In this section, we remind the reader the basic aspects of the SW holographic model [1]. For the sake of simplicity we will consider the simplest Abelian version of this model that is defined by the 5D action

$$S=\int d^4x\,dz\,\sqrt{g}\,e^{-az^2}\left(-\frac{1}{4g_5^2}F_{MN}F^{MN}\right), \qquad (1)$$

where $g=|\det g_{MN}|$, $F_{MN}=\partial_M V_N-\partial_N V_M$, $M,N=0,1,2,3,4$, in the AdS background space whose metrics can be parametrized as

$$g_{MN}\,dx^M\,dx^N=\frac{R^2}{z^2}\left(\eta_{\mu\nu}\,dx^\mu\,dx^\nu-dz^2\right). \qquad (2)$$

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Here $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, R denotes the AdS radius, and $z > 0$ is the holographic coordinate having the physical sense of inverse energy scale. The boundary $z = 0$ represents the 4D Minkowski space.

The gauge invariance of the action (1) permits to choose the axial gauge, $V_z = 0$, in which the equation of motion is simplified. The spectrum m_n of the physical vector mesons emerges from the Kaluza–Klein decomposition of the field V_μ,

$$V_\mu(x,z) = \sum_{n=0}^{\infty} V_\mu^{(n)}(x) v_n(z). \tag{3}$$

For calculating the mass spectrum one should find the normalizable solutions of the equation of motion for the 4D Fourier transform $V_\mu^T(q,z)$ of the transverse components ($\partial^\mu V_\mu^T = 0$). The normalizable eigenfunctions $v_n(z)$ exist only for discrete values of 4D momentum $q_n^2 = m_n^2$. The corresponding equation that follows from the action (1) reads as follows

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v_n\right) + m_n^2 \frac{e^{-az^2}}{z} v_n = 0. \tag{4}$$

This is a typical Sturm–Liouville problem. It is convenient to make the substitution

$$v_n = \sqrt{z} e^{az^2/2} \psi_n, \tag{5}$$

which transforms Eq. (4) into a Schrödinger equation

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \tag{6}$$

$$V(z) = a^2 z^2 + \frac{3}{4z^2}, \tag{7}$$

where the prime stays for ∂_z. The eigenvalues of Eq. (6) yield the mass spectrum of the model

$$m_n^2 = 4|a|(n+1). \tag{8}$$

Although the spectrum (8) does not depend on the sign of a, the choice $a < 0$ leads to unphysical zero mode [10]. For this reason we will assume $a > 0$ in what follows.

In a more general situation (for other spins) the potential (7) has two additional parameters

$$V(z) = a^2 z^2 + \frac{m^2 - 1/4}{z^2} + 4ab, \tag{9}$$

and results in the spectrum

$$m_n^2 = 2a(2n + m + 1 + 2b). \tag{10}$$

3. Generalized Soft Wall model

We wish to derive an exactly solvable generalization of the vector SW model that has an arbitrary intercept in the mass spectrum,

$$m_n^2 = 4a(n + 1 + b). \tag{11}$$

Our generalization must not spoil neither ultraviolet (UV) nor infrared (IR) asymptotics of the original SW model. We are going to show that this requirement fixes unambiguously the form of the background in the 5D action.

Let us write the holographic action in the form

$$S = \int d^4x\, dz\, f^2 \left(-\frac{1}{4g_5^2} F_{MN}^2\right), \tag{12}$$

with the unknown function $f(z)$ to be determined. The conformal symmetry dictates the following UV asymptotics for this function,

$$f(z)|_{z\to 0} \sim \frac{1}{\sqrt{z}}. \tag{13}$$

If the condition (13) is satisfied then in the UV limit the action (12) (written in the covariant form) has a form of the action (1).

The equation for the mass spectrum is

$$\left(f^2 v_n'\right)' + f^2 m_n^2 v_n = 0. \tag{14}$$

The substitution

$$v_n = \frac{\psi_n}{f} \tag{15}$$

brings Eq. (14) into the form of a Schrödinger equation

$$-\psi_n'' + \frac{f''}{f}\psi_n = m_n^2 \psi_n. \tag{16}$$

From (10) it follows that for obtaining a shift in the intercept the potential $\frac{f''}{f}$ must have the form of (9). However, the choice $m^2 \neq 1$ will lead to a wrong UV asymptotics in the vector SW model. The only possibility is to find the function f from the condition

$$\frac{f''}{f} = a^2 z^2 + \frac{3}{4z^2} + 4ab, \tag{17}$$

which has the form of Eq. (6) with m_n^2 replaced by $m_n^2 - 4ab$. This condition ensures the spectrum (11) we are looking for.

Eq. (17) has two solutions – an exponentially decreasing and an exponentially growing one. To comply with the IR asymptotics of the SW model (dictated by the absence of massless mode and by the correct spectrum for the higher spin mesons [10]) we must select the decreasing solution. Neglecting also the cases $b = -1, -2, \ldots$ (since we do not want to have any massless or tachyonic modes) the corresponding solution is

$$f = \Gamma(1+b)\frac{e^{-az^2/2}}{\sqrt{z}} U\left(b, 0; az^2\right), \tag{18}$$

where U is the Tricomi confluent hypergeometric function and Γ is the Gamma function. We have chosen the normalization $\frac{f^2}{z} = 1$ at $z = 0$.

Thus the action of the generalized SW model reads

$$S = \int d^4x\, dz \sqrt{g}\, e^{-az^2} U^2\left(b, 0; az^2\right)\left(-\frac{1}{4g_5^2} F_{MN} F^{MN}\right). \tag{19}$$

This is our main result. Since $U(b, 0; 0) = \Gamma^{-1}(1+b) = \mathrm{const}$ and $U(b, 0; az^2) \to (az^2)^{-b}$ at $z \to \infty$, the obtained modification of the 5D background does not affect neither UV nor leading IR asymptotics. If for some reason one considers the SW model with inverse dilaton background, $a < 0$, the argument of the Tricomi function must be changed to $|a|z^2$ (the function U is complex at negative argument).

It should be emphasized that the action (19) is purely phenomenological. The obtained background does not necessarily follow from a dynamical solution of Einstein's equation. The model can be also regarded as a compact five-dimensional writing of the planar QCD sum rules in the spirit of Ref. [11].

4. Vector correlator

The introduction of arbitrary shift in the spectrum brings qualitatively new properties to the analytical structure of the correlation functions. Following the standard recipe for the holographic calculation of the correlators [3,4], first we should find the solution $v(q,z)$ (q is the 4D momentum) of equation of motion which is subject to the boundary condition $v(q,0)=1$. For the action (19) the corresponding solution is

$$v(q,z)=\frac{\Gamma(1+b-\tilde{q}^2)U(b-\tilde{q}^2,0;az^2)}{\Gamma(1+b)U(b,0;az^2)}, \tag{20}$$

where the dimensionless momentum has been introduced, $\tilde{q}^2\equiv\frac{q^2}{4a}$. The two-point correlation function of vector currents J_μ,

$$\int d^4x\, e^{iqx}\langle J_\mu(x)J_\nu(0)\rangle=(q_\mu q_\nu-q^2g_{\mu\nu})\Pi_V(q^2), \tag{21}$$

can be expressed via $v(q,z)$ [12,13],

$$\Pi_V(q^2)=\frac{R}{g_5^2}\frac{\partial_z v}{q^2z}\bigg|_{z\to0}. \tag{22}$$

The substitution of (20) to (22) gives the expression

$$\Pi_V(q^2)=\frac{R}{2g_5^2}\bigg\{-\psi(1+b-\tilde{q}^2) + \frac{b}{\tilde{q}^2}\big[\psi(1+b-\tilde{q}^2)-\psi(1+b)\big]\bigg\}, \tag{23}$$

where ψ denotes the digamma function. Applying the decomposition

$$\psi(1+x)=-\sum_{n=0}^{\infty}\frac{1}{x+n+1}+\text{const}, \tag{24}$$

we arrive at the spectral representation for the correlator under consideration,

$$\Pi_V(q^2)=-\sum_{n=0}^{\infty}\frac{F_n^2}{q^2-4a(n+1+b)}, \tag{25}$$

$$F_n^2=\frac{2aR}{g_5^2}\left(1-\frac{b}{n+1+b}\right). \tag{26}$$

The poles of the correlator yield the mass spectrum (11). At $b\neq0$ the residues (they determine the electromagnetic decay width, see (34)) acquire a dependence on n. Choosing $b<0$ this new feature allows to mimic a kind of the vector dominance in the case of the ρ-mesons: The lightest vector state possesses the largest value of residue. In the next section, we demonstrate that in the vector sector indeed $b<0$.

The expansion of the correlator (23) at large Euclidean momentum $Q^2=-q^2$ leads to

$$\Pi_V|_{Q^2\to\infty}=\frac{R}{2g_5^2}\bigg\{\log\left(\frac{4a}{Q^2}\right)-\frac{4a}{Q^2}\bigg[\frac{1}{2} + b\left(\log\left(\frac{4a}{Q^2}\right)+1-\psi(1+b)\right)\bigg] + \frac{1}{2}\left(\frac{4a}{Q^2}\right)^2\left(\frac{1}{6}-b^2\right) + \frac{1}{6}\left(\frac{4a}{Q^2}\right)^3 b\left(b^2-\frac{1}{2}\right)+\mathcal{O}(Q^{-8})\bigg\}. \tag{27}$$

The expansion (27) can be matched to the Operator Product Expansion for the vector two-point correlator [14],

$$\Pi_V^{(\mathrm{OPE})}=\frac{N_c}{24\pi^2}\log\left(\frac{\mu_{\mathrm{ren}}^2}{Q^2}\right)+\frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4}-\frac{14\pi\alpha_s}{9}\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}(Q^{-8}). \tag{28}$$

The matching of coefficients in front of the leading logarithm provides the standard normalization factor,

$$\frac{R}{g_5^2}=\frac{N_c}{12\pi^2}. \tag{29}$$

5. Fits and estimations

In principle, the free parameters of the model – the slope $4a$ and the (dimensionless) contribution to the intercept b – can be fixed by matching the $\mathcal{O}(Q^{-4})$ and $\mathcal{O}(Q^{-6})$ terms. For the typical phenomenological values of the gluon and quark condensates, $\frac{\alpha_s}{\pi}\langle G^2\rangle=(360\text{ MeV})^4$ and $\langle\bar{q}q\rangle=-(235\text{ MeV})^3$, one obtains $4a=(905\text{ MeV})^2$ and $b=0.046$. Taking into account the qualitative character of the model, these estimates look reasonable.

A more conservative point of view on the $\mathcal{O}(Q^{-6})$ term would be to consider it as non-reliable for numerical fits because of the asymptotic nature of the expansion. Within the standard SW model, $b=0$, taking a typical phenomenological value for the slope of meson trajectories [15], $4a\approx(1.1\text{ GeV})^2$, the matching of $\mathcal{O}(Q^{-4})$ terms in the expansions (27) and (28) predicts an unrealistically large value for the gluon condensate, $\frac{\alpha_s}{\pi}\langle G^2\rangle\approx(440\text{ MeV})^4$. The parameter b allows to remove this drawback: It can be fixed from the condition to have a realistic gluon condensate in the expansions (27),

$$b^2=\frac{1}{6}-\frac{2\pi^2\frac{\alpha_s}{\pi}\langle G^2\rangle}{N_c(4a)^2}. \tag{30}$$

Substituting the physical values for the slope and gluon condensate to the condition (30), we arrive at the estimate

$$|b|\approx0.3. \tag{31}$$

Below we show that this value is reasonable from the phenomenological point of view.

To compare the obtained estimate for the intercept parameter b with the phenomenology we must make a fit of experimental masses by the linear trajectory. The crucial point here consists in the choice of data. By construction, the model describes the isoscalar vector states, *i.e.* we should consider the ω-mesons in the vector sector and the f_1-mesons in the axial-vector one. According to the Particle Data [16], there are only three well-established ω-mesons: $\omega(782)$, $\omega(1420)$, and $\omega(1650)$. Taking their masses from [16] and ascribing them the "radial" quantum numbers $n=0,1,2$, we obtain the fit (in GeV^2)

$$m_\omega^2(n)\approx1.1(n+0.7). \tag{32}$$

We do not write more accurate numbers because they would exceed the accuracy of the large-N_c limit – typically about 10%. In the isoscalar axial-vector sector, there is only one well-established state $f_1(1285)$ (another one, $f_1(1420)$, consists mostly of the strange quarks). The best we can do is to use the non-confirmed states $f_1(1970)$ and $f_1(2230)$ [15,16]. We ascribe them the "radial" quantum numbers $n=2,3$ (the state corresponding to $n=1$ – the isoscalar partner of $a_1(1640)$ [16] – is not known). This gives the fit

$$m_{f_1}^2(n)\approx1.1(n+1.5), \tag{33}$$

which should be regarded as a guess. It is quite remarkable that the slopes in the linear spectra (32) and (33) are approximately equal.

The spectrum (32) corresponds to $b \approx -0.3$ (see (11)) that perfectly agrees with our estimate (31) from the OPE. The data on axial-vector mesons seems to predict the opposite sign for b. A very interesting feature of the generalized SW model is that it allows the opposite signs of parameter b for the parity partners provided that the absolute value is the same. In particular, the value $b = 0.3$ may be compatible with the future data because it leads to a spectrum which is (taking into account the experimental errors) close to the guessed spectrum (33).

An independent estimate for the parameter b comes from the calculation of electromagnetic decay width of vector mesons,

$$\Gamma_{V \to e^+e^-} = C_V \frac{4\pi\alpha^2 F_V^2}{3m_V}, \tag{34}$$

where α is the fine structure constant, $\alpha = \frac{1}{137}$, and the quantity F_V^2 is given by (26) combined with (29). The factor C_V reflects the quark content of a given vector meson. Within the quark model, the amplitude of electromagnetic decay is proportional to the electric charge in the quark vertex. The quark content of the ρ^0-meson is $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ and of the ω-meson is $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$. As $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$ one has $\Gamma_{\rho^0 \to e^+e^-} \sim (\frac{2}{3} + \frac{1}{3})^2 = 1$ and $\Gamma_{\omega \to e^+e^-} \sim (\frac{2}{3} - \frac{1}{3})^2 = \frac{1}{9}$. By definition $C_\rho = 1$, hence $C_\omega = \frac{1}{9}$. The value of a in (26) follows from the fit (32), $4a \approx 1.1\ \text{GeV}^2$. Experimentally [16] $\Gamma_{\omega \to e^+e^-} = 0.60 \pm 0.02$ keV. The electromagnetic decay widths of the excited ω-mesons ($n > 0$) and of f_1 are not known. Substituting $n = 0$ and $b = 0$ we get $\Gamma_{\omega \to e^+e^-} \approx 0.4$ keV. The observable value is achieved if $b \approx -0.3$ – the same estimate as obtained above.

One can extend the analysis to the isovector case – the ρ and a_1 mesons – by considering the $SU(2)$ Yang–Mills field in the action (19). If, as usual, the mass spectrum is defined by the quadratic part of the holographic action, all formulas of the previous section will be the same. The ensuing numerical fits and conclusions turn out to be very similar.

6. Discussions

The vector correlator of the SW model contains the $\mathcal{O}(Q^{-2})$ term in the expansion at large Euclidean momentum. Such a term is absent in the OPE (28) by virtue of the absence of dim2 local gauge-invariant operator in QCD (although there are many speculations about the phenomenological relevance of dim2 condensate [17]). Unfortunately, the generalized SW model cannot solve this problem because of the logarithm in the numerator of $\mathcal{O}(Q^{-2})$ term in the expansion (27). More precisely, the problem can be partly resolved if one eliminates the constant part in this numerator by fine-tuning the parameter b (this would give $b \approx -0.24$). A possible physical origin of the residual $\frac{\log Q^2}{Q^2}$ term in the OPE remains however unclear. This problem seems to require a further modification of the generalized SW model which is left for future.

The positivity of the $\mathcal{O}(Q^{-4})$ term in the OPE leads to the constraint $|b| < 1/\sqrt{6}$ in the expansion (27). In addition, since this term is universal in the OPE for the vector and axial-vector channels [14] and depends quadratically on b, one has an intriguing possibility for a self-consistent mass splitting between the vector (V) and axial (A) states: The corresponding spectra have universal absolute value of b but opposite sign,

$$m_{V,A}^2(n) = 4a\big(n + 1 \mp |b|\big). \tag{35}$$

As we have seen in the previous section, this possibility seems to be indeed realized in Nature with the absolute value $|b| \approx 0.3$.

A nonzero value of parameter b generates the $\mathcal{O}(Q^{-6})$ term in the OPE and this represents a new feature in comparison with the usual SW model. In the latter, the term $\mathcal{O}(Q^{-6})$ is absent because the intercept (in units of the slope) is equal to unity. It is well known [18] that this is one of values of intercept at which the term $\mathcal{O}(Q^{-6})$ disappears in the OPE of the two-point correlators saturated by the narrow resonances with linearly rising spectrum. It is interesting to note that in the model (35), the $\mathcal{O}(Q^{-6})$ term in the V and A correlators differ by sign only (see Eq. (27)). This is close to the real OPE where these terms differ by the factor $-\frac{7}{11}$ [14]. In this sense, the opposite sign of b for the V and A mesons follows from the OPE. The factor $-\frac{7}{11}$ can be reproduced only for different values of b in the V and A sectors. But this would destroy the universality of $\mathcal{O}(Q^{-4})$ term in the OPE which is related to the gluon condensate. In view of the asymptotic nature of the OPE, the $\mathcal{O}(Q^{-4})$ term is more reliable than the next $\mathcal{O}(Q^{-6})$ one. For this reason we prefer to keep the universality and use the $\mathcal{O}(Q^{-6})$ term for qualitative conclusions at best. In our generalized SW model, the $\mathcal{O}(Q^{-4})$ term in the OPE fixes the absolute value of the intercept parameter b and the $\mathcal{O}(Q^{-6})$ term suggests the opposite sign for the V and A mesons. At the present stage, we cannot derive the sign for say the V mesons theoretically. Only the phenomenology tells us that $b < 0$ for the V mesons and $b > 0$ for the A states.

The SW model can be rewritten in an alternative form – redefining the vector field $V_M = e^{az^2/2}\tilde{V}_M$ in the action (1) leads to elimination of exponential background [19] (see also [7]). The price to pay is the appearance of an effective potential, namely the z-dependent mass term,

$$S = \int d^4x\,dz\,\sqrt{g}\left\{-\frac{1}{4g_5^2}\tilde{F}_{MN}\tilde{F}^{MN} + \frac{a^2 z^4}{2R^2 g_5^2}\tilde{V}_M\tilde{V}^M\right\}. \tag{36}$$

This mass term may be introduced in a gauge-invariant way via the Higgs mechanism [19]: The action

$$S = \int d^4x\,dz\,\sqrt{g}\left\{|D_M\varphi|^2 - m_\varphi^2\varphi^2 - \frac{1}{4g_5^2}\tilde{F}_{MN}\tilde{F}^{MN}\right\}, \tag{37}$$

where $D_M = \partial_M - i\tilde{V}_M$ and the scalar field φ is subjected to the free equation of motion in the AdS space,

$$-\partial_z\left(\frac{\partial_z\varphi}{z^3}\right) + \frac{m_\varphi^2 R^2\varphi}{z^5} = 0, \tag{38}$$

yields the action (36) if Eq. (38) has the solution $\varphi_0 \sim z^2$, *i.e.* if the scalar mass is $m_\varphi^2 R^2 = -4$. According to the AdS/CFT dictionary [3,4], the scalar mass is given by $m_\varphi^2 R^2 = \Delta(\Delta - 4)$, where Δ is the canonical dimension of the corresponding scalar operator in CFT. In the case under consideration, such a scalar field should be dual to a local QCD operator of dimension two. In the light of this observation a question appears how the generalized SW model looks like if we redefine it in a similar way? Making the substitution $V_M = e^{az^2/2}U^{-1}(b,0;az^2)\tilde{V}_M$ in the action (19), we obtain

$$S = \int d^4x\,dz\,\sqrt{g}\left\{-\frac{1}{4g_5^2}\tilde{F}_{MN}\tilde{F}^{MN} + \frac{a^2}{2R^2g_5^2}\left[z^2 + 2\frac{b}{a} - \frac{2U(b-1,0;az^2)}{aU(b,0;az^2)}\right]^2\tilde{V}_M\tilde{V}^M\right\}. \tag{39}$$

At $b = 0$ the action (39) coincides with (36) since $U(-1,0;az^2) = az^2U(0,0;az^2)$. If we now rewrite (39) in the form of (37), the solution for φ leads to a z-dependent mass m_φ^2. The physical interpretation above emerges only in the deep infrared region, $z \to \infty$. The leading contribution to φ_0 in this region is $\varphi_0 \sim z^2 + b$ that would correspond to the 5D mass $m_\varphi^2 R^2 = -\frac{4z^2}{z^2+b}$.

The extension of the SW model to the higher spin (S) mesons leads to a nice relation $m_{n,S}^2 = 4a(n+S)$ [1] which is compatible with other approaches (the Nambu–Goto strings, Veneziano amplitudes). The background in the action (19) does not lead to a simple shift in the spectrum for higher S (for the latter purpose one would need a different background for each spin). One can show that introducing the higher spin states according to the scheme of Ref. [1], the potential (17) of a Schrödinger equation in the background (19) becomes

$$V(z) = a^2 z^2 + 2a(S-1) + \frac{S^2 - 1/4}{z^2} + 4ab\left[1 + (S-1)\zeta(z)\right], \tag{40}$$

where the function $\zeta(z)$,

$$\zeta(z) = \frac{U(1+b, 1; az^2)}{U(b, 0; az^2)}, \tag{41}$$

behaves as $\zeta(z \to 0) \sim -\log z$ and $\zeta(z \to \infty) \sim z^{-2}$ in the UV and IR limits. Consequently the contribution due to nonzero b does not affect the UV and IR asymptotics of the potential (40). This contribution will cause a slight deviation from linearity of the Regge like spectrum.

7. Conclusions

We have shown how to introduce an arbitrary intercept to the linear spectrum of the Soft Wall holographic model in a self-consistent way. The spectrum of vector states becomes $m_n^2 \sim n + 1 + b$, where $n = 0, 1, 2, \dots$ and the case $b = 0$ corresponds to the usual SW model. The obtained generalization of the SW model remains exactly solvable. The resulting freedom in the choice of the intercept entails a sizeable modification of the two-point correlator, specifically the residues of meson poles cease to be universal for all states and a contribution related to the quark condensate is generated. The latter signifies that the parameter b is related to the chiral symmetry breaking.

The Operator Product Expansion of two-point vector correlators dictates the universal value of $|b|$ for the vector and axial-vector particles but simultaneously indicates that the sign of b must be different in the vector and axial sectors. This introduces the mass splitting between the vector and axial particles. The phenomenological value of the gluon condensate and of the slope of radial trajectories leads to the estimate $|b| \approx 0.3$. We studied the sector of isoscalar vector states. The value $b \approx -0.3$ is in a perfect agreement both with the well-established spectrum of ω-mesons and with the electromagnetic decay width of the $\omega(782)$-meson. The value $b \approx 0.3$ seems to be compatible with the spectrum of excited axial f_1-mesons which is not yet well-established experimentally.

In view of recent phenomenological applications of the SW model and attempts to derive it from a more fundamental setup, its generalization presented in this work may be useful.

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