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On the description of exclusive processes beyond the leading twist approximation

I.V. Anikin [a], D.Yu. Ivanov [b], B. Pire [c,*], L. Szymanowski [d], S. Wallon [e,f]

[a] Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia
[b] Institute of Mathematics, 630090 Novosibirsk, Russia
[c] CPhT, École Polytechnique, CNRS, F-91128 Palaiseau, France
[d] Sołtan Institute for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland
[e] LPT, Université d'Orsay, CNRS, 91404 Orsay, France
[f] UPMC Univ. Paris 6, Faculté de Physique, 4 place Jussieu, 75252 Paris Cedex 05, France

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ABSTRACT

We describe hard exclusive processes beyond the leading twist approximation in a framework based on the Taylor expansion of the amplitude around the dominant light-cone directions. This naturally introduces an appropriate set of non-perturbative correlators whose number is minimalized after taking into account QCD equations of motion and the invariance under rotation on the light-cone. We exemplify this method at the twist 3 level and show that the coordinate and momentum space descriptions are fully equivalent.

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Exclusive reactions have been the scene of much recent progress in the domain of perturbative QCD physics. Collinear factorization of the amplitude for vector meson production has been demonstrated at the leading twist level [1]. In this case the meson field appears through twist 2 distribution amplitudes (DAs) which are defined as the Fourier transforms of the matrix element of nonlocal two particle operators $\mathcal{O}(x)$, such as $\bar{\psi}(x)\Gamma\psi(0)$ for fermionic partons, Γ being an appropriate Dirac structure. Generalizing this factorization procedure to the twist 3 case is mandatory in a number of cases (provided the value of the hard scale is sufficiently high to justify phenomenologically the twist expansion), and in particular for the understanding of the production of transversally polarized vector mesons [2], since their twist 2 DAs are often decoupled because of their chiral-odd nature [3].

Following the strategy outlined in [4], we here generalize the approaches [4,5] in order to get a thorough description of the exclusive processes. In particular, we discuss the role of the light-cone direction independency condition and fully explore its consequences in the analysis of genuine higher twist effects, in particular beyond the Wandzura–Wilczek approximation. We show that the present framework is equivalent to the coordinate description developed in [6,7].

In this Letter we summarize only the main ideas of our developments, which are to a big extent valid for arbitrary exclusive processes. However, for illustration of the main steps of the derivation, in particular of the implementation of the invariance under rotation on the light-cone of the physical amplitude, we will rely on the specific meson impact factor case at twist 3, when needed. This quantity is illustrated in Fig. 1, it enters the description of transversely polarized vector mesons production at small x, i.e. at large energy, in $\gamma^*\gamma^*$ and $\gamma^* p$ collisions. For the aims of present Letter it is enough to understand that it represents a part of the amplitudes of above mentioned processes which describes the transition of a virtual photon to a vector meson mediated by two virtual gluons being in the colorless state. Full details of the derivations and explicit definition of impact factor are given in [8].

Thus, we start with the amplitude presented in Fig. 1, written as (with $\{d^4\ell\}_2 = d^4\ell_1\, d^4\ell_2$)

$$\mathcal{A} = \int d^4\ell\, \mathrm{tr}\big[H(\ell)\Phi(\ell)\big] + \int \{d^4\ell\}_2\, \mathrm{tr}\big[H_\mu(\ell)\Phi^\mu(\ell)\big] + \cdots \quad (1)$$

where H and H_μ are the hard parts with two parton legs and two parton and one gluon legs, respectively. It is important to note that it is assumed that virtualities of t-channel gluons in Fig. 1 are large, being of order of photon virtuality Q^2. Thus, all non-perturbative, soft momenta part of the subprocess described by the impact factor is included in the blobs Φ and Φ^μ. In (1), the soft parts are given by the Fourier-transformed two or three body

* Corresponding author.
E-mail address: pire@cpht.polytechnique.fr (B. Pire).

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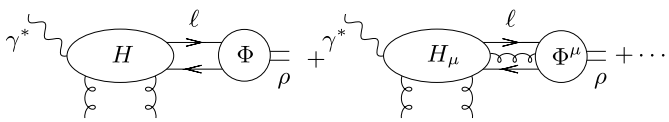

Fig. 1. 2- and 3-body correlators attached to a hard scattering amplitude in the example of the $\gamma^* \to \rho$ impact factor.

correlators. Inclusion of 4-body correlators would lead to twist $\geqslant 4$ contributions, which are suppressed by additional powers of $1/Q$. Besides, here we consider the case of vanishing transverse momentum transfer in the $\gamma^* \to \rho$ transition and we neglect corrections of the order m_ρ/Q.

We have to parameterize momenta and polarization vector of the transversely polarized meson (e). In our subprocess $\gamma^*(q) \to \rho(p_\rho)$, $q^2 = -Q^2$, $p_\rho^2 = 0$,

$$q_\mu = p_\mu - \frac{Q^2}{2}\tilde{n}_\mu, \qquad p_{\rho\mu} = p_\mu. \tag{2}$$

Here p ($\tilde{n}$) be a "plus" ("minus") light-cone vector, respectively (normalized as $p \cdot \tilde{n} = 1$). Such light-cone vectors can be easily constructed from the 4-vectors describing the kinematics of the process in question.

The polarization vector of transversely polarized meson obeys

$$e \cdot p = 0, \qquad e \cdot \tilde{n} = 0. \tag{3}$$

This vector coincides with the physical polarization vector in the infinite ρ-meson momentum frame.

The amplitude (1) is not factorized yet because the hard and soft parts are related by the 4-dimensional integration in the momentum space and by the summation over the Dirac and/or Lorentz indices. To factorize the amplitude, we need to select the dominant direction around which we intend to decompose our relevant momenta and the hard part. For this purpose we choose another pair of light-cone vectors: p and n (normalized again as $p \cdot n = 1$). It is very important to note that here we take the freedom to choose n arbitrary, having no direct relationship with the parametrization of external momenta, $n \neq \tilde{n}$. We carry out an expansion of ℓ in the basis defined by p and n:

$$\ell_{i\mu} = y_i p_\mu + (\ell_i \cdot p) n_\mu + \ell_{i\mu}^\perp \tag{4}$$

with $y_i = \ell_i \cdot n$, and rewrite the integration measure in (1) as $d^4\ell_i \to d^4\ell_i\, dy_i\, \delta(y_i - \ell \cdot n)$. We also decompose the hard part around the dominant "plus" direction:

$$H(\ell) = H(yp) + \frac{\partial H(\ell)}{\partial \ell_\alpha}\bigg|_{\ell = yp} (\ell - yp)_\alpha + \cdots, \tag{5}$$

with $(\ell - yp)_\alpha \approx \ell_\alpha^\perp$ up to twist 3 accuracy, where α denotes transverse components with respect to vectors p and n.

The $l^\perp$ dependence of the hard part looks first as an excursion out of the collinear framework. The factorized expression is obtained as the result of an integration by parts which replaces $\ell_\alpha^\perp$ by $\partial_\alpha^\perp$ acting on the soft correlator. This leads to new operators $\mathcal{O}^\perp$ which contain transverse derivatives, such as $\bar{\psi}\partial^\perp\psi$, and thus to the necessity of considering additional correlators $\Phi^\perp(l)$. This procedure leads to the factorization of the amplitude in momentum space. Factorization in the Dirac space can be achieved by the Fierz decomposition. For example, in the case of two fermions, one should project out the Dirac matrix $\psi_\alpha(0)\bar{\psi}_\beta(z)$ which appears in the soft part of the amplitude on the relevant Γ matrices. Thus, the amplitude finally takes the simple factorized form:

$$\mathcal{A} = \int_0^1 dy\, \mathrm{tr}\big[H(y)\Gamma\big]\Phi^\Gamma(y) + \int_0^1 dy_1\, dy_2\, \mathrm{tr}\big[H^\mu(y_1, y_2)\Gamma\big]\Phi_\mu^\Gamma(y_1, y_2). \tag{6}$$

The first term of Eq. (6) requires the introduction of the soft correlators with quark–antiquark nonlocal operators. Generically,[1]

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho f_\rho\big[\varphi_1(y)(e^* \cdot n)p_\mu + \varphi_3(y)e_\mu^{*T}\big], \tag{7}$$

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho f_\rho i\varphi_A(y)\varepsilon_{\mu\lambda\beta\delta}e_\lambda^{*T}p_\beta n_\delta, \tag{8}$$

where $\overset{\mathcal{F}_1}{=}$ denotes the Fourier transformation $\int_0^1 dy \exp[iyp \cdot z]$. The light-like separation $z = \lambda n$ involves an arbitrary light-like vector n with $n \cdot p = 1$ which already appeared in the Sudakov decomposition (4). The above correlators are defined in the axial light-like gauge $n \cdot A = 0$, which allows to get rid of Wilson lines. Thus, n plays a triple role: it first fixes the axial gauge, second it defines the transverse momentum of partons forming the ρ in the collinear expansion, and last it determines the notion e^T of transverse polarization of the ρ in the above equations. Note that e^T should not be confused with the physical polarization vector of transverse meson, defined by the conditions (3).

The second term of Eq. (6) requires the introduction of matrix elements of quark–transverse gluon operators

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho f_{3\rho}^V B(y_1, y_2)p_\mu e_\alpha^{*T}, \tag{9}$$

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho f_{3\rho}^A iD(y_1, y_2)p_\mu\varepsilon_{\alpha\lambda\beta\delta}e_\lambda^{*T}p_\beta n_\delta, \tag{10}$$

where $\int_0^1 dy_1 \int_0^1 dy_2 \exp[iy_1 p \cdot z_1 + i(y_2 - y_1)p \cdot z_2]$ is denoted by $\overset{\mathcal{F}_2}{=}$. Note that the positivity of the gluon light-cone momentum fraction imposes that quark–gluon parameterizing functions have the form

$$B/D(y_1, y_2) \equiv \mathcal{B}/\mathcal{D}(y_1, y_2; y_2 - y_1)\theta(y_1 \leqslant y_2 \leqslant 1).$$

Here, the light-cone fractions of the quark, antiquark and gluon are respectively y_1, $1 - y_2$ and $y_2 - y_1$.

Finally, the second term of (5) leads to the introduction of the correlators

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu i\overleftrightarrow{\partial}_\alpha^\perp\psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho f_\rho\varphi_1^T(y)p_\mu e_\alpha^{*T}, \tag{11}$$

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_\alpha^\perp\psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho f_\rho i\varphi_A^T(y)p_\mu\varepsilon_{\alpha\lambda\beta\delta}e_\lambda^{*T}p_\beta n_\delta. \tag{12}$$

Let us recall the symmetry properties of DAs [4] coming from C invariance (with $\bar{y} \equiv 1 - y$):

[1] We use $\epsilon^{0123} = -\epsilon_{0123} = 1$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

$$\varphi_1(y)=\varphi_1(\bar{y}),\qquad \varphi_3(y)=\varphi_3(\bar{y}),\qquad \varphi_A(y)=-\varphi_A(\bar{y}),$$
$$\varphi_1^T(y)=-\varphi_1^T(\bar{y}),\qquad \varphi_A^T(y)=\varphi_A^T(\bar{y}),$$
$$\mathcal{B}(y_1,y_2;y_g)=-\mathcal{B}(\bar{y}_2,\bar{y}_1;y_g),$$
$$\mathcal{D}(y_1,y_2;y_g)=\mathcal{D}(\bar{y}_2,\bar{y}_1;y_g). \tag{13}$$

The correlators introduced above are not independent, thanks to the QCD equations of motion for the field operators entering them (see, for example, [4]). In the simplest case of fermionic fields, they follow from the vanishing matrix elements $\langle(i\hat{D}(0)\psi(0))_\alpha\bar{\psi}_\beta(z)\rangle=0$ and $\langle\psi_\alpha(0)i(\hat{D}(z)\bar{\psi}(z))_\beta\rangle=0$ due to the Dirac equation, then projected on different Fierz structure. Denoting $\zeta_{3,\rho}^{V,A}=f_{3\rho}^{V,A}/f_\rho$, we obtain

$$\bar{y}_1\varphi_3(y_1)+\bar{y}_1\varphi_A(y_1)+\varphi_1^T(y_1)+\varphi_A^T(y_1)$$
$$=-\int_0^1 dy_2\left[\zeta_3^V B(y_1,y_2)+\zeta_3^A D(y_1,y_2)\right], \tag{14}$$

$$y_1\varphi_3(y_1)-y_1\varphi_A(y_1)-\varphi_1^T(y_1)+\varphi_A^T(y_1)$$
$$=-\int_0^1 dy_2\left[-\zeta_3^V B(y_2,y_1)+\zeta_3^A D(y_2,y_1)\right]. \tag{15}$$

Note that Eq. (15) can be obtained by the replacement $y_1\to\bar{y}_1$ in (14) and the use of symmetry properties (13).

We stress again that contrarily to the light-cone vector p related to the out-going meson momentum, the second light-cone vector n, required for the parametrization of the correlators (7)–(12), is completely arbitrary and connected in no way to the momenta of particles of the process in question. Alternatively, the notions of transverse momentum, of transverse polarization and of axial gauge, lead to a description of the factorization procedure which is non-covariant. One should thus impose, due to Lorentz invariance, that any physical observables do not depend on the specific choice of n, e.g. the scattering amplitudes should be n-independent. It turns out that this requirement leads to additional non-trivial constraints between the non-perturbative correlators entering the factorized amplitude. This n-independency property was crucial for the understanding of inclusive structure functions properties at the twist three level [5] and its relevance for some exclusive processes was pointed out in [4]. We show now that this condition expressed at the level of the *full amplitude* of any process can be reduced to a set of conditions involving only the soft correlators. The strategy relies on the power of the Ward identities to relate firstly amplitudes with different number of legs and secondly higher order coefficients in the Taylor expansion (5) to lower order ones.

At first sight the n-independency condition of the process amplitude could not lead to some process independent results. Indeed, not only the parametrizations of the hadronic matrix elements but also the process dependent hard scattering coefficient H depends (indirectly) on n, since n defines the direction of the collinear expansion, see Eq. (5). With this respect we remind that using integration by parts in the d^4l integral (1) one replaces $\ell_\alpha^\perp$ by $\partial_\alpha^\perp$ acting on the soft correlator. Therefore the n-independency of the process amplitude can be reduced entirely in considered approximation (at twist-3) to the consideration of the n-dependency of the soft correlators, see Eqs. (7)–(12), which parametrizations depend on n both explicitly and implicitly through the n-dependency of the polarization vector $e_\perp^T$.

The light-cone vector n satisfying $n\cdot p=1$ may be parameterized by its transverse components $n_\perp$ defined with respect to the light-cone vectors p and $\tilde{n}$ fixed by the external kinematics (see Eq. (2))

$$n^\alpha=-\frac{n_\perp^2}{2}p^\alpha+\tilde{n}^\alpha+n_\perp^\alpha. \tag{16}$$

Thus the n-independence of $\mathcal{A}$ for an arbitrary fixed polarization vector e is expressed by the condition[2]

$$\frac{d}{dn_\perp^\mu}\mathcal{A}=0. \tag{17}$$

On the other hand, the scattering amplitude $\mathcal{A}$ receives contributions from the vector correlators, which result into $\mathcal{A}^{vector}$, and from the axial vector correlators, which lead to $\mathcal{A}^{axial}$ part of $\mathcal{A}$. Due to different parity properties of the vector and the axial-vector correlators, the condition (17) means effectively two separate conditions:

$$\frac{d}{dn_\perp^\mu}\mathcal{A}^{vector}=0 \tag{18}$$

and

$$\frac{d}{dn_\perp^\mu}\mathcal{A}^{axial}=0. \tag{19}$$

The dependence of $\mathcal{A}$ on the vector $n_\perp$ is obtained through the dependence of $\mathcal{A}$ on the full vector n. This dependence on n is different in $\mathcal{A}^{axial}$ and in $\mathcal{A}^{vector}$ parts.

The dependence of $\mathcal{A}^{axial}$ on the vector n enters only through the expression $\varepsilon^{pn\beta\gamma}$ involving the contraction with the momentum p and in which the indices β and γ are contracted with some other vectors. Thus the condition (19) is equivalent to

$$\frac{\partial n^\alpha}{\partial n_\perp^\mu}\frac{\partial}{\partial n^\alpha}\mathcal{A}^{axial}=\left[-n_\perp^\mu p^\alpha+g_\perp^{\alpha\mu}\right]\frac{\partial}{\partial n^\alpha}\mathcal{A}^{axial}$$
$$=\frac{\partial}{\partial n_\perp^\mu}\mathcal{A}^{axial}=0, \tag{20}$$

where we took into account the peculiar dependence of $\mathcal{A}^{axial}$ on n discussed above. This will lead at the level of DAs to the equation

$$\frac{d}{dy_1}\varphi_A^T(y_1)=\varphi_A(y_1)-\zeta_3^A\int_0^1\frac{dy_2}{y_2-y_1}$$
$$\times\left(D(y_1,y_2)+D(y_2,y_1)\right). \tag{21}$$

The polarization vector for transverse ρ which enters in the parametrization of twist 3 correlators is defined as

$$e_T^\mu=e^\mu-p^\mu e\cdot n. \tag{22}$$

It turns out, as one can check explicitly in this example, that the dependence of $\mathcal{A}^{vector}$ on the vector n enters only through the scalar product $e^*\cdot n$, and Eq. (18) can be written as

$$\frac{d}{dn_\perp^\mu}\mathcal{A}^{vector}=e_T^{*\mu}\frac{\partial}{\partial(e^*\cdot n)}\mathcal{A}^{vector}=0, \tag{23}$$

[2] The appearance of the total derivative may be interpreted as a (vector) analog to the renormalization group (RG) invariance when the dependences on the renormalization parameter coming from various sources cancel. One can view this as a RG-like flow in the space of light-cone directions of contributions to the amplitude where the polarization vector plays the role of a beta function.

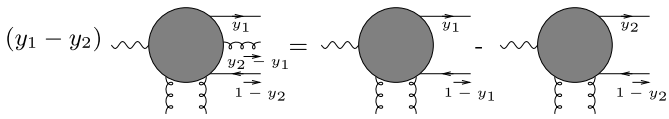

Fig. 2. Reduction of 3-body correlators to 2-body correlators through Ward identity.

from which follows

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1) - \zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1} \times \big(B(y_1, y_2) + B(y_2, y_1)\big). \tag{24}$$

We now sketch the derivation of Eq. (24), which corresponds to the vector correlator contributions, in the peculiar case of the $\gamma^* \to \rho$ impact factor, which is the building block of the description of the $\gamma^* p \to \rho p$ process at large s. The 3-body $(q\bar{q}g)$ contribution and the 2-body contribution involving $\Phi^\perp$ to $\mathcal{A}$ can be reduced to the convolution of the leading order hard 2 particle contributions with linear combination of correlators, thanks to the use of the Ward identity. In the case of the 3-body vector correlator (9), due to (22) the dependency of e_T on $n_\perp$ enters linearly and only through the scalar product $e^* \cdot n$. Using the fact that the amplitude $\mathcal{A}$ depends linearly on the polarization vector e, one finds that the action on the amplitude of the derivative $\frac{\partial}{\partial(e^* \cdot n)}$ in (23) can be extracted by the replacement $e^*_\alpha \to -p_\alpha$. After using the Ward identity, it reads

$$(y_1 - y_2)\,\mathrm{tr}\big[H_\rho(y_1, y_2)p^\rho \not{p}\big] = \mathrm{tr}\big[H(y_1)\not{p}\big] - \mathrm{tr}\big[H(y_2)\not{p}\big],$$

as illustrated by Fig. 2. This results in the last term of r.h.s. of Eq. (24). A similar treatment of 2-body correlators with transverse derivative whose contribution can be viewed as 3-body processes with vanishing gluon momentum leads to the l.h.s. of Eq. (24). The first term with φ_1 of the r.h.s. of Eq. (24) originates from the 2-body vector correlator and corresponds to the contribution of $e_L \sim p$. The second term with φ_3 corresponds to the contribution of the same correlator for ρ_T supplemented by Eq. (22). A similar treatment for axial correlators leads to Eq. (21).

Since we rely on the n-independency of the amplitude, one may wonder about the effect of the gauge choice, which is fixed by n, on the hard part. The QCD Ward identities require the vanishing of the amplitude in which polarization vector of a gluon is replaced by its momentum provided all other partons are on the mass shell. In the framework of the k_T-factorization the t-channel gluons are off the mass-shell. Therefore the replacement of the s-channel gluon polarization vector by its momentum leads to the vanishing of scattering amplitude up to terms proportional to $k_\perp^2/s$ where $k_\perp$ are transverse momenta of t-channel gluons. From the point of view of the t-channel, the gauge invariance of the impact-factor means that it should vanish when the transverse momentum of any t-channel gluon vanishes. To achieve this property it is necessary to include in a consistent way not only DAs with lowest Fock state containing only quarks but also those involving quarks and gluon, see [8].

In practice, we here check this invariance by contracting the s-channel emitted gluon vertex in the hard part with the momentum, which in collinear factorization is proportional to the ρ-meson momentum, this leads to simplifications in the use of (collinear) Ward identities. This means that the hard part is gauge invariant.

In order to prove this, one should first project on the various color Casimir structure. In the case of the impact factor, this means to distinguish N_c and C_F terms. In this case, C_F terms arise from 2-partons diagrams and from 3-partons diagrams where the emitted gluon is attached to a quark line, while N_c terms are obtained from 3-partons diagrams where the emitted gluon is attached to a quark line only between 2 t-channel exchanged gluons or from diagrams involving at least one triple gluon vertex. Using Ward identity, the result looks like the product of the 2-parton diagrams contributing to the transition $\gamma_T^* \to \rho_L$. One can readily check at Born order that this transition vanishes, in our twist 3 treatment. In the case of contribution proportional to N_c, one can prove that this gives zero. Note that terms proportional to the virtuality $\underline{k}^2$ remains, but they are of the form $\underline{k}^2/s$, which are higher order with respect to the dominant s-power contributions to the impact factor (which scales like s^0) involved in the k_T-factorization approach. This is thus in agreement with Ward identity in perturbative Regge factorization.

Although our implementation of factorization and n-independence condition was illustrated on the particular example of the impact factor at twist 3, we expect that this procedure is more general and that the above method can be applied for other exclusive processes, the key tool being the collinear Ward identity. This means in particular that each building block (soft and hard part, for each structure which lead to the introduction of a DA) are separately gauge invariant. This fact simplifies dramatically the use of the n-independence principle.

We thus emphasize that the equations obtained above for the special case of impact factor are expected to be universal, i.e. not to refer to any specific hard process. The equations of motion (14), (15) together with Eqs. (24), (21) following from n-independence are four equations which constrain seven DAs $(\varphi_1, \varphi_3, \varphi_1^T, \varphi_A, \varphi_A^T, B, D)$. Thus, any hard process involving the exclusive production of a ρ can be expressed, at twist 3, in terms of the three independent DAs which we choose as φ_1, B and D. The solution of Eqs. (14), (15), (24), (21) with vanishing B and D DAs corresponds to the Wandzura–Wilczek approximation and takes the form[3]

$$\varphi_{3/A}^{WW}(y) = \frac{1}{2}\left[\int_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm \int_y^1 \frac{dv}{v}\varphi_1(v)\right], \tag{25}$$

$$\varphi_{1/A}^{TWW}(y) = \frac{1}{2}\left[-\bar{y}\int_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm y\int_y^1 \frac{dv}{v}\varphi_1(v)\right]. \tag{26}$$

The remaining genuine twist 3 DAs read:

$$\begin{aligned}\varphi_3^{gen}(y) = -\frac{1}{2}\int_y^1 \frac{du}{u}\Bigg[&\int_0^u dy_2 \frac{d}{du}\big(\zeta_3^V B - \zeta_3^A D\big)(y_2, u) \\ &- \int_u^1 \frac{dy_2}{y_2 - u}\big(\zeta_3^V B - \zeta_3^A D\big)(u, y_2) \\ &- \int_0^u \frac{dy_2}{y_2 - u}\big(\zeta_3^V B - \zeta_3^A D\big)(y_2, u)\Bigg] \\ &- \frac{1}{2}\int_0^{y_1} \frac{du}{\bar{u}}\Bigg[\int_u^1 dy_2 \frac{d}{du}\big(\zeta_3^V B + \zeta_3^A D\big)(u, y_2)\end{aligned}$$

[3] We use the generic notation $\varphi = \varphi^{WW} + \varphi^{gen}$.

$$-\int_u^1 \frac{dy_2}{y_2-u}\big(\zeta_3^V B+\zeta_3^A D\big)(u,y_2)$$
$$-\int_0^u \frac{dy_2}{y_2-u}\big(\zeta_3^V B+\zeta_3^A D\big)(y_2,u)\Bigg], \tag{27}$$

$$\varphi_1^{T\,gen}(y)=\int_0^y du\,\varphi_3^{gen}(u)-\zeta_3^V\int_0^y dy_1\int_y^1 dy_2\frac{B(y_1,y_2)}{y_2-y_1}, \tag{28}$$

while the corresponding expressions for $\varphi_A^{gen}(y)$ and $\varphi_A^{T\,gen}(y)$ are obtained by the substitutions:

$$\varphi_A^{gen}(y)\overset{\zeta_3^V B\leftrightarrow\zeta_3^A D}{\longleftrightarrow}\varphi_3^{gen}(y), \tag{29}$$

$$\varphi_A^{T\,gen}(y)\overset{\zeta_3^V B\leftrightarrow\zeta_3^A D}{\longleftrightarrow}\varphi_1^{T\,gen}(y). \tag{30}$$

Let us now discuss an equivalent way for obtaining the result (28). We start with the following operator equation [6] written in a gauge-invariant form, where on the right-hand side we only keep the contributions of vector correlators

$$\frac{\partial}{\partial z_\alpha}\big[\bar\psi(z)\gamma_\mu[z,-z]\psi(-z)\big]$$
$$=-\bar\psi(z)\gamma_\mu[z,-z]\overrightarrow{D_\alpha}\psi(-z)+\bar\psi(z)\overleftarrow{D_\alpha}\gamma_\mu[z,-z]\psi(-z)$$
$$-ig\int_{-1}^1 dv\,v\bar\psi(z)[z,vz]z_\nu G_{\nu\alpha}(vz)\gamma_\mu[vz,-z]\psi(-z), \tag{31}$$

where [ ] denotes the Wilson line and $\overrightarrow{D_\alpha}=\overrightarrow{\partial_\alpha}-igA_\alpha(-z)$ and $\overleftarrow{D_\alpha}=\overleftarrow{\partial_\alpha}+igA_\alpha(z)$. Note that in (31), the derivatives act on the arguments of the fermion fields. In principle, the path in the Wilson line can be an arbitrary one, provided it lies entirely on the light-cone. However, for simplicity, we choose the path to be a straight line. We now compute the matrix elements of the various part of Eq. (31).

First, using the parametrization (9), the matrix element of the quark–gluon part on r.h.s. of Eq. (31) reads

$$\big\langle\rho(p)\big|\bar\psi(z)\gamma_\mu\big\{[z,-z]A_\alpha(-z)+A_\alpha(z)[z,-z]\big\}\psi(-z)|0\rangle$$
$$-\int_{-1}^1 dv\,v\big\langle\rho(p)\big|\bar\psi(z)[z,vz]z_\nu G_{\nu\alpha}(vz)\gamma_\mu[vz,-z]\psi(-z)|0\rangle$$
$$=2m_\rho f_{3\rho}^V p_\mu e_\alpha^{*T}\int_0^1 dy\,e^{i(y-\bar y)p\cdot z}\int_0^y dx_1\int_y^1 dx_2\,\frac{B(x_1,x_2)}{x_2-x_1}, \tag{32}$$

where in the last stage integration by parts has been used.

Further, we turn on the quark–antiquark operator in (31) and its vacuum-to-meson correlator. Using (22), the correlator (7) can be rewritten as

$$\big\langle\rho(p)\big|\bar\psi(z)\gamma_\mu[z,-z]\psi(-z)|0\rangle$$
$$=-2im_\rho f_\rho p_\mu\big(e^*\cdot z\big)\int_0^1 dy\,e^{i(y-\bar y)p\cdot z}h(y)+\cdots, \tag{33}$$

where

$$h(y)=\int_0^y du\big(\varphi_1(u)-\varphi_3(u)\big), \tag{34}$$

where the ellipsis denotes terms which are responsible for corrections of m_ρ^2-order and, therefore, they are part of twist 4 corrections [7], which we omit in this Letter.

Up to twist 3, we have

$$\frac{\partial}{\partial z_\alpha}\big[\big\langle\rho(p)\big|\bar\psi(z)\gamma_\mu[z,-z]\psi(-z)|0\rangle\big]$$
$$=-2im_\rho f_\rho p_\mu e_\alpha^*\int_0^1 dy\,e^{i(y-\bar y)p\cdot z}h(y). \tag{35}$$

Finally, the pure quark part of r.h.s. of Eq. (31) together with the parametrization (11) and the translation invariance yields,

$$\big\langle\rho(p)\big|\bar\psi(z)\gamma_\mu\big[\overrightarrow{\partial_\alpha^\perp}-\overleftarrow{\partial_\alpha^\perp}\big]\psi(-z)|0\rangle$$
$$=-2im_\rho f_\rho p_\mu e_\alpha^{*T}\int_0^1 dy\,e^{i(y-\bar y)p\cdot z}\varphi_1^T(y). \tag{36}$$

Thus, using (32), (35), (36), the matrix element of Eq. (31) leads to

$$h(y)=-\varphi_1^T(y)-\zeta_3^V\int_0^y dx_1\int_y^1 dx_2\,\frac{B(x_1,x_2)}{x_2-x_1} \tag{37}$$

which reproduces the results (25)–(27) obtained previously. This can be seen most easily by inserting in Eq. (37) the expressions (28) for $\varphi_1^{T\,gen}$ and (26) for $\varphi_1^{T\,WW}$. The B terms then cancel out and what remains to be shown is that the r.h.s. of (34) is reproduced. The φ_3^{gen} is readily obtained and the integral of φ_1 and of φ_3^{WW} in the r.h.s. of (34) is correctly reproduced after interchange of integrations. This operator approach can be applied similarly to the axial part.

We now relate our results (27), (29), with those obtained in studies of DAs based on the conformal expansion on the (coordinate space) light cone [7]. Comparison between (9), (10) and similar expressions of Ref. [7] in the light-cone $n\cdot A=0$ gauge leads to the identification

$$B(y_1,y_2)=-\frac{1}{m_\rho}\frac{V(y_1,1-y_2,y_2-y_1)}{y_2-y_1}, \tag{38}$$

$$D(y_1,y_2)=-\frac{1}{m_\rho}\frac{A(y_1,1-y_2,y_2-y_1)}{y_2-y_1}. \tag{39}$$

Using these expressions for V and A one can now recover our results (27), (29) after straightforward but tedious manipulations, provided the important identification is done, for the vector projection:

$$\varphi_1(y)=\phi_\parallel(y),\qquad \varphi_3(y)=g^{(v)}(y), \tag{40}$$

and for the axial projection

$$\varphi_A(y)=-\frac{1}{4}\frac{\partial g^{(a)}(y)}{\partial y}. \tag{41}$$

In summary, we describe consistently exclusive processes at higher twist. The specific $\gamma^*\to\rho$ impact factor will indeed be shown to be gauge invariant and free from end-point singularities in a separate publication [8]. The n-independency condition generalized up to the dynamical twist 3 plays a crucial role for the consistency of this approach with the studies of DAs performed in [7].

The present framework opens the way to a systematic and consistent treatment of hard exclusive processes. This does not preclude the solution of the well-known end-point singularity problem [2,9] which may still require a separate treatment. Let us

finally emphasize the simplicity of the present method to perform practical calculations [8].

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