Heterotic string backgrounds and CP violation

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Abstract

In the framework of Z_N orbifolds, we discuss effects of heterotic string backgrounds including discrete Wilson lines on the Yukawa matrices and their connection to CP violation.

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1. Introduction

The heterotic string [1] compactified on an orbifold [2] has a number of phenomenologically attractive features. These include the possibility of obtaining realistic gauge groups and a small number of chiral generations at low energies [3,4]. Also, the Yukawa couplings of the twisted states exhibit a hierarchy [5,6] which is a highly desirable feature from the phenomenological perspective.

Realistic models require the presence of backgrounds [3,4]—the Wilson lines [7] and the antisymmetric background field. The former are required if we are to obtain a realistic gauge group and 3 chiral generations, while the latter is suggested by the observed CP violation. In this Letter, we study the connection between CP violation and the backgrounds. In particular, we analyze the effects of the backgrounds on the Yukawa couplings and whether these effects may lead

to physical CP violation. We build on earlier work [8,9] and now include the effects of discrete Wilson lines and the U-moduli. We also discuss certain subtleties appearing in the definition of the CP symmetry from a higher-dimensional perspective.

The relevant to the discussion of CP violation part of the low energy Lagrangian is given by

$$\Delta\mathcal{L} = Y_{ij}^u H^u Q_i U_j^c + Y_{ij}^d H^d Q_i D_j^c, \tag{1}$$

where $Y_{ij}^{u,d}$ are the Yukawa matrices and i, j are the generational indices labeling the three chiral families. Here we exhibit the Yukawa interactions in the two Higgs doublet model, while for the Standard Model the two doublets are related: $H^u \sim (H^d)^c$. When the Higgses develop vacuum expectation values, these interactions are responsible for generating the quark mass matrices. These may have complex phases, which can be absorbed into the definition of the quark fields if $Y_{ij}^u = |Y_{ij}^u|e^{i(\alpha_i + \beta_j^u)}$, $Y_{ij}^d = |Y_{ij}^d|e^{i(\alpha_i + \beta_j^d)}$, but otherwise lead to observable CP violation. We will study the Yukawa matrices in Z_N orbifolds which are defined by dividing a 6D torus by a space group which

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consists of the Z_N twists and lattice shifts. We will focus on the Yukawa couplings among twisted states which reside at the orbifold fixed points f defined by

$$\theta f = f + l, \tag{2}$$

where θ is the orbifold twist and l is a torus lattice shift. The untwisted couplings are moduli independent and do not lead to CP violation. We will deal mostly with renormalizable couplings although some statements about nonrenormalizable couplings will be made as well.

The Letter is organized as follows. In the next section, we define the CP symmetry and discuss under what circumstances it is violated by the backgrounds. In Section 3, we present the full moduli dependence of the heterotic Yukawa couplings. Section 4 is devoted to the effects of discrete Wilson lines. Section 5 presents our conclusions.

2. CP symmetry and the string action

The bosonic part of the heterotic string action in the presence of constant backgrounds is given by[1]

$$S = \frac{1}{2\pi} \int d\tau \, d\sigma \big(G_{ij} \partial^\alpha X^i \partial_\alpha X^j - B_{ij} \epsilon^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j$$
$$+ A_{iI} \epsilon^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^I \big). \tag{3}$$

Here X^i, $i = 1, \ldots, 10$ are the space–time coordinates, X^I, $I = 1, \ldots, 16$ are the gauge space left-moving coordinates, and G_{ij}, B_{ij}, and A_{iI} are the background metric, the antisymmetric field, and the Wilson line, respectively.

In what follows we will discuss CP properties of the twisted Yukawa couplings which presumably are the source of the observed CP violation in the quark sector. CP violation in the Yukawa couplings $Y_{\alpha\beta\gamma}$ is directly related to CP properties of the string action since

$$Y_{\alpha\beta\gamma} = \text{const} \times \sum_{X_{\text{cl}}} e^{-S_{\text{cl}}}, \tag{4}$$

where X_{cl} are solutions to the string equations of motion in the presence of the twist fields, S_{cl} is the *Euclidean* action, and α, β, γ label the twisted sectors.

Further, we will study a class of the Z_N orbifold compactifications of the $E_8 \times E_8$ heterotic string which admit the decomposition $T^2 \oplus T^2 \oplus T^2$ such that the backgrounds have a block-diagonal form. To discuss the CP symmetry, one introduces the orthogonal coordinates $X' = O^{-1} X$ in which the metric is diagonal

$$G \to O^{\text{T}} G O = \eta, \tag{5}$$

with η being the Minkowski metric. Henceforth, we will work in this basis and will omit the prime. The bosonic part of the CP transformation can be defined as [10,11][2]

$$X^I \to -X^I, \quad I = 1, \ldots, 16,$$
$$X^i \to X^i, \quad i = 1, 5, 7, 9,$$
$$X^i \to -X^i, \quad i = 2, 3, 4, 6, 8, 10. \tag{6}$$

This is a combination of the conventional reflection of the three spatial coordinates, a reflection of three of the compactified coordinates, and a reversal of the gauge charges. Although a four-dimensional P operation is not a proper Lorentz transformation, it becomes one when supplemented by the reflection of the compactified coordinates. In terms of the complex coordinates of the internal manifold, this amounts to the complex conjugation $Z^i \to Z^{i*}$ [10]. The reversal of the gauge charges is an automorphism of the $E_8 \times E_8$ group and therefore is also a symmetry of the system (when no backgrounds are present).

The above definition does not appear to be unique in the sense that one can extend a conventional P operation to a proper Lorentz transformation in a number of ways, for instance, through a reflection of only one of the compactified coordinates. Such "truncated CP" appears as a well-defined symmetry at the classical bosonic action level, but it is *not* a symmetry of the theory as a whole. In particular, since it acts in the 6D subspace, it is not a gauge symmetry in the fermionic sector [11]. Also, it is not a symmetry of the compactification: it transforms the twist $(\theta_1, \theta_2, \theta_3)$ into $(\theta_1^*, \theta_2, \theta_3)$ which does not belong to a subgroup of $SU(3)$ (unless $\theta_1 = e^{i\pi}$), which leads to a non-supersymmetric orbifold. Similar arguments apply to Calabi–Yau compactifications.

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[1] We omit the pure gauge part of the action which does not pertain to our considerations.

[2] See also [12,13].

The presence of string backgrounds B_{ij} and A_{iI} *generally* violates the CP symmetry as defined in Eq. (6). First, consider the antisymmetric background B_{ij}. Under our factorization assumption, it can be written as

$$B = B_{(1)} \oplus B_{(2)} \oplus B_{(3)}, \tag{7}$$

where $B_{(i)}$ corresponds to the ith compactified plane. The invariance of the action under twisting

$$\theta^{\mathrm{T}} B \theta = B, \tag{8}$$

where θ is the orbifold twist, and the antisymmetry require

$$B_{(i)} = \begin{pmatrix} 0 & b_{(i)} \\ -b_{(i)} & 0 \end{pmatrix}. \tag{9}$$

Then, clearly if $b_{(i)} \neq 0$, the symmetry (6) is violated. On the other hand, this background preserves the block-diagonal part of the (proper) Lorentz symmetry. Indeed, the orbifold twist splits as

$$\theta = \theta_{(1)} \oplus \theta_{(2)} \oplus \theta_{(3)}, \tag{10}$$

where

$$\theta_{(i)} = \pm \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} \tag{11}$$

rotates the ith plane by θ_i. This is a proper Lorentz transformation, so Eq. (8) signifies the invariance of the B-background under this class of Lorentz transformations. However, the reflection symmetry of the background-free action

$$S_0 = \frac{1}{2\pi} \int d\tau \, d\sigma \, \partial^\alpha X^i \partial_\alpha X^i \tag{12}$$

is lost. Note that this action is invariant under the full Lorentz group including orientation changing transformations

$$X^i \rightarrow -X^i \tag{13}$$

for any i.

This observation raises the question "What is the higher-dimensional symmetry that ensures that the low-energy 4D Yukawa couplings conserve CP?" To answer this question, let us first note that 4D field theory tells us that complex Yukawa couplings break conventional CP (if the phases cannot be rotated away) while real ones conserve it. From Eq. (4) it is clear that CP is broken when the Euclidean action

has a nonvanishing imaginary part. The $\partial^\alpha X^i \partial_\alpha X^i$ piece always gives a real contribution, while the B_{ij} contribution is imaginary. The difference arises from the τ-dependence combined with the Wick's rotation: a quadratic ∂_τ dependence gives a real result while a linear one produces a factor of i, i.e., we have

$$\partial^\alpha \partial_\alpha \quad \text{vs} \quad \epsilon^{\alpha\beta} \partial_\alpha \partial_\beta.$$

The $\epsilon^{\alpha\beta}$-piece leads to antisymmetric with respect to the Lorentz (or Lorentz-gauge) indices contributions to the action. Such contributions necessarily break some reflection symmetries (13). We thus conclude that the *parity symmetry* (13) ensures that the Yukawa couplings conserve CP. Under our factorization $T^2 \oplus T^2 \oplus T^2$ assumption, this is equivalent to requiring the CP symmetry (6).

Now, it is clear that, from the bosonic action perspective, the "truncated CP" does not reduce to the conventional 4D CP symmetry. Consider, for instance, $B_{ij} \neq 0$ in the third plane only. This configuration conserves "pseudo-CP" defined with the reflection of one axis in the first plane only. Yet, the Yukawa couplings violate CP in the usual sense.

Nonvanishing B_{ij} does not necessarily result in observable CP violation since at isolated points in the parameter space the effect of B_{ij} may simply amount to $S \rightarrow S + 2\pi i n$. Further constraints come from[3]

(i) flavor-dependence,
(ii) modular invariance.

The first of them means that, if we associate quark flavors with orbifold fixed points, the CP violating phases produced by B_{ij} should depend on the relative positions of the fixed points in such a way that they could not be eliminated by a redefinition of the quark fields [16]. This amounts to nonvanishing of the Jarlskog invariant[4] [14]. The second requirement (which is related to the first one) means that the CP phases cannot be eliminated by a modular transformation whenever the system possesses a modular sym-

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[3] Apart from producing CP violating Yukawa couplings, B_{ij} (with 4D indices) also couples to $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ which violates CP. This term is constrained to be extremely small experimentally leading to the notorious strong CP problem.

[4] In supersymmetric models, CP violation is governed by a class of K- and L-invariants in addition to the Jarlskog invariant [15].

metry [16,17]. Whether these requirements are satisfied or not depends on the orbifold and the relevant moduli, but in principle observable CP violation can be achieved even at the renormalizable level if $B_{ij} \neq 0$ [16].

The discussion of the Wilson lines proceeds (at first) along similar lines. Consider the case of continuous Wilson lines. A continuous Wilson line is realized through the correspondence between the space group and rotations and shifts of the gauge $E_8 \times E_8$ lattice [4]

$$(\theta, l) \rightarrow (\Theta, a), \tag{14}$$

where θ is a point group element, Θ is a rotation of the $E_8 \times E_8$ lattice, and l and a are related by

$$l^i = \sum_\alpha n_\alpha e_\alpha^i, \qquad a^I = \sum_\alpha n_\alpha A_\alpha^I. \tag{15}$$

Here n_α are some integers, e_α^i are the torus basis vectors, and A_α^I are the Wilson lines.

For the *standard* embedding ($\theta = \Theta$), we have

$$A = A_{(1)} \oplus A_{(2)} \oplus A_{(3)}, \tag{16}$$

where $A_{(i)}$ are 2×2 blocks. The twist invariance

$$\theta^{\mathrm{T}} A \theta = A \tag{17}$$

requires

$$A_{(i)} = \begin{pmatrix} a_{(i)} & b_{(i)} \\ -b_{(i)} & a_{(i)} \end{pmatrix}. \tag{18}$$

A priori, $a_{(i)} \neq 0$ or $b_{(i)} \neq 0$ violate the symmetry (6). However, embedding of the space group into the gauge group imposes additional constraints. In particular, on shell, effectively we have [9]

$$\partial_\alpha X_L^i \sim \partial_\alpha X^I \tag{19}$$

for each of the 2×2 blocks (one can, for instance, take $i = I$), while X_R^i decouple from the Wilson lines. This identification restores the CP invariance so that S_{cl} gives a CP conserving contribution to the Yukawa couplings. These arguments equally apply to a class of nonstandard embeddings for which an orbifold twist is associated with a rotation of more than one planes in the gauge space. The explicit formulae will be given in the next section.

The case of discrete Wilson lines is more complicated and will be dealt with separately in one of the subsequent sections.

Similar discussion applies to the nonrenormalizable couplings. The relevant n-point amplitude is given by [18]

$$A \propto \sum_{X_{\mathrm{cl}}} \partial X_{\mathrm{cl}}^i \cdots e^{-S_{\mathrm{cl}}}. \tag{20}$$

The classical solutions X_{cl}^i are not affected by B_{ij} and A_{iI} since they enter neither the equations of motion nor the boundary conditions. Thus CP violation arises through $e^{-S_{\mathrm{cl}}}$ and the arguments above apply. We note that these couplings are exponentially suppressed by the radii of the compactified dimensions in symmetric orbifolds [18].

3. Moduli dependence of the Yukawa couplings

To make our arguments more explicit, here we present the full moduli dependence of the heterotic Yukawa couplings. The Yukawa couplings are calculated via pairing two real coordinates in each plane into a complex one (see [19] and [8] for details). The action is then written as

$$\begin{aligned}
S_{\mathrm{cl}} = {}& \frac{1}{2\pi} \int d^2z \left(\partial Z^i \bar{\partial} \overline{Z}^i + \bar{\partial} Z^i \partial \overline{Z}^i \right) \\
& - \frac{B_{i,i+1}}{2\pi} \int d^2z \left(\partial Z^i \bar{\partial} \overline{Z}^i - \bar{\partial} Z^i \partial \overline{Z}^i \right) \\
& + \frac{1}{2\pi} \int d^2z \left[\mathcal{A}_{iI} \left(\partial Z^i \bar{\partial} Z^I - \bar{\partial} Z^i \partial Z^I \right) \right. \\
& \qquad\qquad + \mathcal{A}'_{iI} \left(\partial Z^i \bar{\partial} \overline{Z}^I - \bar{\partial} Z^i \partial \overline{Z}^I \right) \\
& \qquad\qquad \left. - \mathrm{h.c.} \right],
\end{aligned} \tag{21}$$

where $z = e^{-2(\tau + i\sigma)}$, $Z^i = X^i + iX^{i+1}$, $i = 1, 3, 5$; $Z^I = X^I + iX^{I+1}$, $I = 1, 3, \ldots, 15$; and

$$\mathcal{A}_{iI} = \frac{1}{4}(A_{iI} - A_{i+1,I+1} - iA_{i+1,I} - iA_{i,I+1}),$$

$$\mathcal{A}'_{iI} = \frac{1}{4}(A_{iI} + A_{i+1,I+1} - iA_{i+1,I} + iA_{i,I+1}). \tag{22}$$

Here h.c. replaces a quantity with the corresponding barred one and conjugates $\mathcal{A}, \mathcal{A}'$. We omit the pure gauge contribution to the action since it vanishes on shell.

The classical solutions are completely determined by their singular behavior at the twist operator insertion points and the boundary conditions. If the twist field of order k/N is placed at the point z_1 on the world

sheet, another twist field of order l/N is placed at z_2, etc., the solution to the equations of motion has the form

$$\partial Z = c(z - z_1)^{-(1-k/N)}(z - z_2)^{-(1-l/N)}$$
$$\times (z - z_3)^{-k/N - l/N},$$
$$\bar{\partial}\overline{Z} = \bar{c}(\bar{z} - \bar{z}_1)^{-(1-k/N)}(\bar{z} - \bar{z}_2)^{-(1-l/N)}$$
$$\times (\bar{z} - \bar{z}_3)^{-k/N - l/N},$$
$$\bar{\partial} Z = d(\bar{z} - \bar{z}_1)^{-k/N}(\bar{z} - \bar{z}_2)^{-l/N}$$
$$\times (\bar{z} - \bar{z}_3)^{-(1-k/N-l/N)},$$
$$\partial \overline{Z} = \bar{d}(z - z_1)^{-k/N}(z - z_2)^{-l/N}$$
$$\times (z - z_3)^{-(1-k/N-l/N)} \qquad (23)$$

for each complex plane. The constants c, d are to be determined by the following *monodromy* conditions

$$\Delta Z^i = \int_{\mathcal{C}} dz\, \partial Z^i + \int_{\mathcal{C}} d\bar{z}\, \bar{\partial} Z^i = v^i,$$
$$\Delta \overline{Z}^i = \int_{\mathcal{C}} dz\, \partial \overline{Z}^i + \int_{\mathcal{C}} d\bar{z}\, \bar{\partial} \overline{Z}^i = \bar{v}^i, \qquad (24)$$

with the complex lattice vector v^i defined by $v^i = v^{(i)} + i v^{(i+1)}$ and $\bar{v}^i = v^{(i)} - i v^{(i+1)}$. Here the contour $\mathcal{C}$ is chosen such that Z^i gets shifted but not rotated upon going around $\mathcal{C}$. These equations allow to solve for c, d in terms of the winding vectors v^i.

In the case of the standard embedding, $\bar{\partial} \overline{Z}^i$ and $\bar{\partial} \overline{Z}^I$ have the same structure [9]. In particular, they have the same singular behavior at the twist operator insertion points:

$$\bar{\partial}\overline{Z}^I = \bar{c}'(\bar{z} - \bar{z}_1)^{-(1-k/N)}(\bar{z} - \bar{z}_2)^{-(1-l/N)}$$
$$\times (\bar{z} - \bar{z}_3)^{-k/N - l/N}, \qquad (25)$$

while $\partial \overline{Z}^I = \partial Z^I = 0$ since X^I are left moving. The constant c' is determined by the monodromy condition for Z^I:

$$\int_{\mathcal{C}} d\bar{z}\, \bar{\partial} \overline{Z}^I = \bar{u}. \qquad (26)$$

Here u is the gauge space representation of the space group element v which appears in the monodromy condition for Z^i (see [9] for a detailed discussion).

It has been shown [5,6] that the Yukawa couplings are determined by the holomorphic instantons, i.e.,

classical solutions with holomorphic Z^i and antiholomorphic $\overline{Z}^i$ (i.e., $d = 0$). Omitting the intermediate details, let us give the final result [9,20]

$$Y_{\alpha\beta\gamma} = \text{const} \times \sum_{n_i, m_i \in \mathbf{Z}} \exp\Bigg[- \sum_{i=1,3,5} \frac{T_i + A_i \bar{A}_i}{\text{Re}\, U_i}$$
$$\times \left(n_i^2 - 2 n_i m_i \, \text{Im}\, U_i + m_i^2 \right)$$
$$\times \frac{\pi |\sin(k_i \pi/N)||\sin(l_i \pi/N)|}{2\sin^2(k_i l_i \pi/N)|\sin((k_i + l_i)\pi/N)|} \Bigg]. \qquad (27)$$

Here we have used the following definitions of the moduli [20]

$$T = \frac{\sqrt{\det g}}{2\pi^2}(1 - ib),$$
$$U = \frac{1}{g_{11}}\left(\sqrt{\det g} - i g_{12}\right),$$
$$A\bar{A} = \frac{\sqrt{\det g}}{4\pi^2}|a + ib|^2 \qquad (28)$$

for each of the three planes. Here the integers n_i, m_i are determined by the space group selection rule. The background parameters a, b are given by Eqs. (9), (18). The orbifold metric g_{ab} is

$$g_{ab} = e_a \cdot e_b \qquad (29)$$

and we assume $e_1^2 = e_2^2 = R^2$ and $e_1 \cdot e_2 = R^2 \cos\phi$, where R is the compactification radius and ϕ is an angle between e_1 and e_2.

A comment about the U-dependence is in order. Although we display the U-dependence explicitly, in all relevant cases the value of the U-modulus is fixed once the orbifold is specified ($U = -ie^{i\phi}$). It is a continuous modulus only if the orbifold possesses a Z_2 plane. In this case the allowed Yukawa coupling is of the form $\theta\theta\theta^2$ in this plane, which reduces to a 2-point twist–antitwist correlator. This gives just a multiplicative constant irrelevant to our discussion.

It is clear from Eq. (27) that the only potential source of CP violation is the T-moduli (apart from, possibly, discrete Wilson lines which will be discussed in the next section). The U- and A-moduli only affect the magnitudes of the Yukawa couplings. In particular, the presence of continuous Wilson lines always makes the hierarchy among the Yukawa couplings stronger.

The flavor-dependence necessary for physical CP violation comes from the dependence of n_i, m_i on the

positions of the fixed points (for analogous discussion of type I models, see [21]). The space group selection rule requires $ne_1 + me_2 = (1 - \theta^{kl})(f_1 - f_2 + \Lambda)$ in each plane, where $f_{1,2}$ are the fixed points where the fields are placed and Λ is a lattice vector. If the fixed points do not coincide, n or m do not start from zero and the coupling is suppressed by the distance between the fixed points. The consequent CP phases depend on the relative positions of the fixed points. Given a favorable configuration, observable CP violation can result at the renormalizable level [16]. This only occurs in the even order orbifolds where the space group selection rule is not diagonal. In the odd order orbifolds, CP violation, if it occurs at the renormalizable level, has to result from either "mixing" of the fixed points due to an anomalous $U(1)$ or a nonminimal Higgs sector (e.g., 6 Higgs doublets, etc.). Of course, it can also come entirely from non-renormalizable operators in which case not much can be said quantitatively.[5]

Concerning the effect of the target space modular symmetries, one can show that, due to the axionic shift invariance $T_i \to T_i + i$, the CP phases can be rotated away if $\mathrm{Im}\, T_i = \pm 1/2$ [16]. No CP violation occurs in this case. We note that the axionic shift symmetry is unbroken by the presence of Wilson lines, unlike the duality symmetry, so it is a symmetry of many realistic models. Thus, under the above assumptions, the T-moduli have to be stabilized away from the lines $\mathrm{Im}\, T_i = \pm 1/2$ which include the fixed points of the modular group. This imposes nontrivial constraints on realistic models [23] since the moduli are often stabilized at the fixed points $T = 1, e^{\pm i\pi/6}$ (as suggested by symmetries of the scalar potential).

4. Discrete Wilson lines and CP violation

A priori, the presence of discrete Wilson lines violates CP. In this section, we show that this *does not* occur at least at the renormalizable level if the low energy physics is described by the Standard Model (or its minimal supersymmetric version).

A discrete Wilson line is realized through an Abelian embedding of the space group into the gauge

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[5] See [22] on related subjects.

group of the orbifold [3]:

$$(\theta, l) \to (1, v + a), \tag{30}$$

where v is a shift of the $E_8 \times E_8$ lattice, and l and a are related by (15). If we associate a Wilson line a_1 with e_1, then the same Wilson line is also associated with θe_1. This can be seen as follows: to respect the group multiplication rules, one has to associate $(\theta, e_1)(\theta^{-1}, e_1) = (1, e_1 + \theta e_1)$ with $(1, v + a_1)(1, -v + a_1) = (1, 2a_1)$ from which the above statement follows. Thus we have

$$a^I(e_i) = a^I(\theta e_i), \tag{31}$$

up to a gauge lattice vector. Further, since $(\theta, l)^N = (1, 0)$, such Wilson lines have to be *discrete*:

$$Na^I = 0 \tag{32}$$

up to a lattice vector.

Consider an example of the Z_3 orbifold. Denote the two $SU(3)$ root vectors by e_1 and e_2. The twist θ rotates them as follows:

$$\theta e_1 = e_2, \qquad \theta e_2 = -e_1 - e_2. \tag{33}$$

Then the Wilson lines satisfy the following conditions:

$$a_1 = a_2, \qquad 3a_1 = 0, \tag{34}$$

up to a lattice vector. Here we use the shorthand notation $a_i \equiv a^I(e_i)$.

Similarly, one can study constraints on discrete Wilson lines for other orbifolds [24,25]. The results are presented in Table 1. The second column shows the orbifold twists, while the third column gives an example of a 6D Lie lattice which realizes the orbifold twist by its Coxeter element (in general, there are more than one 6D lattices realizing the same orbifold twist). The constraints on the discrete Wilson lines depend on which 6D lattice is used. The 6D lattice shown in the third column is the one leading to most degrees of freedom for the discrete Wilson lines (see [24,25] for other lattices). The constraints are shown in the fourth column, where each equation is meant to be satisfied up to a gauge lattice vector.

To discuss the discrete Wilson lines further, we will need some facts about the selection rules for the Yukawa couplings. With every fixed point let us associate a space group element $(\theta^k, n_i e^i)$ according

Table 1
Allowed discrete Wilson lines

Orbifold	Twist	6D Lie lattice	Wilson line	Further constraints
Z_3	$(1, 1, -2)/3$	$SU(3)^3$	$3a_{1,3,5} = 0$	$a_{i+1} = a_i$ $(i = 1, 3, 5)$
Z_4	$(1, 1, -2)/4$	$SO(5)^2 \times SU(2)^2$	$2a_{2,4,5,6} = 0$	$a_1 = a_3 = 0$
Z_6-I	$(1, 1, -2)/6$	$SU(3) \times G_2^2$	$3a_1 = 0$	$a_1 = a_2, a_{3-6} = 0$
Z_6-II	$(1, 2, -3)/6$	$SU(3) \times SU(2)^2 \times G_2$	$3a_1 = 2a_{3,4} = 0$	$a_1 = a_2, a_{5,6} = 0$
Z_7	$(1, 2, -3)/7$	$SU(7)$	$7a_1 = 0$	$a_1 = a_{2-6}$
Z_8-I	$(1, 2, -3)/8$	$SO(9) \times SO(5)$	$2a_{4,6} = 0$	$a_{1,2,3,5} = 0$
Z_8-II	$(1, 3, -4)/8$	$SO(9) \times SU(2)^2$	$2a_{4-6} = 0$	$a_{1-3} = 0$
Z_{12}-I	$(1, 4, -5)/12$	$SU(3) \times F_4$	$3a_1 = 0$	$a_1 = a_2, a_{3-6} = 0$
Z_{12}-II	$(1, 5, -6)/12$	$SU(2)^2 \times F_4$	$2a_{1,2} = 0$	$a_{3-6} = 0$

to

$$\theta^k f_{k,n_i e^i} = f_{k,n_i e^i} + n_i e^i, \tag{35}$$

where n_i is an integer. Henceforth, we will denote a fixed point by its space group element $(\theta^k, n_i e^i)$. The fixed points $(\theta^p, \ell_i e^i + (1 - \theta^p)\Lambda)$ with different Λ are equivalent to each other. The twisted states of the orbifold are located at the fixed points (in even orbifolds a physical state corresponds to a linear combination of the fixed points in the same twisted sector [25,26]).

The Yukawa couplings among the three states corresponding to the fixed points $(\theta^p, \ell_i e^i)$, $(\theta^q, m_i e^i)$ and $(\theta^r, n_i e^i)$ are allowed only if certain selection rules are satisfied. In particular, the space group invariance requires that

$$\begin{aligned}
&(\theta^p, \ell_i e^i)(\theta^q, m_i e^i)(\theta^r, n_i e^i) \\
&= \left(1, \sum_{s=p,q,r} (1 - \theta^s)\Lambda\right).
\end{aligned} \tag{36}$$

We note that the right-hand side of this equation is equivalent to $(1, 0)$.

As an example, consider this selection rule for the Z_3 orbifold. Each 2D Z_3 orbifold has three fixed points 0, $e^1/3 + 2e^2/3$, and $2e^1/3 + e^2/3$, or in our notation, $(\theta, 0)$, $(\theta, -e^1 - e^2)$, and $(\theta, -e^1)$, respectively. Since the fixed points shifted by $(1 - \theta)\Lambda = ne^1 + (3m - n)e^2$ are equivalent, we can denote these fixed points by (θ, ke^1) with $k = 0, 1, 2$. Then the Yukawa coupling of the states corresponding to the fixed points $(\theta, \ell e^1)$, (θ, me^1) and (θ, ne^1) is allowed if

$$\ell + m + n = 0 \bmod 3, \tag{37}$$

and similarly for the other 2D planes. This selection rule implies that once ℓ and m are specified, n is fixed uniquely (mod 3). This sort of a selection rule is diagonal in the sense that the positions of the two of the fixed points determine the third one uniquely. The resulting quark Yukawa matrices are diagonal. Note that this orbifold allows for nontrivial Wilson lines (Table 1).

The space group selection rule is not always diagonal [25,27]. Consider the G_2 plane with the twist $e^{2\pi i/6}$ of the Z_6-I orbifold. Denote the lattice basis vectors as e_3 and e_4. The θ twist has one fixed point $(\theta, 0)$, θ^2 has three fixed points (θ^2, pe_3) $(p = 0, 1, 2)$, and θ^3 has four fixed points (θ^3, ne_3) $(n = 0, 1, 2, 3)$. It is easy to show that the coupling $(\theta, 0)(\theta^2, pe_3) \times (\theta^3, ne_3)$ is allowed for any p and n. That means that the positions of the two fixed points do not determine the position of the third one uniquely and the Yukawa matrices can have off-diagonal elements. On the other hand, discrete Wilson lines in this plane are not allowed (Table 1).

These two examples suggest that whenever off-diagonal Yukawa matrix elements are allowed, the discrete Wilson lines are *forbidden*. This is indeed true as can be checked by inspecting all orbifolds. The reason for that is as follows. A space group element $(\theta^p, n_i e^i)$ is embedded into the gauge space as $(1, pv + n_i a^i)$. The selection rule (36) then implies

$$\begin{aligned}
&(1, pv + \ell_i a^i)(1, qv + m_i a^i)(1, rv + n_i a^i) \\
&= (1, \Lambda_{E_8 \times E_8}).
\end{aligned} \tag{38}$$

Since $p + q + r = 0 \bmod N$ and Nv is a lattice vector, we have

$$(\ell_i + m_i + n_i)a^i = \Lambda_{E_8 \times E_8}. \tag{39}$$

Suppose now that $\ell_1 = 0, 1$ are both allowed such that the selection rule is not diagonal. Taking the difference of (39) with $\ell_1 = 0$ and $\ell_1 = 1$, we obtain

$$a^1 = \Lambda_{E_8 \times E_8} \tag{40}$$

which is equivalent to zero.

This result has important implications. Suppose that the presence of discrete Wilson lines results in CP phases in the Yukawa couplings. Since in this case the Yukawa matrices are bound to be diagonal, these CP phases can always be rotated away by a redefinition of the right handed quarks. Thus, at the renormalizable level, discrete Wilson lines do not lead to CP violation.[6]

This result may be altered by the presence of non-renormalizable operators contributing to the Yukawa matrices. Although they involve exponentially small factors [18], they are not necessarily small numerically [29] and thus can have a nonnegligible effect. The difficulty with an explicit calculation of the discrete Wilson line contribution is that the standard action (3) is not invariant (or does not transform as $S \to S + 2\pi n$) under an $E_8 \times E_8$ lattice shift of the Wilson line for *arbitrary* ∂X^i. For the same reason, it is not generally twist-invariant. This problem remains open.

5. Conclusion

We have studied a connection between heterotic string backgrounds and CP violation in the Yukawa couplings. We find that only the antisymmetric background field B_{ij} is a viable candidate for the source of the observed CP violation. The continuous Wilson lines and the U-moduli conserve CP, whereas the discrete Wilson lines do not lead to physical CP phases at least at the renormalizable level.

Acknowledgements

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[6] This argument may not apply if there are many Higgs doublets in the low energy theory (as in Ref. [28]).

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