Research on Dynamic Fracture Toughness of Granite and Finite Element Analysis

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Abstract

In order to investigate the dynamic fracture toughness of granite, the Brazilian disk specimens were subjected to diametral impact by using the split Hopkinson pressure bar. Based on three-dimensional model of SHPB system, the test loading was introduced into Finite element model, the dynamic stress intensity factor k(t) was determined directly from the specimens at the crack tip, and then get the dynamic fracture toughness of material. The maximum average test load was substituted into the quasi-static equation to get the value of dynamic fracture toughness comparing with the results from Finite element calculation. After comparison, it is feasible to extent the quasi-static equation into dynamic situation.

Keywords: rock mechanics; SHPB; fracture; toughness; finite element analysis

1. Introduction

In many mining and civil engineering applications, such as Explosion, the impact of ground pressure and so on, Rocks are loaded dynamically. In order to prevent the emergence of rock structures damage, it is necessary to understand the dynamics of the rock failure mechanism, the study of dynamic mechanical properties of rock materials has an important practical significance[1].

Rock dynamic fracture toughness is considered as the critical parameters in rock mechanical properties. Accurate measurement of dynamic fracture toughness are crucial in engineering field. Until now, Various of methods have been proposed in the literature to measure the dynamic fracture toughness of rock including semi-circular bend[2, 3], Brazilian disc[4], Cracked chevron notched brazilian disc(CCNBD)[5]and Cracked straight through brazilian disc(CSTBD)[6]. International Society of Rock
Mechanics also proposed cracked chevron notched brazilian disc as a sample to test the dynamic fracture toughness in 1995[5].

At present, many scholars have already started testing the dynamic fracture toughness of rock. Tang[7] tried to measure dynamic fracture toughness of rocks by three point impact using a single Hopkinson bar. Zhang[7][9] employed the split Hopkinson pressure bar (SHPB) technique to measure the rock dynamic fracture toughness with short rod specimens. The two attempts with Hopkinson to investigate the evolution of stress intensity factor, and the fracture toughness were calculated using a quasi-static analysis without full consideration of the loading inertial effect, but the inertial effect in dynamic result in unreliable data[10]. Wang[11] used finite element analysis with the dynamic load recorded in the experiment as the input for the numerical analysis, and then the dynamic fracture toughness is determined by crack initiation time, however, this requires lots of post-processing time.

In this work, we measure the dynamic fracture toughness using the central sharp-notched circular disk specimen, loaded dynamically with the split Hopkinson bar system. A three-dimensional model of SHPB system is build to study the dynamic fracture toughness. After the comparison of the experimental and numerical results, it is feasible to promotion the quasi-static formular to dynamic situation.

2. Basic theory of SHPB technique

Fig. 1 shows the major components of the conventional of SHPB experimental set-up. The conventional SHPB technique mainly consists of a stricker bar, pressure bar and transmitted bar. A compressed-air gun is used to accelerate the stricker bar to impact the pressure bar, and the impact results in an incident elastic wave (\( \varepsilon_i \)) generated at the impact face of the incident bar. The incident elastic compressive wave travels in the incident bar toward the specimen. Due to the impedance mismatch between the specimen and the pressure bars, part of the incident elastic compressive wave is reflected (\( \varepsilon_r \)) and returns to the impact face. Part of the incident elastic compressive wave transmits through the specimen into the transmitted bar. A strain gauge mounted on the incident bar and transmitted bar measure the incident pulse (\( \varepsilon_i \)), reflect pulse (\( \varepsilon_r \)) and transmitted pulse (\( \varepsilon_t \)). Based on the one-dimensional elastic wave propagation theory with the assumption of homogeneous deformation of specimen, the forces on both ends of the specimen can be calculated as:

\[
\begin{align*}
    P_L(t) &= EA[\varepsilon_i(t) + \varepsilon_r(t)] \\
    P_R(t) &= EA\varepsilon_i(t) \\
    \bar{P}(t) &= \frac{P_L(t) + P_R(t)}{2} = \frac{EA}{2} \left[\varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t(t)\right]
\end{align*}
\]

Where \( A \) is cross section area and \( E \) is elastic modulus of the bar. \( \bar{P}(t) \) is the average load of specimen.

![Fig.1 The schematics of Hopkinson pressure bar](image-url)
3. Experimental work

3.1. The Experimental Setup

The dynamic test was conducted on a 37mm diameter split Hopkinson pressure bar (SHPB) system, taking into account the low wave impedance of rock materials, both of the incident bar and the transmitted bar made of Aluminum in order to obtain accurate results. The material’s mechanical parameters were: elastic modulus 70 GPa, Poisson’s ratio 0.3, density 2700 Kg/m3. The length of the incident bar is 2000mm, the length of the transmission bar is 2000mm. Strain gauges were mounted on the incident bar at a distance of 1000mm and at the transmission bar at a distance of 400mm to the end contacting the surface of specimen respectively.

A material for the pulse shaper is selected so that the pulse shaper deforms in a designed manner impact, effectively controlling the shape of the incident pulse of the incident bar. The gradually increasing pulse shaper area upon impact of the striker bar allows greater momentum to transfer from the striker bar to the incident bar. The proper choice of the pulse shaper material and dimensions control the profile of the incident pulse. In this paper, a thin circular plane of plastic film of 1 mm and a diameter of 37 mm is used as pulse shaper.

3.2. Specimen

In this study, an isotropic fine-grained granitic rock is chosen for this research (Fig.2). The granite is taken from the Fuding city, Fujian province China. Rock cores with a nominal diameter of 50 mm are first drilled from a rock block and then sliced to obtain discs with an average thickness of 13 mm. All the disc samples are polished. Then the sample is fixed in the jig, center crack is made by the high-pressure water jet. The diameter of the water jet is 0.6mm. By measuring the notch is approximately 1mm thickness. Sufficient crack tip sharpness is necessary for accurately measuring fracture initiation toughness[12]. In our experiments, we first make approximately 1 mm wide notch and then sharpen the crack tip with a diamond wire saw to achieve a tip diameter of 0.35 mm, so the diameter of the crack tip is similar to the thickness of naturally formed cracks. This will ensure accurate measurements of fracture toughness.

Fig 2  The schematics of central sharp-notched circular disk

4. Experimental results and dynamic finite element analysis

The commercial software ABAQUS is used for the finite element analysis. Due to symmetry, quarter model is employed to construct the finite element model. Element C3D8R is used in the analysis. To better simulate the stress singularity of the crack tip, refined elements[13] are applied to the vicinity of the crack.
tip in the mesh of the finite element model. The nearest two nodes in the singular element are 0.05mm and 0.1mm away from the crack tip. The entire model has 4,0690 elements and 5,2684 nodes. Fig 3.

![Finite element model of SHPB](image)

**Fig 3. Finite element model of SHPB**

In the local crack tip coordinate system Fig 4, assuming plain strain, the near-tip crack SIF for a stationary crack under static loading can be calculated [14]:

\[
K_1 = \frac{2G\sqrt{2\pi u_y}}{(1 + \kappa)\sqrt{r}}
\]

(4)

Where \( G \) is shear modulus, \( \mu \) is Poisson ratio, \( \kappa = 3 - 4\mu \).

![Crack tip coordinate system](image)

**Fig 4. Crack tip coordinate system**

The load \( P(t) \) used in finite element computation is an average load of \( P_L(t) \) and \( P_R(t) \). Dynamic finite element analysis is carried out to determine the SIF evolution by solving the equation of motion with an implicit reduced integration method with the integration time step of 2 µs in ABAQUS. According to crack initiation time the dynamic fracture toughness of granite were determined. The crack initiation time \( t_f \) is determined by the maximum value for the average load with respect to time. The signal of the strain gauge mounted on the incident bar and transmitted bar is depicted on the Fig 5. It shows that the finite element simulation results match the test well. 

According to the analysis of Dong [15], the mode I stress intensity factor at the crack tip in the central cracked circular disk \( K_I \) can be expressed by

\[
K_I = \sigma \sqrt{\pi a F_1} = \frac{P}{\pi BR} \sqrt{\pi a F_1}
\]

(5)
Where \( B \) is the disk thickness. \( R \) is the radius of the specimen. \( F_I \) is a non-dimensional functional that only depends on the disk geometry, such as relative crack length, \( a/R \) and loading angle.

\[
I_{dK} = \text{calculated by employing the quasi-static formula, and } K_{id} \text{ is finite element simulation results.}
\]

Table 1. Granite dynamic test data

<table>
<thead>
<tr>
<th>Label of specimen</th>
<th>( P_{\text{max}} ) /KN</th>
<th>( K ) / (MPa ( \cdot ) m(^{1/2} ) s(^{-1} ))</th>
<th>( K_{id} ) / (MPa ( \cdot ) m(^{1/2} ))</th>
<th>( K_{id}^* ) / (MPa ( \cdot ) m(^{1/2} ))</th>
<th>Error / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>28.19</td>
<td>6.79 ( \times 10^4 )</td>
<td>7.81</td>
<td>7.34</td>
<td>6.0</td>
</tr>
<tr>
<td>D2</td>
<td>21.96</td>
<td>6.05 ( \times 10^4 )</td>
<td>6.18</td>
<td>5.85</td>
<td>5.34</td>
</tr>
<tr>
<td>D3</td>
<td>26.92</td>
<td>6.45 ( \times 10^4 )</td>
<td>7.61</td>
<td>7.1</td>
<td>6.7</td>
</tr>
<tr>
<td>D4</td>
<td>28.75</td>
<td>6.97 ( \times 10^4 )</td>
<td>8.09</td>
<td>7.54</td>
<td>6.8</td>
</tr>
<tr>
<td>Average</td>
<td>26.46</td>
<td>6.57 ( \times 10^4 )</td>
<td>7.42</td>
<td>6.96</td>
<td>6.2</td>
</tr>
</tbody>
</table>

From the above table, with the careful process, the dynamic fracture toughness can be deduced from the peak far-field average force by virtue of quasi-static equation. In spite of an average error of 6.2%, the possible reason may be this: First, the loading condition is not very accurate due to the
oscillation. Secondly, Rock itself is a complex material. For the case, the dynamic fracture toughness is 7.42 MPa·m^{1/2}, with the loading rate of 6.57×10^4 MPa·m s^{-1}.

5. Conclusion

In this paper, We promote the equation of quasi-static stress intensity factor for dynamic fracture toughness calculation. The maximum load under the balance dynamic force was substituted into the equation to get the value of dynamic fracture toughness. And then a three-dimensional model of SHPB system is built. The load used in finite element computation is an average load. From the experimental and simulation results we can deduce that the promotion of equation to dynamic is feasible.

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References