Physics Letters B 697 (2011) 497–499

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Note

Note on "Domain wall universe in the Einstein–Born–Infeld theory" [Phys. Lett. B 679 (2009) 160]

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ARTICLE INFO

Article history:
Received 31 August 2010
Received in revised form 27 December 2010
Accepted 11 February 2011
Available online 23 February 2011
Editor: T. Yanagida

Keywords:
Domain wall
Born–Infeld
Einstein–Maxwell
Non-linear electromagnetism

ABSTRACT

The interaction between bulk and dynamic domain wall in the presence of a linear/non-linear electromagnetism make energy density, tension and pressure on the wall all variables, depending on the wall position. In Lee et al. (2009) [1] this fact seems to be ignored.

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The $(n+1)$-dimensional bulk space time with Z_2 symmetry can equivalently be chosen as (i.e. Eq. (4) of Ref. [1])

$$ds^2 = -f(R)\,dT^2 + \frac{dR^2}{f(R)} + R^2\,d\Omega_{n-1}^2 \tag{1}$$

in which $d\Omega_{n-1}^2$ is the line element on S^{n-1}. The n-dimensional domain wall (DW) in the FRW form is

$$ds^2 = -d\tau^2 + a(\tau)^2\,d\Omega_{n-1}^2 \tag{2}$$

with the constraint

$$f(a)\dot{T}^2 - \frac{\dot{a}^2}{f(a)} = 1 \tag{3}$$

in which a dot implies $\frac{d}{d\tau}$.

The Israel junction condition

$$[K_{\mu\nu} - g_{\mu\nu}K] = -\kappa_{n+1}^2 S_{\mu\nu} \tag{4}$$

leads to (with Z_2 symmetry)

$$-\frac{2(n-1)}{a}\sqrt{f+\dot{a}^2} = \kappa_{n+1}^2(\rho+\sigma), \quad \text{for } \tau\tau \text{ component}, \tag{5}$$

$$\frac{2(n-2)}{a}\sqrt{f+\dot{a}^2} + \frac{f'+2\ddot{a}}{\sqrt{f+\dot{a}^2}} = \kappa_{n+1}^2(p-\sigma),$$

for $\theta_i\theta_i$ components. (6)

As considered in Ref. [1] the DW energy momentum $S_\mu^\nu = \mathrm{diag}(-\rho-\sigma, p-\sigma, \dots, p-\sigma)$ is given by

$$S^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\int d^n x\,\sqrt{-g}(-\sigma+\mathcal{L}_m) \tag{7}$$

in which (Eq. (22) of Ref. [1])

$$\mathcal{L}_m = \mathcal{L}_0 + \frac{C}{a^{n-1}}\bar{A}_\tau \tag{8}$$

and $C = \pm\frac{q\sqrt{2(n-1)(n-2)}}{\kappa_{n+1}^2}$. By using (7) one finds

$$S_{\mu\nu} = -2\frac{\delta\mathcal{L}_{DW}}{\delta g^{\mu\nu}} + \mathcal{L}_{DW}\,g_{\mu\nu} \tag{9}$$

for $\mathcal{L}_{DW} = (-\sigma+\mathcal{L}_m)$. The latter equation implies (see Appendix A)

$$S_\tau^\tau = \frac{2C}{a^{n-1}}\bar{A}_\tau + \mathcal{L}_0 - \sigma, \tag{10}$$

and

$$S_{\theta_i}^{\theta_i} = \mathcal{L}_0 - \sigma \quad (i = 1, \dots, n-1). \tag{11}$$

Comparison with the general form of S_μ^ν implies that the induced electrostatic energy density on the DW is

DOI of original article: 10.1016/j.physletb.2009.07.026.
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doi:10.1016/j.physletb.2011.02.034

$$\rho = -\frac{2C}{a^{n-1}}\bar{A}_\tau - \mathcal{L}_0 \tag{12}$$

while the pressure is

$$p = \mathcal{L}_0. \tag{13}$$

Now, taking into account Eqs. (5) and (6), we get two equations to be satisfied simultaneously, i.e.

$$-\frac{2(n-1)}{a}\sqrt{f+\dot{a}^2} = \kappa_{n+1}^2\left(-\frac{2C}{a^{n-1}}\bar{A}_\tau - \mathcal{L}_0 + \sigma\right) \tag{14}$$

and

$$\frac{2(n-2)}{a}\sqrt{f+\dot{a}^2} + \frac{f'+2\ddot{a}}{\sqrt{f+\dot{a}^2}} = \kappa_{n+1}^2(\mathcal{L}_0 - \sigma). \tag{15}$$

Herein $\bar{A}_\tau$ is given in terms of the bulk potential and metric function by

$$\bar{A}_\tau = \bar{A}_T\frac{\sqrt{f+\dot{a}^2}}{f}. \tag{16}$$

The angular part of Israel equation admits

$$\kappa_{n+1}^2(\sigma - \mathcal{L}_0) = -\left(\frac{2(n-2)}{a}\sqrt{f+\dot{a}^2} + \frac{f'+2\ddot{a}}{\sqrt{f+\dot{a}^2}}\right), \tag{17}$$

which is clearly not a constant. In Ref. [1] the authors consider a new constant parameter $\chi^2 = \kappa_{n+1}^2(\sigma\ell)^2/4(n-1)^2$ and by setting $\mathcal{L}_0 = 0$ (i.e. zero pressure) they find an equation of motion for the dynamic domain wall, based only on Eq. (14), which reads

$$\dot{a}^2 + V(a) = 0. \tag{18}$$

Plotting rescaled form of $V(a)$ for fixed values of χ (namely $\chi = 1.1$) is the last stage of Ref. [1]. Based on our argument on the other hand setting χ to a constant value is equivalent to setting $\sigma = const.$ which is obviously in contradiction with the form of σ we found in Eq. (17) above. In other words, choosing $\sigma = const.$ does not satisfy both of the Israel junction conditions at the same time.

Unlike this case, if we neglect the interaction between the bulk and domain wall in the form of Nambu–Goto action, i.e.

$$S_{DW} = -\sigma_\circ\int_\Sigma d^n x\sqrt{-g} \tag{19}$$

we observe that

$$S_{\mu\nu} = -2\frac{\delta\mathcal{L}_{DW}}{\delta g^{\mu\nu}} + \mathcal{L}_{DW}g_{\mu\nu} = -\sigma_\circ g_{\mu\nu}. \tag{20}$$

This means from $S_\mu^\nu = \mathrm{diag}(-\rho-\sigma, p-\sigma, \ldots, p-\sigma) = \mathrm{diag}(-\sigma_\circ, -\sigma_\circ, \ldots, -\sigma_\circ)$ that $-\rho = p = const.$ (which is set to zero for simplicity). As a result the two Israel junction conditions are consistent, i.e.

$$-\frac{2(n-1)}{a}\sqrt{f+\dot{a}^2} = \kappa_{n+1}^2\sigma_\circ, \tag{21}$$

$$\frac{2(n-2)}{a}\sqrt{f+\dot{a}^2} + \frac{f'+2\ddot{a}}{\sqrt{f+\dot{a}^2}} = -\kappa_{n+1}^2\sigma_\circ. \tag{22}$$

By differentiating (21) one obtains

$$\frac{f'+2\ddot{a}}{\sqrt{f+\dot{a}^2}} = \frac{2\sqrt{f+\dot{a}^2}}{a}, \tag{23}$$

which reduces (22) to (21). Therefore these two equations amount to the single equation (21).

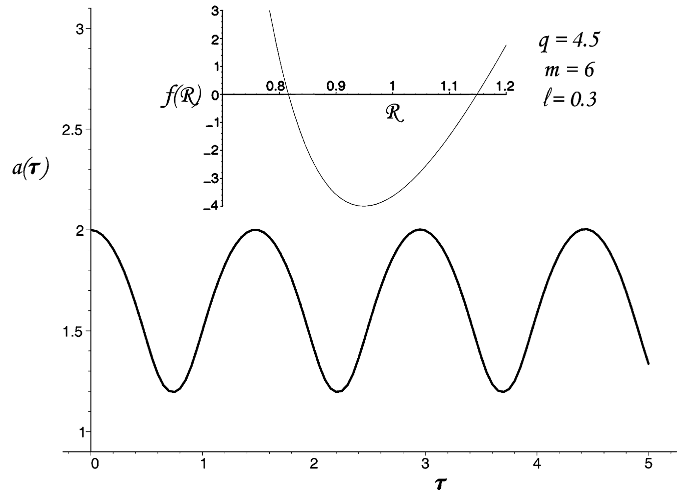

Fig. 1. The plot of radius $a(\tau)$ of the FRW universe for $n = 4$, on the domain wall as a function of proper time. The oscillatory behavior reveals a bounce at a distance greater than the horizon ($a > r_h$). The choice of parameters is: $C < 0$, $q = 4.5$, $m = 6$ and $\ell = 0.3$. The exact location of the event horizon (r_h) is shown in the smaller figure for $f(r)$.

Our conclusion to this problem simply implies a more complicated equation of motion for the dynamic domain wall that emerges from the substitution of Eqs. (17) into (14), i.e.,

$$\ddot{a} + \frac{(G-1)}{a}\dot{a}^2 + \frac{(G-1)f}{a} + \frac{f'}{2} = 0, \tag{24}$$

in which

$$G = \kappa_{n+1}^2\left(\frac{C}{a^{n-2}}\bar{A}_T\frac{1}{f}\right). \tag{25}$$

Given the complexities of $f(R)$ and $\bar{A}_T$ for the Einstein–Born–Infeld theory [1], Eq. (24) is a rather difficult differential equation to be solved. To give an idea about its structure yet we resort to the 5-dimensional cosmological Einstein–Maxwell theory ($n = 4$ and $\beta \to \infty$ limit of Ref. [1]). Solution for $f(R)$ and $\bar{A}_T$ are given (from Eqs. (12) and (16) of [1] with $\beta \to \infty$) by

$$f(R) = 1 + \frac{R^2}{\ell^2} - \frac{m^2}{R^2} + \frac{q^2}{R^4}, \tag{26}$$

$$\bar{A}_T = \frac{\sqrt{3}}{2}\frac{q}{R^2}. \tag{27}$$

Plugging these expressions with (25) into (24) (for $\kappa_{n+1}^2 = 1$, $C = -2\sqrt{3}q$ and $R = a(\tau)$) plots the $f(R)$ which in turn determine numerical integrations of (24) for specific parameters. We remark, that depending on the initial conditions and parameters falling into black hole or escaping to infinity and any possibility in between those two extremes are available. We plot, for instance in Fig. 1 the bouncing property of $a(\tau)$ with the choice $C < 0$. It should be remarked that with the choice $C > 0$, there is no bounce.

Appendix A

To find S_τ^τ and $S_{\theta_i}^{\theta_i}$ we use the formula (9), and consider

$$\mathcal{L}_{DW} = (-\sigma + \mathcal{L}_m), \tag{28}$$

in which

$$\mathcal{L}_m = \mathcal{L}_0 + \frac{C}{a^{n-1}}\bar{A}_\tau = \mathcal{L}_0 + \frac{C}{a^{n-1}}\bar{A}^\mu g_{\mu\tau}. \tag{29}$$

Now, in our variational principle we assume σ to be independent of $g_{\mu\nu}$. Variation of $\mathcal{L}_{DW}$ with respect to the canonical variable $g^{\mu\nu}$ leads accordingly to

$$\delta\mathcal{L}_{DW} = \frac{C}{a^{n-1}}\bar{A}^{\mu}\delta g_{\mu\tau} + \delta\left(\frac{C}{a^{n-1}}\right)\bar{A}^{\mu}g_{\mu\tau} = \frac{C}{a^{n-1}}\bar{A}^{\mu}\left(\frac{1}{2}g_{\mu\tau}g_{\alpha\beta} - g_{\mu\alpha}g_{\tau\beta}\right)\delta g^{\alpha\beta}, \tag{30}$$

which, after substitution into (9), it implies

$$S_{\alpha\beta} = -2\frac{\delta\mathcal{L}_{DW}}{\delta g^{\alpha\beta}} + \mathcal{L}_{DW}g_{\alpha\beta} = -2\frac{C}{a^{n-1}}\bar{A}^{\mu}\left(\frac{1}{2}g_{\mu\tau}g_{\alpha\beta} - g_{\mu\alpha}g_{\tau\beta}\right) + \left(-\sigma + \mathcal{L}_0 + \frac{C}{a^{n-1}}\bar{A}_{\tau}\right)g_{\alpha\beta}. \tag{31}$$

One obtains

$$S_{\tau\tau} = -\frac{C}{a^{n-1}}\bar{A}_{\tau} - \left(-\sigma + \mathcal{L}_0 + \frac{C}{a^{n-1}}\bar{A}_{\tau}\right) = \left(-2\frac{C}{a^{n-1}}\bar{A}_{\tau} + \sigma - \mathcal{L}_0\right) \tag{32}$$

or equivalently

$$S^{\tau}_{\tau} = 2\frac{C}{a^{n-1}}\bar{A}_{\tau} + \mathcal{L}_0 - \sigma. \tag{33}$$

In the same manner one finds

$$S^{\theta_i}_{\theta_i} = \mathcal{L}_0 - \sigma. \tag{34}$$

References

[1] B. H Lee, W. Lee, M. Minamitsuji, Phys. Lett. B 679 (2009) 160.