ABSTRACT

Thin walled steel members, such as cold-formed steel (CFS) members, are susceptible for local buckling at low loads. However, initiation of local buckling of elements does not necessarily mean the ultimate limit state of the member, and thin-walled plate elements can exhibit substantial post-buckling strength. International cold-formed steel design specifications recognize the post-buckling strength of uniformly compressed stiffened and unstiffened elements, and the post-buckling strength of stiffened and unstiffened webs in flexure. However, potential post-buckling shear strength of webs is not considered in CFS design specifications, though post-buckling shear strength of stiffened webs is considered in structural steel plate girder design. The objective of this study is to quantify the post-buckling shear strength of thin-walled cold-formed steel members. This numerical investigation is based on finite element method.

This paper presents the finite element modelling details associated with the thick/thin plates representing the web of a cold-formed steel member. The study considered simply-supported rectangular plates subjected to in-plane shear loadings experiencing buckling, post-buckling, and yielding until failure. The plate was modelled using geometrically non-linear quadrilateral shell elements, and non-linear steel stress-strain relationship derived from experiments. Total Langrangian with large displacement/small strain formulation was used for such analyses. The model also considered the initial geometric imperfections. The first part of the investigation established the ultimate shear strength of plates having different parametric dimensions. The next part compared such results with the strength values based on the current code provisions. For slender plates, the current code uses the shear buckling load as the strength, whereas the analyses indicated significant post-buckling strength. The third part of this paper establishes the shear design equations incorporating the post-buckling strength of plates.

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1. INTRODUCTION

Thin walled steel members, such as cold-formed steel (CFS) members, are susceptible for local buckling at low loads. Figure 1 indicates a plate subjected to pure shear loads along the edges, representing a web of a steel bending member. At the initial loading stage, prior to shear buckling when the plate is perfectly flat, equal tensile and compressive principal stresses are developed. As the applied shear loads increase, the compressive and tensile stresses within the plate increase equally until the shear buckling load is reached. In plated structural elements, initiation of local buckling of elements does not necessarily mean the ultimate state of member, and thin-walled plate elements can exhibit substantial post-buckling strength. Beyond shear buckling, plates cannot take any additional compressive stresses. However, any additional shear loads are then resisted by the tension field action (tensile membrane stress) only. The edge zones of the plate serve as an anchor to the diagonal tension field. Therefore, the rigidity of edge members greatly influences the magnitude of the post-buckling strength of the plate. If the edge elements are flexible, the edge member will bend inward, plate failure will be initiated by forming plastic hinges at the edge members. If the edge elements are rigid, plate failure will be governed by frame action. The objective of this study is to quantify the post-buckling shear strength of thin steel plates.

Figure 1: Simply supported plate subjected to pure shear load

2. SHEAR BUCKLING STRENGTH OF PLATES

The critical shear buckling stress of a rectangular plate with length $a$, width $h$ and thickness $t$ subjected to uniform shear loads can be formulated as (Timoshenko and Gere 1961);

$$\tau_{cr} = k_v \frac{\pi^2 E}{12(1-\nu^2)(h/t)^2} \quad (1)$$

Where, $k_v$ is the non-dimensional shear-buckling coefficient, which depends on the aspect ratio and the boundary conditions of plates. For plates with all edges simply supported;

$$k_v = 5.34 + \frac{4}{(a/h)^2}, \quad \text{for} \quad a/h \geq 1 \quad (2)$$

Obviously the critical buckling stress depends on the slenderness ratio $h/t$ of the plate. As $h/t$ decreases, $\tau_{cr}$ can be as high as the shear yield stress $\tau_y$ (ignoring strain hardening). It can be shown that, in theory, when the slenderness ratio $h/t$ is greater than $1.25\sqrt{E/\rho\tau_y}$, the plate will shear buckle before it yields.
The North American Specification (AISI 2007) provides a procedure to estimate the ultimate shear capacity of solid web plates, which is as given below:

The nominal shear strength [resistance], \( V_n \), of the web element shall be calculated as follows:

\[
V_n = A_w F_v
\]  
(3)

(a)- THICK PLATE: For \( h/t \geq \sqrt{E_k / F_y} ; F_v = 0.60 F_y \)

(b)- MODERATE PLATE: For \( \sqrt{E_k / F_y} < h/t \leq 1.51 \sqrt{E_k / F_y} ; \)

\[
F_v = \frac{0.60 \sqrt{E_k F_y}}{(h/t)}
\]  
(4a)

(c)- THIN PLATE: For \( h/t > 1.51 \sqrt{E_k / F_y} \; ; \)

\[
F_v = \frac{\pi^2 E_k}{12(1 - \nu^2)(h/t)^2} = 0.904 E_k / (h / t)^2
\]  
(4b)

According to the AISI (2007) procedure the cold-formed steel web plates can be divided into three cases, namely, (a) thick, (b) moderate thick and (c) thin. Plates falling in region (a) will fully yield first, since the corresponding shear buckling stress is higher than the yield strength. Region (c) is for plates having their critical shear buckling stress less than the proportional limit in shear \((0.8 \tau_y = 0.8 * F_y / \sqrt{3})\). Such plates undergo elastic buckling behavior. For plates with \( h/t \) ratio falling in case (b), which is the transition zone between elastic buckling and yielding, plates undergo inelastic buckling. The ultimate strength of such plates were established as the geometric mean of their shear buckling stress and 0.8 times the shear yield stress, which is, \( F_v = \sqrt{\tau_y (0.8 \tau_y)} \). It is evident that the procedure provided in AISI (2007) does not incorporate the post-buckling behavior of web plates into its design shear capacity.

3. FINITE ELEMENT MODEL FOR A PLATE UNDER SHEAR LOADS

In this section, a general finite element model for analyzing the post-buckling behavior of a simply supported rectangular cold-formed steel plate subjected to pure shear loads is presented. Figure 1 shows the dimensions and the loading conditions of plates under consideration. The quadrilateral four-node shell elements were used in this model. Unless otherwise specified, all nodes were fixed in the z-direction rotation \((\theta_z)\) to restrain the rigid body rotation about z-axis and were free in all the other degree of freedoms. In order to simulate a simply supported boundary condition, the nodes along the four edges of the plate were fixed in z-direction translation. In addition to the z direction translation, one point was also fixed in the x and y translations to eliminate the rigid body movement of the plate in x and y directions. Another point was also fixed in x-translation to avoid the rigid body rotation of the plate about z axis. To simulate a pure shear loading state, uniformly distributed line loads were applied along the four edges of the plate. The shear loads were directly applied to the nodes as a system of conservative forces and were kept tangential to the edges of the plate during the deformation process. The proposed finite element model used an idealized multi-linear stress-strain relationship for cold-formed steel proposed by Sivakumaran and Abdel-Rahman (1997). Since this study focuses only on the flat plates, the stress-strain relationship corresponding to the flat area was used for all models. The analyses considered commonly used steel having yield strength of 350MPa. The von Mises yield criterion was adopted as the yielding criterion for steel. Isotropic hardening rule was used. The model has included the effects of the geometrical initial imperfection, and has ignored the effects of the residual stress. Geometrical imperfection is a function of plate width, thickness, forming process, installation etc. For this study, a double sine function was used to represent the distribution of the geometrical imperfection in a web plate. Without going into details the amplitude of the imperfection was taken as \( w_0 = 0.003h \), where \( h \) is the width of the main plate. Residual stresses are stresses that exist in steel sections as a result of the deformations during the cold-forming fabricating processes, and the thermal gradients that are induced in
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the welding process. Rodal (1992), Abdel-Rahman and Sivakumaran (1997) and Schafer and Peköz (1998) have experimentally and analytically studied the changing of the yield strength and the residual stresses in cold-formed steel sections due to cold-working. It was found by these researches that cold-formed steel sections have elevated yield strengths in the corner regions due to the cold-working process. The increase of yield strength and the induced residual stresses tend to compensate each another. Also, due to modeling complications and lack of systematic data, residual stresses are often ignored in numerical analysis. Based on above reasons, the residual stresses were neglected in the current study. The automatic-time-step (ATS) was used as the analysis method. At least 100 load steps were used for each loading process to ensure sufficient numbers of data points were available for the load-displacement diagram. Based on convergence studies a 24x24 mesh configuration with a total of 576 elements was used to model plates with an aspect ratio of 1, and the mesh was gradually increased to 24x120 to model plates with $a/h = 5$.

4. PARAMETRIC STUDIES ON PLATES SUBJECTED TO SHEAR LOADS

Forty simply supported plate models covering five different aspect ratios ($a/h = 1, 2, 3, 4, 5$), and eight slenderness values were studied. The AISI (2007) limits the slenderness value $h/t$ for an unreinforced flexural web panel of a cold-formed steel member to a maximum value of 200, thus the $h/t$ ratios selected in this study started from $h/t = 50$, and increased at every 25 interval until $h/t = 200$. However, one extra $h/t$ value, namely $h/t = 250$, was also studied in order to obtain a better view on how the ultimate shear strength of a plate changes as the $h/t$ ratio is changed. Thus, this study considered eight slenderness values, using a fixed width of the plates of $h = 100$mm. Figure 2 shows the relationship between the average applied shear stress and the average shear strain for plates with aspect ratios of 1 and 5. It can be seen from these figures that the ultimate shear strength of solid plates decreases at a decreasing rate as the $h/t$ values increase. It can also be found that the ultimate shear strength of solid plates decreases as the $a/h$ ratios increase. However, as the aspect ratio becomes large, the effect of the $a/h$ value on the ultimate shear capacity of plates fades out. In practice, cold-formed steel members are normally used without intermediate stiffener, which means that the $a/h$ for the web panels tend to be very large ($a/h >> 5$). Thus, the effect on the ultimate shear strength of web panels caused from the aspect ratio $a/h$ can be normally neglected.

![Figure 2: Shear stress Vs Shear strain for plates with aspect ratio $a/h = 1$ and $5$](image1)

Figure 3 shows the ultimate shear strength of plates as a function of slenderness ratio obtained from both the finite element analysis and the AISI (2007) method for plates with $a/h = 1$ and 5. Regions (a), (b) and (c) in this figure correspond to the thick, moderately thick, and thin web panels defined earlier. In regions (a) and (b), the ultimate shear capacities obtained from the finite element analysis are somewhat
lower than those calculated following the AISI (2007) specification. For plates in region (a), the AISI (2007) uses $I_J y = 0.60 F_y$ instead of $I_J y = F_y / \sqrt{3} \approx 0.577 F_y$ when calculating the ultimate shear capacity of thick webs. Thus the nominal ultimate shear strength for plates using the AISI (2007) method in region (a) tends to be over-estimated. The major difference in the ultimate shear strengths between the finite element analysis results and the AISI (2007) results occurs for case (c) plates (thin plates). The shear capacities obtained from the finite element study for thin plates are higher than those calculated from the AISI (2007) equations. This is because that the AISI (2007) calculates the ultimate shear strength of plates based on either the shear yield stress or the shear-buckling stress of web plates. Any additional post-buckling strength due to tension field action that may exist after the web has buckled is neglected. However, appreciable amounts of post-buckling strengths can build up for thin plates. Thus the current AISI specification has conservatively estimated the ultimate shear strength of thin shear webs. Thus, modifications may be made to the AISI equations to include the post-buckling strength of thin web plates.

![Figure 3: Ultimate strength Vs Slenderness for plates with aspect ratio a/h = 1 and 5](image)

### 5. SHEAR POST-BUCKLING STRENGTH OF PLATES

Parametric studies indicated considerable post-buckling shear capacity in thin plates. The post-buckling strength of plates $\tau_p$ may be quantified by subtracting the elastic buckling stress from the ultimate shear strength of plates. Elastic buckling stress of plates $\tau_{cr}$ was given in equation (1). In order to establish the impact of various parameters on the post-buckling capacity, first we established a dimensionless parameter $\frac{\tau_p}{\tau_{cr}}$, which is essentially the post-buckling strength divided by the shear buckling stress of the corresponding plates. Figure 4[A] shows the relationship between the post-buckling strength normalized by the shear buckling strength and the slenderness ratios up to $h/t = 200$ for plates with varying aspect ratios. Figure 4[B] shows the relationship between the post-buckling strength normalized by the shear buckling strength and the aspect ratios up to $a/h = 5$ for plates with varying slenderness ratios.

It can be seen that the ratio $\frac{\tau_p}{\tau_{cr}}$ increases approximately linearly as the $h/t$ ratio increases. Thus the new proposed equation could use a linear equation to represent the relationship between the ratio $\frac{\tau_p}{\tau_{cr}}$ and their slenderness ratios $h/t$. It can also be seen that when $a/h \leq 2$, the ratio $\frac{\tau_p}{\tau_{cr}}$ increases for plates with $a/h = 1$ to plates with $a/h = 2$. However, the real function between the ratio $\frac{\tau_p}{\tau_{cr}}$ and the aspect ratio $a/h$ is unknown for plates with $a/h \leq 2$ since only two data points ($a/h = 1$ and $a/h = 2$) are available. In this study, a linear relationship is assumed as an approximate estimation of the relationship between the ratio $\frac{\tau_p}{\tau_{cr}}$ and the aspect ratio $a/h$ for plates with $a/h \leq 2$. When $a/h > 2$, the ratio $\frac{\tau_p}{\tau_{cr}}$ decreases approximately linearly as the $a/h$ ratio increases. As a result, one linear equation could be used in each $a/h$ range to include the effects on the post-buckling strength of plates caused by changing of the aspect ratio.
Based on above discussion, this paper proposes a modified shear strength equation for thin plates (Region c).

Accordingly, for plates having $h/t > 1.51 \sqrt{Ek_y / F_y}$ the design shear stress could be taken as:

$$F_v = \tau_{cr} + \tau_p$$  \hspace{1cm} (5)

where

$$\tau_{cr} = \frac{\pi^2 Ek_y}{12(1-\nu^2)(h/t)^2}$$

and

$$\tau_p = \{0.048(h/t - 1.51 \sqrt{Ek_y / F_y} \cdot f(a/h)) \cdot \tau_{cr}\}$$

and, where $f(a/h) = 0.01*(a/h) + 0.2$, for $1 \leq a/h \leq 2$ and $f(a/h) = -0.02*(a/h) + 0.26$, for $a/h > 2$

6. CONCLUDING REMARKS

This paper studied the ultimate shear capacity of simply supported plates subjected to pure edge shear loading, through finite element analysis. The relationships between the shear capacity, the slenderness ratio and the aspect ratio of plates were obtained. It was shown that the ultimate shear capacity of a plate decreases as the slenderness ratio $h/t$ of the plate increases. The ultimate shear capacity also decreases, but to a lesser extent, as the aspect ratio $a/h$ of the plate increases. By comparing the shear strength obtained from the finite element analyses with that obtained from the AISI (2007) equations, the results indicated that by ignoring the post-buckling strength of slender plates, the AISI (2007) has underestimated the shear strength of thin plates, since the post-buckling strength can be many times larger than the shear-buckling strength of thin plates. This paper quantified the post-buckling strength and had proposed a modified design equation for thin plates which incorporates the beneficial effects of post-buckling shear strength of such plates. The material included in this paper is part of the thesis by Chen (2009).

References


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