Computers & Operations Research 38 (2011) 1199–1209

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Tabu search with path relinking for an integrated production–distribution problem

V.A. Armentano [a,*], A.L. Shiguemoto [b], A. Løkketangen [c]

[a] Faculdade de Engenharia Elétrica e de Computação, Universidade Estadual de Campinas, Dpt. de Engenharia de Sistemas, Campinas, São Paulo 13081-852, Brazil
[b] Departamento de Estatística e Matemática Aplicada, Universidade Federal do Ceará, Campus do Pici, Bloco 910, Fortaleza, Ceará 60455-760, Brazil
[c] Molde University College, Specialized University in Logistics, Britveien 2, 6411, Molde, Norway

A R T I C L E   I N F O

Available online 27 October 2010

Keywords:
Supply chain management
Production–distribution problem
Tabu search
Path relinking

A B S T R A C T

This paper deals with the problem of integrating production and distribution planning over periods of a finite horizon. We consider a capacity-constrained plant that produces a number of items distributed by a fleet of homogenous vehicles to customers with known demand for each item in each period. The production planning defines the amount of each item produced in every period, while the distribution planning defines when customers should be visited, the amount of each item that should be delivered to customers, and the vehicle routes. The objective is to minimize production and inventory costs at the plant, inventory costs at the customers and distribution costs. We propose two tabu search variants for this problem, one that involves construction and a short-term memory, and one that incorporates a longer term memory used to integrate a path relinking procedure to the first variant. The proposed tabu search variants are tested on generated instances with up to ten items and on instances from the literature involving a single item.

1. Introduction

The characteristics of high competition in the global market, such as the introduction of items with short life cycles, increased service level, higher efficiency and lower costs, have led companies to focus attention on the management of their supply chains. The definition of a supply chain provided by Simchi-Levi et al. [1] emphasizes the importance of integrating decisions of different functions such as outsourcing, procurement, production planning, inventory and distribution management so as to obtain an optimal strategy that minimizes total costs for the entire company. However, due to the complexity of the supply chain, it is usually not viable to build a model that encompasses the decisions of all functions. For this reason, there has been increased interest on optimization models that integrate smaller sections of the supply chain.

This paper addresses the following integrated production–distribution problem over periods of a finite horizon [2]. A plant with capacity constraints produces several items, and a homogeneous fleet with an unrestricted or restricted number of vehicles is available for the distribution of the items in order to meet the customers' demands. In each period, the production problem involves determining the amount produced for each item, while the distribution problem consists of defining the quantities of each item to deliver to each customer and the vehicle routes. A production fixed cost and a transportation fixed cost are incurred every period that an item is produced, and every period that a vehicle is used, respectively. If the customer is visited in a given period, we consider that the customer is served by a single vehicle. The objective is to minimize fixed and variable production costs, inventory costs at the plant and customers and transportation costs. The first review on optimization models for tactical and strategic coordinated decisions for supply chain management problems was conducted by Thomas and Griffin [3]. At the tactical level, the authors point out that the three fundamental stages of the supply chain, namely, procurement, production and distribution have been managed independently, buffered by large inventories. The tactical models are then organized in three categories of operational coordination: buyer–vendor, production–distribution and inventory–distribution. The strategic planning models include decisions such as opening or closing a facility, assigning equipment to facilities and selecting locations for manufacturing a new item. Vidal and Goetschalckx [4] present an extensive literature review on domestic and international strategic production-distribution models.

Sarmiento and Nagi [5] present a review on integrated production–distribution systems and stress the importance of simultaneously optimizing decision variables of different functions or stages of the supply chain, as opposed to the traditional decoupled

* Corresponding author. Tel.: +55 19 3788 3705; fax: +55 19 3289 1395.
E-mail addresses: vinicius@densis.fee.unicamp.br (V.A. Armentano), shiguemoto@ufc.br (A.L. Shiguemoto), Arne.Lokketangen@hiMolde.no (A. Løkketangen).

0305-0548 © 2010 Elsevier Ltd. Open access under the Elsevier OA license.
doi:10.1016/j.cor.2010.10.026

optimization, in which the optimized output of one stage becomes the input to another stage, such as setting production plans and then planning the distribution. The review focuses on models that consider the transportation system explicitly in order to evaluate the way it is integrated and the resulting competitive advantage. Erengüç et al. [6] identify relevant decisions in the supplier, plant and distribution stages that need to be considered in the integrated planning of a supply chain, but do not emphasize models that achieve such an integration.

Chen [7] states that production and distribution are the most important operational functions in a supply chain and provides a comprehensive review of models that explicitly integrate such functions. At the tactical level, the problems involve joint lot sizing and finished product delivery models with infinite horizon or a finite horizon with discrete time periods. Such models are concerned with decisions such as, how much and when to produce, how much and when ship to customers, how much of inventory to keep at the plant and at the customers. The models are classified into five classes according to the following factors: (i) tactical or operational level; (ii) integration structure involving inbound transportation, production and outbound transportation; (iii) length of planning horizon and constant rate or dynamic demand. A number of real world problems belong to one of these classes, and the only reported application that requires vehicle routing is production and distribution of newspapers ([8,9]; see also [10]).

According to the above classification, the integrated production–distribution problem addressed in this paper is at the tactical level, characterized by a joint capacitated lot sizing production and vehicle routing distribution planning.

The literature related to this problem is rather scarce. Chandra and Fisher [11] propose two heuristics for solving the problem. The first heuristic is based on the decoupled approach, in which the production planning problem is optimally solved by the mixed integer optimization (MINTO) software and then the distribution planning problem is solved by means of constructive heuristics followed by 3-opt local search. The second heuristic follows an integrated approach in which the amount of each item delivered to each customer in a given period is anticipated to previous periods in which the item is produced. An optimal production plan is recomputed for the ten vehicle capacity feasible moves that yield the greatest reduction in distribution cost. This process is repeated until no improving move takes place. The decoupled and the integrated heuristics are applied to a set of 132 instances with up to 10 items, 50 customers, 10 periods and the cost savings obtained by the integrated approach ranges between 3% and 20%.

Fumero and Vercellis [12] propose a mathematical model for the problem and develop a Lagrangian heuristic that is applied to 20 instances involving up to 10 items, 12 customers and 8 periods. A cost reduction from 8% to 10% is obtained by the integrated approach relative to the decoupled approach.

Boudia et al. [13] consider a production–distribution problem with a single item and capacity constraints, and suggest a reactive GRASP procedure for solving the problem. A path relinking procedure is also proposed in order to link any two solutions from a pool of elite solutions, as a post-optimization phase, or to link a GRASP local optimum with a solution from the pool. Boudia and Prins [14] develop a memetic algorithm with population diversification management for the same problem. Diversification is achieved by including a new solution in the population only if its distance to the population is greater than or equal to a threshold. This approach outperformed the GRASP procedure on all instances generated by Boudia et al. [13]. For the same problem, Bard and Nananukul [15] propose a reactive tabu search and a path relinking procedure to connect solutions from a pool of elite solutions while being feasible at all times, and compare their results with those of Boudia et al. [13]. Lei et al. [16] deal with a complex real-life integrated, inventory and distribution routing problem involving the production at several plants, demands of distribution centers and heterogeneous fleet. The problem is solved in two phases. In phase I, the routing is ignored and considered as direct shipments from the plants to distribution centers and phase II deals with consolidation of loads and vehicle routing.

Tabu search has been applied with a high degree of success to a variety of hard combinatorial problems, as for example in vehicle routing problems, [17], but its application to solving production–distribution and inventory-routing problems is scant. Two variants of the tabu search approach are proposed here. The first variant has a short-term memory and the search is guided by the objective function aided by the violation of production capacity and vehicle capacity. Therefore, the search has feasible and infeasible trajectories. The second variant has a longer term memory that is used to integrate a path relinking procedure with the first tabu search variant, such that selected solutions from the trajectories are linked with a solution from the pool of elite solutions. The performance of our methods are assessed by a set of instances with up to 10 items generated by the authors, and a set of single item instances generated by Boudia et al. [13].

The remainder of the paper is organized as follows. Section 2 introduces the problem description. Section 3 describes the construction of an initial solution and the tabu search procedure. The path relinking procedure is presented in Section 4. Computational results and analyzes are reported in Section 5 and conclusions and suggestions are outlined in Section 6.

2. Problem description

The integrated production–distribution problem (IPDP) is defined on a complete graph $G=(W, E)$, where $W=\{0, 1, \ldots, N\}$ is the set of nodes and $E=\{(k,l): k,l \in W, k \neq l\}$ is the set of edges. Node $k=0$ represents the plant that produces a set of items $j \in \{1,\ldots,J\}$ shipped to a set of customers corresponding to nodes $k \in \{1,\ldots,N\}$ by a set of homogeneous vehicles $v \in \{1,\ldots,V\}$ with capacity C in periods of time $t \in \{1,\ldots,T\}$. The number of vehicles can be restricted or unrestricted. In each period, each vehicle can perform at most one route of length limited to L, and each customer can be visited by a single vehicle. The capacity of the plant in time units is B and the time required to produce one unit of item j is b_j. The unit inventory cost of item j at the plant is h_{j0}, the unit production cost of item j is c_j^p and if item j is produced in period t, a setup cost f_j^p is incurred. The demand of item j of customer k in period t is d_{jkt} and the unit inventory cost at the customer is h_{jk}. To each item j and each customer k a minimum inventory lower bound L_{jk} and a maximum inventory upper bound U_{jk} are associated. Transportation costs include a fixed cost f^v if a vehicle v is used in period t and a cost c_{kl}^v for traveling from node k to node l. Let M denote a large number, as for example, $\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{N} d_{jkt}$.

Consider the following variables:

p_{jt} = quantity of item j produced in period t;
I_{jkt} = inventory of item j of customer k at the end of period t;

$$y_{jt} = \begin{cases} 1 & \text{if product } j \text{ is produced in period } t \\ 0 & \text{otherwise;} \end{cases}$$

q_{jkt}^v = quantity of item j delivered to customer k by vehicle v in period t;
x_{jklt}^v = quantity of item j transported on edge (k, l) by vehicle v in period t;

$$z_{klt}^v = \begin{cases} 1 & \text{if vehicle } v \text{ travels along edge } (k,l) \text{ in period } t \\ 0 & \text{otherwise;} \end{cases}$$

The following formulation is based on the mathematical model proposed by Fumero and Vercellis [12] for the IPDP.

$$\min \sum_{t=1}^{T}\left\{\sum_{j=1}^{J}[\sum_{k=0}^{N} h_{jk}I_{jkt}+c_j^p p_{jt}+f_j^p y_{jt}]+\sum_{v=1}^{V}\left[\sum_{l=1}^{N} f^v z_{0lt}^v+\sum_{l=0,k\neq l}^{N} c_{kl}^v z_{klt}^v\right]\right\} \quad (1)$$

$$p_{jt}+I_{j0,t-1}-I_{j0t}=\sum_{k=1}^{N}\sum_{v=1}^{V} q_{jkt}^v \quad t=1,\ldots,T; \quad j=1,\ldots J \quad (2)$$

$$q_{jkt}^v+I_{jk,t-1}-I_{jkt}=d_{jkt} \quad t=1,\ldots,T; \quad v=1,\ldots,V; \quad j=1,\ldots J; \quad k=1,\ldots,N \quad (3)$$

$$\sum_{j=1}^{J} b_j p_{jt} \le B \quad t=1,\ldots,T \quad (4)$$

$$p_{jt} \le M y_{jt} \quad t=1,\ldots,T; \quad j=1,\ldots J \quad (5)$$

$$\sum_{\substack{i=0\\ i\neq k}}^{N} x_{jikt}^v-\sum_{\substack{m=0\\ m\neq k}}^{N} x_{jkmt}^v=q_{jkt}^v \quad t=1,\ldots,T; \quad v=1,\ldots,V; \quad j=1,\ldots J; \quad k=1,\ldots,N \quad (6)$$

$$\sum_{i=1}^{N}\sum_{v=1}^{V} x_{ji0t}^v-\sum_{m=1}^{N}\sum_{v=1}^{V} x_{j0mt}^v=-\sum_{k=1}^{N}\sum_{v=1}^{V} q_{jkt}^v \quad t=1,\ldots,T; \quad j=1,\ldots J \quad (7)$$

$$\sum_{j=1}^{J} x_{jklt}^v \le C z_{klt}^v \quad t=1,\ldots,T; \quad v=1,\ldots,V; \quad k,l=0,\ldots,N; \quad k\neq l \quad (8)$$

$$\sum_{k=0}^{N}\sum_{l=0}^{N} c_{kl}^v z_{klt}^v \le L \quad t=1,\ldots,T; \quad v=1,\ldots,V; \quad k\neq l \quad (9)$$

$$\sum_{k=1}^{N} z_{0kt}^v \le 1 \quad t=1,\ldots,T; \quad v=1,\ldots,V \quad (10)$$

$$\sum_{\substack{i=0\\ i\neq k}}^{N} z_{ikt}^v-\sum_{\substack{m=0\\ m\neq k}}^{N} z_{kmt}^v=0 \quad t=1,\ldots,T; \quad v=1,\ldots,V; \quad k=1,\ldots,N \quad (11)$$

$$\sum_{k=0}^{N}\sum_{v=1}^{V} z_{klt}^v \le 1 \quad t=1,\ldots,T; \quad l=1,\ldots,N \quad (12)$$

$$L_{jk} \le I_{jkt} \le U_{jk}, \quad p_{jt}\ge 0, \quad q_{jkt}^v \ge 0, \quad x_{jklt}^v \ge 0, \quad y_{jt}\in\{0,1\}, \quad z_{klt}^v\in\{0,1\}, \forall j,k,l,t \quad (13)$$

The objective function (1) expresses the minimization of production costs, inventory costs at the plant and customers and transportation costs. Constraints (2) represent the balance among production, inventory and deliveries at the plant, and constraints (3) represent the balance between deliveries, inventory and the demand at the customers. Constraints (4) limit the production, and constraints (5) ensure that a setup cost is incurred only if there is production. Constraints (6) and (7) express the commodity conservation flow at the customers and at the plant. Constraints (8) and (9) impose limits on vehicle capacity and route length, respectively. Constraints (10) ensure that, in each period, each vehicle is assigned to at most one route, while constraints (11) guarantee that each vehicle returns to the plant at the end of the route. Constraints (12) force that no more than one vehicle visit a customer in every period. Constraints (13) indicate the type of the variables, with lower and upper bounds for inventory levels. Fumero and Vercellis [12] point out that the demand fulfillment in constraints (6) and (7) precludes the existence of subtours.

The above IPDP model integrates the production and distribution decisions by means of constraints (2). As mentioned in Section 1, a common approach to tackle the PDP is to first solve the production problem and then the distribution problem, which is called the decoupled production–distribution problem (DPDP). In this case, the right-hand side of constraint (2) becomes $\sum_{k=1}^{N} d_{jkt}$ and the production problem involves the determination of the production quantities p_{jt} in order to minimize the production costs subject to constraints (2), (4), (5) and (13). Such a problem is known in the literature as the capacitated lot sizing problem, see Karimi et al. [18] for a review. The decision variables of the distribution problem, I_{jkt}, q_{jkt}^v, x_{jklt}^v and z_{klt}^v are determined so that the distribution costs are minimized subject to constraints (3), (6)–(12) and $\sum_{k=1}^{N}\sum_{v=1}^{V} \le I_{j0t}$, $j,=1,\ldots,J$, $t=1,\ldots,T$. In order to assess the quality of a solution for the IPDP in larger instances its cost is compared to that of a solution for the DPDP.

3. A Tabu search approach to the IPDP

Our solution for the IPDP is based on the tabu search methodology, a memory-based local search strategy that transcends local optimality by forbidding certain moves [19]. In the short-term memory, selected attributes that occur in solutions recently visited are stored in a tabu list to prevent some solutions from being revisited, and blocks off a part of the search space. This feature prevents cycling and forces other solutions to be explored. The longer term memory contains a selective history of solutions and/or their attributes found in the search, in order to activate diversification and/or intensification strategies. We propose two tabu search variants for the PDP, one that has two phases, namely, construction and short-term memory, (TS), and one that also incorporates a longer term memory to be used in a path relinking procedure (TSPR).

An important feature of our approach is to allow some infeasible solutions in the tabu search and path relinking procedures. This improves the connectedness of the search space associated with neighborhoods induced by simple types of moves, thus making it easier to proceed to good solutions. The importance of allowing feasible and infeasible search trajectories has been emphasized by Gendreau [20]. The infeasibilities allowed in our searches are all with respect to the capacity constraints.

Let S be the set of solutions. These might be infeasible with respect to the capacity constraints (4), (8) and length constraints (9). For a solution $s\in S$, let $c(s)$ denote its cost according to (1) and let $g(s)=\sum_{t=1}^{T}\max\{\sum_{j=1}^{J} b_j p_{jt}-B,0\}$, $h(s)=\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{l=0}^{N}\sum_{k=0,k\neq l}^{N}\max\{\sum_{j=1}^{J} x_{jklt}^v-Cz_{klt}^v,0\}$ and $l(s)=\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{l=0}^{N}\sum_{k=0,k\neq l}^{N}\max\{c_{jk}^l z_{klt}^v-L,0\}$ denote the total violation of these constraints. The total violation of production capacity, vehicle capacity and route length are represented by $g(s)$, $h(s)$ and $l(s)$, respectively. Solutions are then evaluated by a cost function $f(s)=c(s)+\alpha g(s)+\beta h(s)+\gamma l(s)$, where α, β and γ are positive parameters that are adjusted during the search in order to facilitate the exploration of the search space.

3.1. The construction phase

The procedure that constructs the initial solution consists of three stages, and is used by both TS and TSPR

Step 1. If the initial inventories at the customers I_{jk0} are greater than zero, $j=1, \ldots, J$; $k=1, \ldots, N$, use such inventories to meet the customers' demand of initial periods, and then deliver the demands of the remaining periods. In other words, identify the first period $\hat{t}$ such that $I_{jk,\hat{t}-1}-d_{jk\hat{t}} < L_{jk}$, and set the delivered quantities to $q_{jkt}=0$ for $t=1,\ldots,\hat{t}-1$, $q_{jk\hat{t}}=L_{jk}-I_{jk\hat{t}}+d_{jk,\hat{t}-1}$, and $q_{jkt}=d_{jkt}$ for $t=\hat{t}+1,\ldots,T; j=1,\ldots J;\ k=1,\ldots,N$.

Step 2. Use the parallel version of the Clarke and Wright [21] algorithm to determine the routes for each period $t=1, \ldots, T$. If a maximum number of vehicles is given then the last constructed route might be capacity infeasible.

Step 3. Determine a production plan for each item $j=1, \ldots, J$ by applying Evans [22] efficient implementation of the Wagner and Whitin [23] algorithm. This plan might be capacity infeasible.

3.2. The basic tabu search, TS

The method TS starts from the initial solution. It is a standard tabu search, using the move evaluation function presented in the start of Section 3, enabling the search to visit capacity-infeasible solutions.

Of special interest is also the neighborhood $N(s)$ of a solution s. This neighborhood is defined by a composite move with three components doing the following:

- *Move Component 1*. Transference of the maximum quantity $r_{jktt'} \le q_{jkt}$ from period t to period $t' \ne t$ without violating the bounds on the inventory levels I_{jkt} and $I_{jkt'}$.
- *Move Component 2*. Insertion of the quantity $r_{jktt'}$ in one route of period t'.
- *Move Component 3*. Determination of a new production plan.

The details of each of these components are described in the following.

Move Component 1. Transference of the maximum quantity $r_{jktt'} \le q_{jkt}$ from period t to period $t' \ne t$ without violating the bounds on the inventory levels I_{jkt} and $I_{jkt'}$.

From constraints (2) and (3) it follows that the shift of the quantity $r_{jktt'}$ from period t to a period $t' < t$ causes the inventory levels $I_{j0\tau}$ and $I_{jk\tau}$, $\tau=t', \ldots, t-1$ to be decreased and increased, respectively, by $r_{jktt'}$. Similarly, the transference of the quantity $r_{jktt'}$ from period t to a period $t' > t$ increases the inventory levels $I_{j0\tau}$ and decreases the inventory levels $I_{jk\tau}$, $\tau=t', \ldots, t-1$ by $r_{jktt'}$. Therefore, if $t' < t$

$$r_{jktt'} = \min\{q_{jkt}, \min_{\tau}\{U_{jk}-I_{jk\tau}, I_{j0\tau}\}\} \quad \tau = t', \ldots, t-1$$

and if $t' > t$

$$r_{jktt'} = \min\{q_{jkt}, \min_{\tau}\{I_{jk\tau}-L_{jk}\}\} \quad \tau = t, \ldots, t'-1.$$

Move Component 2. Insertion of the quantity $r_{jktt'}$ in one route of period t'.

A customer is allowed to be visited by a single vehicle in each period: if customer k is visited in period t', then we add the quantity $r_{jktt'}$ to be delivered to customer k and maintain the same route, regardless if the vehicle capacity is violated or not. If customer k is not visited in period t', we evaluate the following alternatives and select the most economical one: (i) insert customer k in all positions of the routes in period t' and select the cheapest insertion as measured by the cost function $f(s)$; (ii) open a new route for customer k, as long as the maximum number of vehicles is not exceeded in the restricted case. If there are no routes in period t', then we open a new route for customer k.

Move Component 3. Determination of a new production plan.

After shifting $r_{jktt'}$, and assigning it to a vehicle v in period t', we have a new delivery quantity $q^v_{jkt'} \leftarrow q^v_{jkt'} + r_{jktt'}$, and a new production plan for item j is determined due to the change in the right-hand side of constraints (2). This plan is determined by applying Evans [22] algorithm to solve the following penalized production subproblem:

$$\min \sum_{t=1}^{T} (\alpha b_j + c^p_j) p_{jt} + h_{jk} I_{jkt} + f^p_j y_{jt}$$

subject to

$$p_{jt} + I_{j0,t-1} - I_{j0t} = \sum_{k=1}^{N} \sum_{v=1}^{V} q^v_{jkt} \quad t = 1, \ldots, T$$

$$p_{jt} \ge 0, \quad I^0_{j0t} \ge 0, t = 1, \ldots, T.$$

The composite move is examined for each item j, each customer k, every pair of periods t and $t' < t$, $t \ne t'$, and the one that results in the least total cost is chosen. The pair (j, t') associated with such a move is stored in a matrix to indicate that the shift of any quantity of item j from period t' is tabu for γ iterations, where the discrete value of γ is randomly selected from an interval $[a,b]$ with uniform distribution. As aspiration criterion we adopt the most commonly used, i.e., the tabu status of the move is revoked whenever the move leads to a solution that is better than the best solution recorded during the search so far.

The search procedure terminates when it reaches $\delta \times N \times J \times T \times V$ iterations or when $\eta \times N \times J \times T \times V$ iterations have elapsed without updating the incumbent solution, where δ and η are parameters.

4. Path relinking

Path relinking was originally proposed by Glover [24] as an approach to integrate intensification and diversification strategies in the context of tabu search and scatter search [25–27]. This approach generates new solutions by exploring trajectories that connect high-quality solutions (intensification), or solutions that come from different regions or that exhibit contrasting features (diversification). Starting from one of these solutions, called *initiating solution*, a path composed of capacity feasible and infeasible solutions is generated in a restricted neighborhood space that leads toward the other solution, called *guiding solution*. This is accomplished by selecting moves that introduce attributes contained in the guiding solutions. The best move that increases the number of attributes that are present in the guiding solution is executed. The previously described neighborhood search is applied to every path local minimum, which is a solution that is both immediately preceded and succeeded in the path by strictly worse solutions, as measured by the cost function $f(s)$.

In our implementation, one of the solutions, s_1, is a tabu search local minimum, the other solution, s_2, is selected from a set of elite solutions E that contains the e best solutions found during the search, subject to the restriction of a minimum Hamming distance between the solutions in order to maintain a degree of diversity. Let s_{best} and s_{worst} denote the minimum cost and maximum cost solutions in E. A new solution enters E if it has smaller cost than s_{best} or if it has smaller cost than s_{worst} and increases the distance between solutions in E. In either case, the worst solution leaves the elite set E. The distance between a solution s_1 and a solution s_2 is defined as the number of periods in which the quantities of the items that are delivered to customers are different. The maximum distance between any two solutions is $T \times N \times J$ and the minimum distance is $\lfloor \rho \times T \times N \times J \rfloor$, where ρ is a parameter.

We use a mixed path relinking strategy that is applied for connecting each tabu search local minimum s_1 to the solution s_2 selected from E as being the one farthest away from s_1. The mixed

Fig. 1. Mixed path relinking.

```
0  Procedure path_relinking (s1, s2)
1  While s1 ≠ s2 do
2    For all items j, all customers k, and all periods t do
3      If q^1_jkt in solution s_i is different from q^2_jkt in solution s2 then
4        u_jkt = q^1_jkt − q^2_jkt
5        For all t' ≠ t do
6          If t' < t then
7            w_jktt' = min{|u_jkt|, min_τ{U_jk − I_jkτ, I_j0τ}}, τ = t',...,t−1
8          Else
9            w_jktt' = min{|u_jkt|, min_τ{I_jkτ − L_jk}}, τ = t,...,t'−1
10         Compute the cost of transferring w_jktt' from period t to period t'
11       End For
12     End If
13   End For
     Let s_int be the solution obtained by least cost transference w_jktt' from period t to
14   period t'. If the distance between the initiating and guiding solutions is not increased,
     execute the transference.
15   s1 = s2
16   s2 = s_int
17 End While
```

Fig. 2. Path relinking procedure.

strategy then builds two paths, one starting in s_1 and the other at s_2, and interchanging the role of the initiating and guiding solutions at each step of the path relinking procedure. The paths evolve and eventually meet at some solution s_m between s_1 and s_2 as illustrated in Fig. 1. Resende et al. [28] point out that, in general, there are more high-quality solutions near the solutions of the elite set, because the size of the restricted neighborhood decreases as one moves along the path towards the guiding solution. The mixed path relinking shares the benefits of starting from s_1 and s_2 and it usually takes less time than exploring two paths, initiating at solutions s_1 and s_2.

The path relinking procedure between solutions s_1 and s_2 is shown in Fig. 2. Let q^1_{jkt} and q^2_{jkt} denote the quantity of item j that is delivered to customer k in period t in the initiating solution s_1 and the guiding solution s_2, respectively. While $s_1 \neq s_2$ lines 1 to 16 are executed. For all items, customers and periods the difference $u_{jkt}=q^1_{jkt}-q^2_{jkt}$ is calculated in line 4. For every period $t' \neq t$, we consider in lines 7 and 9 the transference quantity $w_{jktt'}$ from period t to every period t' in solution s_1, such that the inventory bounds are not violated and the distance between solutions is not increased. In line 14 the least cost transference is executed and an intermediate solution s_{int} is obtained. In lines 15 and 16, s_2 becomes the initiating solution and s_{int} is the new guiding solution.

5. Computational experiments

All heuristic algorithms proposed here were coded in C++ by using the version 3.3.3 of the GCC compiler with optimizer option O3 and computational tests were carried out on an Intel Pentium IV 2.8 GHz with Linux Fedora Core 2 operating system. Our heuristics were tested on generated instances with multiple items generated by the authors, (sets S_1 and S_2, see Sections 5.1–5.3) and on instances of set with a single item generated by Boudia et al. [13] (set S_3, see Section 5.4).

Table 1
Generation of instances.

Customers demand	$d_{jkt} \in [10, 100]$
Inventory level lower bound at the customers	$L_{jk} \in [50, 150]$
Inventory level upper bound at the customers	$U_{jk}=L_{jk}+d_{jkt}g_{jk}$, where $g_{jk} \in \{2,3,5,6,10,15,30\}$
Initial inventory at the customers	$I_{jk0} \in [L_{jk}, U_{jk}]$
Initial inventory at the plant	$I_{j00} = \sum_{k \in N} (U_{jk}-L_{jk})$
Unit inventory cost at the customers	$h_{jk} \in [0.1, 1]$
Unit inventory cost at the plant	h_{j0}=0.3 and 0.8
Setup production cost	$f_j^p = 1000h_{j0}$
Unit production time	$b_j = 1$
Production capacity	$B = 3.5(\sum_j\sum_k\sum_t d_{jkt}/T)$
Fixed transportation cost	$f^v = (N+1)\max_{k,l \in N} c^v_{kl}$
Transportation cost	$c^v_{kl} = \lfloor \sqrt{(x_k-x_l)^2+(y_k-y_l)^2} \rfloor$
Coordinates	$x_k \in [0, 500]$ and $y_k \in [0,1000]$
Vehicle capacity	$C = \max_{j \in J, k \in N} U_{jk}$
Maximum route length	$L = r(\max_{k,l \in N} c^v_{kl}),\ r \in [5,20]$

5.1. Instance generation

A set S_1 with small instances and a set S_2 with larger instances were generated by the authors. The number of customers, periods and items in set S_1 is {5, 10, 15}, {7, 14} and {3, 5}, respectively. For each combination of customer, period and item, six instances were created, totaling 72 instances. For the set S_2, the number of customers, periods and items is {30, 50, 100}, {12, 24}, {5, 10}, respectively. For each combination of customer, period and item, nine instances were created, totaling 108 instances with unrestricted number of vehicles V. Except for parameters b_j and B, the remaining parameters in Table 1 were generated as suggested by Bertazzi et al. [29] randomly generate the continuous and discrete parameters from their respective intervals listed in Table 1. The time b_j required to produce one unit of item j is 1, and the production capacity B is generated as in Trigeiro et al. [30] and Toledo and Armentano [31], and corresponds to the sum over all items and periods of the production $p_{jt} = \sum_{k=1}^{N} d_{jkt}$ of item j in period t divided by the number of periods T. The factor 3.5 adjusts the capacity tightness.

5.2. Best search parameter values and strategies

Table 2 presents the selected values and the range of tested values for the parameters η and δ associated with the stopping criteria, the parameter ρ that determines the Hamming distance between two solutions in the path relinking procedure, and the initial values for the search parameters α, β and γ that penalize the violation of production capacity, vehicle capacity and maximum route length. The last three parameters are updated dynamically in the following way. Each parameter is multiplied by 2 every time the next solution is infeasible, and divided by 2 every time the next solution is feasible, with relation to the respective capacity constraint.

Regarding parameter sensitivity, we have applied a variation of up to 20% on the values shown in Table 2. The reduction of the values of η and δ leads to a decrease of computational time and a degradation of up to 1% on the best solution value. On the other hand, an increase of their values results in a larger computational

Table 2
Parameters of the heuristic procedures.

Parameter	Selected value	Range of tested values
η	0.005	[0.0001, 0.5]
δ	50	[15, 75]
ρ	0.3	[0.1, 0.5]
α	1	[1, 5]
β	0.001	[0.00001, 0.5]
γ	1	[1, 5]

time and a negligible variation in the range of -0.05% to 0.05% of the best solution value. The change on the value of parameter ρ causes a negligible impact of less than 0.05% on the best solution value and a change of computational time in the range of -5% to 5%. The variation of the values of parameters α, β and γ, respectively, leads to an increase of the best solution value of less than 0.05%, 1% and 0.62% and a change of computational time between -2% and 2%, -16% and 37%, and -19% and 15%.

The tabu tenure is selected from an interval with uniform distribution that depends on the problem size, given by $[\lceil\sqrt{T\times J\times V}/3\rceil, \lceil\sqrt{T\times J\times V}\rceil]$. This interval was obtained by determining suitable tabu tenures for several instances of different sizes and then applying the square root function. Since the number of vehicles V is unrestricted, its initial value in the tabu tenure interval is made equal to the number of customers. Then at each iteration, the value of V is dynamically updated according to the maximum number of routes over the planning horizon of the current solution. The cardinality of the set of elite solution e was set to 20 for the path relinking procedure.

5.3. Results for the multiple item instances

5.3.1. Results for test set S_1

The first and second columns of Table 3 indicate the number of products J and the number of periods T for the set S_1 of 72 instances. The remaining columns show the average percentage gap over six instances of the cost of solutions obtained by tabu search (TS) and tabu search-path relinking (TSPR) procedures in relation to the optimal cost solutions, which result from solving the mathematical model of Section 2 by the software CPLEX 10. The following terminology is used in the analysis of results. For each instance, the number of vehicles required for distribution is the maximum number of vehicles utilized in a period, over the periods of the planning horizon. The total number of vehicles, which is the sum of the number of vehicles over six instances, is shown in Table 4. Both procedures yield high-quality solutions, but TSPR performs slightly better on the larger instances.

Table 5 shows the average computational time in seconds required by TS, TSPR and CPLEX. Computational time to find an optimal solution by CPLEX increases drastically, reaching more than 4 hours to solve the largest instances with 5 items, 14 periods and 15 customers, whereas TS requires about 22 seconds to obtain good solutions with average gap of 1.91%, and about 2 min is needed for TSPR to reach a better solutions with average gap of 1.48%.

5.3.2. Results for test set S_2

The 108 larger instances of set S_2 were solved by the TS and TSPR procedures, in order to establish a tradeoff between computational time reduction and cost increase of TS with respect to TSPR, which yields a minimum cost solution for all instances.

Table 3
Percentage gap of TS and TSPR relative to optimal solutions on set S_1.

		5 customers		10 customers		15 customers	
J	T	TS	TSPR	TS	TSPR	TS	TSPR
03	7	0	0	0.50	0.12	1.36	0.87
	14	0	0	1.14	0.32	1.51	1.05
05	7	0	0	1.39	0.84	1.65	1.46
	14	0	0	1.63	1.08	1.91	1.48
Overall mean		0	0	1.17	0.59	1.61	1.22

Table 4
Total number of vehicles on set S_1.

		5 customers			10 customers			15 customers		
J	T	TS	TSPR	Optimal	TS	TSPR	Optimal	TS	TSPR	Optimal
03	7	13	13	13	23	22	22	33	32	32
	14	15	15	15	25	25	25	36	36	35
05	7	15	15	15	25	25	25	33	33	32
	14	17	17	17	27	26	26	36	35	35
Overall total		60	60	60	100	98	98	138	136	134

Table 5
Computational time on set S_1.

		5 customers			10 customers			15 customers		
J	T	TS	TSPR	CPLEX	TS	TSPR	CPLEX	TS	TSPR	CPLEX
03	07	1.6	12.9	1588.4	5.1	38.4	2588.7	12.9	118.4	6084.9
	14	3.4	27.0	2887.6	8.2	71.7	4660.1	25.4	152.4	13,293.1
05	07	2.3	16.1	1667.6	5.7	39.6	3017.5	14.6	122.7	6433.5
	14	3.7	28.8	3075.6	10.2	75.3	5424.6	25.9	167.5	14,580.6
Overall mean		2.8	21.2	2304.8	7.3	56.3	3922.7	19.7	140.3	10,098.0

Table 6
Percentage reduction cost of IPDP relative to DPDP on set S_2.

		30 customers		50 customers		100 customers	
J	T	TS	TSPR	TS	TSPR	TS	TSPR
05	12	28.39	29.96	35.82	37.86	50.12	52.31
	24	25.23	27.07	35.78	36.93	49.26	50.88
10	12	27.60	29.06	33.59	35.05	50.42	52.85
	24	27.02	27.97	35.01	36.06	50.64	51.18
Overall mean		27.06	28.51	35.05	36.47	50.11	51.81

The solution of the decoupled production–distribution problem DPDP is obtained by first solving the capacitated lot sizing production problem by a Lagrangian heuristic. (For details, see [31]). The solutions of the production problem have a good quality as evidenced by the average and the maximum gap of the production costs with respect to the Lagrangian lower bound over the 108 instances whose values are 3.78% and 4.67%, respectively. The distribution problem is then solved by applying steps 1 and 2 of the construction phase and components 1 and 2 of the short memory tabu search integrated with path relinking.

Table 6 presents the average percentage reduction cost obtained by the solution approaches TS and TSPR for IPDP in relation to the above solution method for DPDP over nine instances. The overall

Table 7
Total number of vehicles for DPDP and IPDP on set S_2.

		30 customers			50 customers			100 customers		
J	T	DPDP	TS	TSPR	DPDP	TS	TSPR	DPDP	TS	TSPR
05	12	106	100	99	159	155	154	285	280	278
	24	109	106	104	162	157	155	289	286	283
10	12	108	104	103	161	157	155	289	285	282
	24	110	105	105	165	160	158	292	288	285
Overall total		433	415	411	647	629	622	1155	1139	1128

Table 8
Computational time for DPDP and IPDP on set S_2.

		30 customers			50 customers			100 customers		
J	T	DPDP	TS	TSPR	DPDP	TS	TSPR	DPDP	TS	TSPR
05	12	49.7	127.2	858.4	76.6	197.2	1320.1	123.0	822.7	4970.1
	24	103.9	344.7	1243.1	129.0	562.8	1857.8	245.5	1714.9	8950.8
10	12	85.4	195.9	1442.1	174.0	327.2	2205.8	280.0	990.0	6122.4
	24	119.7	565.7	5288.9	241.9	861.2	6728.0	435.7	2300.3	13,400.7
Overall mean		89.7	308.4	2208.1	155.4	487.1	3027.9	271.1	1457.0	8361.0

Table 9
Sensitivity of solution costs relative to setup cost change, set S_2.

	Setup cost			
	$1000h_{j0}$–$100h_{j0}$		$100h_{j0}$–$10h_{j0}$	
	Min	Max	Min	Max
Production cost (−)	1.35	17.41	0.53	3.65
Inventory cost at the plant (−)	2.50	29.98	3.15	26.59
Inventory cost at the customers (−)	1.78	48.75	5.87	27.83
Transportation cost (+)	2.68	40.19	2.97	28.98

mean reduction cost obtained by the integrated approach indicates that DPDP yields poor quality solutions, which gets worse with the increase of number of customers. Table 7 shows that DPDP requires more vehicles and Table 8 shows that the average computational time, in seconds, is more affected by the number of customers and periods than the number of items. As expected, DPDP requires a small computational time compared to TS and TSPR, and in turn TSPR requires a larger computational time than TS. This fact and the reduction in cost in Table 6 indicate that TS and TSPR provide a good tradeoff between finding high-quality solutions and spending computational time.

We also analyzed the sensitivity of the TSPR solution costs and capacity utilization with changes in the setup cost and available capacity. If the setup cost is reduced from $1000h_{j0}$ to $100h_{j0}$ and $10h_{j0}$, the total cost solution over 108 instances diminishes in the range 1.94%–3.61%, while the average computational time increases from 11.54% to 17.01%. Table 9 shows the minimum and maximum component cost variation over 108 instances when the setup cost is reduced from $1000h_{j0}$ to $100h_{j0}$ and from $100h_{j0}$ to $10h_{j0}$. The reduction or the increase of a component cost is indicated by a "−" or "+" sign between parentheses. Production cost undergoes a relatively small decrease when compared to a large decrease of inventory costs at the plant and at the customers. The resulting lower inventories in each period imply that customers are visited more often and consequently the transportation cost increases with a magnitude rather similar to the inventory cost at the customers.

Table 10
Sensitivity of solution costs relative to production capacity, set S_2. Production caapacity.

	Production Capacity			
	$3.5\hat{B}-3.2\hat{B}$		$32\hat{B}-3\hat{B}$	
	Min	Max	Min	Max
Production cost (+)	2.35	14.22	1.03	10.35
Inventory cost at the plant (−)	3.21	15.59	1.68	9.65
Inventory cost at the customers (−)	3.45	24.86	3.23	10.47
Transportation cost (+)	6.83	9.82	1.88	5.06

The reduction of the setup cost leads to a computational time increase in the range 11.54–17.01% and this occurs because the incumbent solution is updated more often and the stopping criteria of $\eta \times N \times J \times T \times V$ iterations without updating the incumbent solution takes more time to be satisfied.

Let $\hat{B} = (\sum_j \sum_k \sum_t d_{jkt}/t)$ and consider the decrease of production capacity from $3.5\hat{B}$ to $3.2\hat{B}$ and $3\hat{B}$. The average utilized capacity for these three production capacities is 76%, 88% and 96%, respectively. The cost solution over 108 instances increases in the range 2.6–3.0%, whereas the computational time is reduced in the range 18.65%–36.04%. Table 10 shows the minimum and maximum component cost change over 108 instances when the production capacity is reduced from $3.5\hat{B}$ to $3.2\hat{B}$ and from $3.2\hat{B}$ to $3\hat{B}$. Such a reduction causes an increase in production cost and a decrease in both inventory cost at the plant and at the customers, which in turn lead to an increase of the transportation cost. As expected, the capacity reduction reduces the space of feasible solution which leads to an increase of the utilized capacity and the incumbent solution stopping criterion is met earlier. The generated instances and solution values are available at http://www.densis.fee.unicamp.br/~vinicius/.

5.4. *Results for the single item instances*

In this section, we test the performance of the tabu search and tabu search-path relinking procedures on the set of instances S_3 generated by Boudia et al. [13]. Our tests are compared to the memetic algorithm with population management (MA|PM) developed by Boudia and Prins [14] and the best results obtained between reactive tabu search (RTS) and path relinking proposed by Bard and Nananukul [15]. Unfortunately, the last authors do not provide a description of their path relinking procedure. S_3 contains three subsets of 30 instances each with 50, 100 and 200 customers, with a limited fleet of 5, 9 and 13 vehicles, respectively, to be used over a planning horizon of 20 periods. This leads to a total limit of 150, 270 and 390 vehicles for each set of 30 instances. The number of vehicles V in the tabu tenure interval is made equal to 5, 9 and 13 according to the subset of instances, and the route length is not limited.

Table 11 presents a summary of the results for a single item. The first and second columns show the number of customers and the method names, respectively. The third, fourth and fifth column show the mean, minimum and maximum percentage deviation, over 30 instances, of the cost obtained by each method relative to the best solution, which is always obtained by TSPR. The sixth column indicates the total number of utilized vehicles over 30 instances for TS and TSPR. This number is not reported for the other methods, and therefore we assume that the total limit of vehicles was used. For nearly half number of the 90 instances TS requires a smaller number of vehicles than TSPR. The proposed heuristics TS and TSPR outperform MA|PM and RTS in all instances, and the solution quality tends to improve with the increase of number of customers, as indicated by mean gaps. The smallest mean gap from TSPR is 1.06% relative to TS for 50 customers, and the largest one is 8.57% relative to RTS for 200 customers.

The last column shows that TS requires a shorter mean computational time, in seconds, for any number of customers, whereas TSPR required about twice as much time.

Tables 12–15 in Appendix show the detailed results.

Table 11
Summary results for the single item instances, set S_3.

Customers	Method	Mean Gap	Min Gap	Max Gap	Total vehicles	Time
	MA|PM[a]	8.53	0.99	18.72	150	172.7
50	RTS[b]	1.98	0.62	6.44	150	330.6
	TS	1.06	0.18	1.99	150	144.9
	TSPR	0	0	0	150	293.8
	MA|PM	4.20	1.14	12.93	270	1108.1
100	RTS	3.85	0.86	8.23	270	975.6
	TS	1.46	0.37	1.98	263	507.9
	TSPR	0	0	0	268	1059.7
	MA|PM	5.05	1.03	11.80	390	4098.5
200	RTS	8.57	4.34	12.29	390	2492.3
	TS	2.10	0.63	4.97	383	1783.7
	TSPR	0	0	0	387	3659.2

[a] MA|PM 2.8 GHz PC.
[b] RTS 2.53 GHz PC.

6. Conclusions

We have considered a tactical integrated multiple-item production–distribution problem over periods of a finite horizon. In each period, the production problem involves determining how much to produce of each item, and the distribution problem consists of defining the quantities of each item to deliver to each customer as well as the vehicle routes. The objective is to minimize production and inventory costs at the plant, inventory costs at the customers and distribution costs.

We have proposed two heuristic approaches to solve this problem that allow trajectories with feasible and infeasible solutions. The first one is a tabu search with short memory that uses a compound move at each iteration involving the shift of an amount of an item delivered in a given period to every preceding and succeeding period, the determination of a new route, and the calculation of a new production plan over the time horizon. The second approach makes use of path relinking that is integrated with tabu search, such that every tabu search local minimum is linked with the farthest solution of a pool of elite solutions.

The approaches were tested on a set of small and large generated instances with multiple items. For small instances, both approaches yield high-quality solutions and the integrated tabu search-path relinking approach always achieves the best solution. For larger problems, this integrated approach again reaches the best solutions, however at the expense of high computational time. The tabu search and the tabu search-path relinking approaches yield good tradeoffs between solution quality and computational time. The approaches were also tested on a set of single item instances proposed by Boudia et al. [13] and they outperformed the memetic algorithm suggested by Boudia and Prins [14] and the

Table 12
Results for 50 customers, single item, set S_3

Instance	MA|PM		RTS		TS			TSPR	
	Cost	Gap%	Cost	Gap%	Cost	Gap%	Vehicles	Cost	Vehicles
1	378,378	1.29	398,795	6.76	377,057.66	0.94	5	373,549.72	5
2	403,913	9.37	373,374	1.10	373,249.66	1.07	5	369,303.34	5
3	409,573	19.08	353,058	2.64	350,227.31	1.82	5	343,960.38	5
4	399,220	12.48	361,176	1.76	360,792.25	1.65	5	354,935.34	5
5	422,279	18.05	364,819	1.98	364,336.81	1.85	5	357,718.38	5
6	407,122	10.80	368,082	0.18	367,966.75	0.15	5	367,425.75	5
7	414,977	14.30	369,963	1.90	368,821.88	1.59	5	363,048.94	5
8	379,744	4.30	370,822	1.85	367,997.88	1.07	5	364,096.09	5
9	407,935	8.99	379,379	1.36	378,215.91	1.05	5	374,300.53	5
10	396,258	9.02	370,655	1.98	369,623.09	1.70	5	363,461.88	5
11	402,475	14.12	354,025	0.38	353,002.69	0.09	5	352,688.88	5
12	358,702	2.03	354,981	0.97	353,161.97	0.45	5	351,572.34	5
13	371,030	2.74	365,432	1.19	364,435.53	0.92	5	361,130.13	5
14	406,114	14.78	363,404	2.71	356,746.78	0.82	5	353,828.59	5
15	373,076	3.28	367,659	1.78	365,955.19	1.31	5	361,234.34	5
16	379,404	6.55	360,534	1.25	359,325.44	0.91	5	356,096.06	5
17	406,353	4.22	398,442	2.19	397,516.12	1.95	5	389,911.81	5
18	401,179	10.86	368,533	1.84	363,586.69	0.47	5	361,887.88	5
19	406,893	10.48	377,073	2.39	375,240.16	1.89	5	368,279.06	5
20	398,508	9.23	372,141	2.01	369,903.97	1.39	5	364,817.69	5
21	397,112	7.12	374,743	1.08	372,477.44	0.47	5	370,730.5	5
22	358,749	6.94	347,329	3.53	340,979.91	1.64	5	335,476.97	5
23	407,369	13.06	362,619	0.64	361,957.16	0.46	5	360,303.09	5
24	369,784	2.80	375,022	4.26	365,428.91	1.59	5	359,697.19	5
25	411,556	13.97	374,682	3.75	367,662.19	1.81	5	361,124.88	5
26	408,704	14.10	366,167	2.22	365,258.06	1.97	5	358,213.31	5
27	366,197	1.52	375,261	4.03	365,018.5	1.19	5	360,714.47	5
28	401,032	10.55	373,155	2.87	370,284.25	2.08	5	362,753.97	5
29	384,282	3.52	379,320	2.19	376,922.91	1.54	5	371,205.41	5
30	369,959	3.44	369,223	3.23	365,533.22	2.20	5	357,664.13	5
Mean/total	393,262.6	8.77	369,662.3	2.20	366,289.54	1.27	150	361,704.37	150

Table 13
Results for 100 customers, single item, set S_3.

Instance	MA|PM		RTS		TS			TSPR	
	Cost	Gap%	Cost	Gap%	Cost	Gap%	Vehicles	Cost	Vehicles
1	714,401	2.27	711,671	1.88	710,344.31	1.69	9	698,537.88	9
2	722,047	8.79	694,694	4.67	676,042.38	1.86	8	663,692.19	9
3	677,598	1.46	683,270	2.31	674,553.50	1.00	9	667,862.19	9
4	710,552	3.17	718,252	4.29	701,164.13	1.81	8	688,698.00	9
5	733,040	3.40	731,260	3.15	721,641.44	1.79	9	708,949.50	9
6	696,146	1.18	744,927	8.26	690,589.22	0.37	9	688,059.81	9
7	705,322	6.27	695,728	4.82	673,210.75	1.43	9	663,718.69	9
8	679,210	2.73	706,058	6.79	672,165.69	1.66	9	661,188.75	9
9	699,518	2.96	705,035	3.77	689,903.06	1.54	9	679,438.50	9
10	705,778	4.31	696,521	2.94	686,892.23	1.52	9	676,606.44	9
11	709,122	1.63	711,895	2.02	703,834.94	0.87	9	697,769.88	8
12	755,726	12.95	703,162	5.10	677,824.44	1.31	9	669,058.75	9
13	695,466	1.94	721,066	5.70	692,988.54	1.58	9	682,207.94	9
14	718,260	6.61	698,548	3.69	684,362.69	1.58	8	673,717.13	9
15	736,041	7.55	711,506	3.97	695,865.19	1.68	9	684,363.38	9
16	715,209	5.34	714,873	5.29	691,734.44	1.88	9	678,968.81	9
17	737,832	6.86	702,314	1.72	696,829.31	0.93	8	690,434.06	9
18	723,413	3.54	720,238	3.08	708,632.31	1.42	9	698,709.25	9
19	720,218	2.83	748,734	6.90	710,978.81	1.51	9	700,400.44	9
20	724,727	2.40	729,099	3.02	719,182.44	1.62	9	707,716.69	9
21	724,328	3.07	738,746	5.12	712,886.63	1.44	9	702,765.81	9
22	701,506	2.39	702,849	2.58	696,728.63	1.69	9	685,148.50	9
23	710,033	2.66	712,717	3.04	702,188.94	1.52	8	691,658.88	8
24	734,327	3.78	727,741	2.85	721,506.31	1.97	8	707,566.25	9
25	725,446	1.97	725,869	2.03	721,826.50	1.46	8	711,438.75	9
26	718,939	3.52	700,719	0.89	697,072.50	0.37	9	694,507.94	9
27	715,068	8.37	686,382	4.02	668,840.94	1.36	9	659,865.00	9
28	685,117	4.88	700,980	7.30	662,994.00	1.49	9	653,259.50	9
29	722,571	2.04	725,030	2.39	71,9397.88	1.59	9	708,137.44	9
30	721,850	5.77	698,942	2.41	695,538.00	1.91	9	682,500.56	9
Mean/total	714,627.0	4.22	712,294.2	3.87	695,924.01	1.46	263	685,898.23	268

Table 14
Results for 200 customers, single item, set S_3

Instance	MA|PM		RTS		TS			TSPR	
	Cost	Gap%	Cost	Gap%	Cost	Gap%	Vehicles	Cost	Vehicles
1	996,151	4.72	1,030,684	8.35	970,113.88	1.98	13	951,277.31	13
2	978,373	8.87	1,010,158	12.41	917,543.69	2.10	12	898,670.56	13
3	986,147	6.10	1,016,681	9.38	949,758.69	2.18	13	929,493.06	13
4	962,937	1.01	1,042,854	9.39	959,083.69	0.60	13	953,348.56	13
5	970,638	3.08	1,023,680	8.72	960,243.63	1.98	13	941,598.19	13
6	965,646	3.21	1,025,262	9.58	955,943.75	2.17	13	935,638.88	13
7	980,562	1.33	1,038,746	7.35	976,149.81	0.88	12	967,666.31	12
8	1,014,809	2.67	1,066,068	7.85	1,001,074.25	1.28	13	988,430.38	13
9	967,738	5.15	1,018,420	10.66	937,101.88	1.82	13	920,346.75	13
10	1,093,230	10.40	1,035,240	4.55	996,755.56	0.66	13	990,212.44	13
11	1,008,080	6.98	1,037,705	10.12	960,246.69	1.90	13	942,340.94	13
12	998,951	7.41	1,035,350	11.33	948,593.06	2.00	12	929,992.13	13
13	984,918	3.74	1,063,024	11.96	967,493.88	1.90	13	949,453.31	13
14	964,301	1.76	1,024,491	8.11	953,588.81	0.63	12	947,613.44	12
15	981,167	3.67	1,026,787	8.49	965,394.81	2.00	13	946,464.00	13
16	1,017,777	5.45	1,033,656	7.10	985,894.69	2.15	13	965,142.94	13
17	1,073,640	10.76	1,022,250	5.46	997,104.31	2.86	13	969,354.38	13
18	1,003,670	3.54	1,063,306	9.69	984,964.00	1.61	13	969,355.13	13
19	997,348	2.98	1,065,705	10.04	985,349.50	1.74	13	968,496.56	13
20	981,788	5.40	1,027,134	10.26	958,566.88	2.90	12	931,517.06	13
21	974,384	1.75	1,044,771	9.10	963,701.00	0.63	13	957,665.56	13
22	1,065,780	8.87	1,045,790	6.83	996,786.00	1.82	13	978,963.44	13
23	1,070,520	9.07	1,027,042	4.64	996,699.69	1.54	13	981,539.31	13
24	978,491	2.72	1,045,014	9.70	973,964.88	2.24	12	952,625.00	12
25	1,029,327	11.67	1,024,239	11.11	940,405.25	2.02	13	921,783.88	13
26	961,728	0.84	1,043,128	9.37	960,674.94	0.73	13	953,756.19	13
27	1,028,006	5.85	1,030,753	6.14	999,996.00	2.97	13	971,152.44	13
28	1,011,689	8.08	1,032,478	10.30	956,877.44	2.22	12	936,095.06	13
29	1,015,741	5.55	1,019,371	5.93	978,860.69	1.72	13	962,295.63	13
30	985,496	5.19	1,027,915	9.72	953,611.13	1.79	13	936,837.81	13
Mean/total	1,001,634.4	5.26	1,034,923.4	8.79	968,418.08	1.77	383	951,637.56	387

Table 15
Computational times, single item, set S_3.

Instance	50 customers				100 customers				200 customers			
	MA|PM[a]	RTS[b]	TS	TSPR	MA|PM[a]	RTS[b]	TS	TSPR	MA|PM[a]	RTS[b]	TS	TSPR
1	170.4	180.0	154.7	322.7	1147.4	1079.0	551.0	1164.8	3633.8	2965.0	1823.6	4043.1
2	149.0	71.0	159.6	329.7	1192.8	240.0	529.1	1165.5	3755.5	1250.0	1986.3	3750.6
3	135.7	234.2	142.9	255.7	925.9	1299.0	548.3	1219.9	3629.6	1200.0	1966.5	3962.2
4	159.7	300.0	153.9	308.6	1097.3	672.0	540.9	1147.7	3851.8	2141.0	1971.5	3739.3
5	193.3	290.0	158.2	340.8	1125.0	335.0	538.7	1190.4	4185.5	2423.0	1882.9	3743.5
6	174.7	467.1	143.3	249.3	1107.0	1143.0	522.9	1043.2	3794.8	2205.0	1921.1	3972.1
7	174.0	404.7	139.1	220.1	1081.8	1250.0	545.2	1145.1	4732.9	2610.0	1926.3	3931.4
8	170.8	329.3	153.8	302.5	1043.1	1008.0	547.2	1131.7	3929.4	2978.0	1910.0	4156.1
9	158.1	36.0	145.3	252.8	1136.1	1125.0	515.8	1107.1	3424.8	1250.0	1929.6	4094.4
10	179.4	260.0	164.2	370.1	976.9	985.0	541.1	1098.9	4600.8	2625.0	1977.5	4150.2
11	178.3	540.0	143.0	250.0	1154.7	835.0	557.4	1267.5	5440.0	3428.0	1899.9	3802.6
12	151.0	180.0	147.3	260.7	1163.3	1129.0	543.1	1131.7	3933.4	2242.0	2007.0	4211.8
13	193.3	208.0	145.2	252.8	1036.6	599.0	548.7	1050.9	4662.0	2719.0	1921.6	4118.2
14	160.8	184.0	165.9	339.5	1153.7	1065.0	519.6	1115.4	3596.1	1920.0	1916.5	3699.3
15	173.4	555.1	181.6	459.7	1252.7	1139.0	565.3	1181.4	4023.7	2408.0	1913.5	4385.7
16	186.2	493.8	150.7	296.1	1210.9	1226.0	560.4	1212.1	4245.7	3200.0	1862.0	3681.3
17	177.5	153.9	157.9	334.4	964.5	1218.0	558.9	1091.2	4355.4	2018.0	1867.3	3921.8
18	163.5	189.0	152.6	336.9	1021.0	720.0	546.5	1232.1	3875.2	4740.0	1907.2	3959.5
19	186.9	488.7	156.2	340.4	1164.4	1349.0	545.6	1130.6	4157.7	2797.0	1849.5	3970.5
20	182.7	635.9	153.8	315.4	1167.4	1131.0	533.3	1127.3	4048.1	2425.0	1822.3	3989.5
21	188.4	160.2	158.4	366.6	1173.5	544.0	536.4	1060.8	4205.0	3900.0	1945.2	3931.8
22	146.1	810.0	156.8	354.2	1179.8	998.0	548.8	1130.7	4465.9	2822.0	1807.9	3645.5
23	188.5	794.7	168.8	336.0	1196.9	1037.0	556.5	1177.6	3147.2	1797.0	1848.3	3804.6
24	180.1	232.3	156.8	354.0	1250.7	1380.0	542.7	1064.2	3308.8	2908.0	1992.3	4100.1
25	192.4	163.0	158.9	349.4	848.4	1240.0	548.5	1269.6	4349.0	2601.0	1959.2	3868.3
26	159.9	129.0	160.1	344.5	1094.4	585.0	549.5	1171.3	3785.8	1673.0	1861.8	4096.5
27	173.1	156.7	160.9	349.0	1033.7	454.0	544.1	1177.3	4086.9	2037.0	1785.1	4300.2
28	171.0	230.7	151.8	282.9	1092.3	1190.0	559.4	1067.9	4456.1	2909.0	1940.0	4024.1
29	198.4	543.8	148.5	281.4	1069.8	993.0	545.5	1180.2	5168.0	2179.0	1964.1	3511.7
30	163.5	495.7	163.5	354.6	1180.3	1300.0	527.4	1175.4	4105.8	2400.0	1999.7	3225.0
Mean	172.7	330.6	155.1	317.0	1108.1	975.6	543.9	1147.6	4098.5	2492.3	1912.2	3926.4

[a] 2.8 GHz PC.
[b] 2.53 GHz PC.

reactive tabu search developed by Bard and Nananukul [15] in all instances.

Our results have shown that tabu search and path relinking can be successfully applied to the addressed problem and more generally to planning problems with a discrete time horizon. It would be very interesting to investigate the application of the proposed approaches to other production–distribution problems and also to inventory-routing problems.

Acknowledgements

This research was partially funded by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and the Fundação de Amparo à Pesquisa (FAPESP). The authors are also grateful to Dr. Mourad Boudia for providing them with the instances.

Appendix

Tables 12–14 show the detailed results for the single item instances, summarized in Table 10. Each table shows the instance number, the solution cost and the gap obtained by each method, which is the percentage deviation of the cost obtained by each method relative to the least solution cost obtained by TSPR. For nearly half number of instances TS requires a smaller number of vehicles than TSPR. In the last line, mean results are shown for solution costs and gaps, while "total" stands for the sum of vehicles under the columns named vehicles. The proposed heuristics TS and TSPR outperform MA|PM and RTS in all instances, and the solution quality improves with the increase of number of customers, as indicated by the mean gaps obtained by TSPR. The smallest mean gap is 2.20% relative to RTS for 50 customers, and the largest one is 8.79% relative to RTS for 200 customers. Table 15 shows that the TS require a short mean computational time for any number of customers.

References

[1] Simchi-Levi D, Kaminskt P, Simchi-Levi E. Managing the Supply Chain. New York: McGraw-Hill; 2004.
[2] Shiguemoto AL. Métodos heurísticos para resolução de problemas integrados de produção, estoque e distribuição. Doctoral thesis (in portuguese), Universidade Estadual de Campinas, SP, Brazil; 2008.
[3] Thomas JD, Griffin PM. Coordinated supply chain management. European Journal of Operational Research 1996;94:1–15.
[4] Vidal CJ, Goetschalckx M. Strategic production–distribution models: a critical review with emphasis on global supply chain models. European Journal of Operational Research 1997;98:1–18.
[5] Sarmiento AM, Nagi R. A review of integrated analysis of production–distribution. IIE Transactions 1999;31:1061–74.
[6] Erengüç SS, Simpson NC, Vakharia AJ. Integrated production/distribution planning in supply chains: an invited review. European Journal of Operational Research 1999;115:219–36.
[7] Chen Z-L. Integrated Production and Distribution Operations: Taxonomy, Models, and Review. In: Simchi-Levi D, Wu D, Chen Z-L, editors. Handbook of Quantitative Supply Chain Analysis: Modelling in the E-Business Era. Boston, MA: Academic Publishers, Kluwer; 2004. p. 711–46.
[8] Hurter AP, Van Buer MG. The newspaper production/distribution problem. Journal of Business Logistics 1996;17:85–107.
[9] Van Buer MG, Woodruff DL, Olson RT. Solving the medium newspaper production/distribution problem. European Journal of Operational Research 1999;115:237–53.

[10] Russell R, Chiang W-C, Zepeda D. Integrating multi-product production and distribution in newspaper logistics. Computers and Operations Research 2008;35:1576–88.
[11] Chandra P, Fisher ML. Coordination of production and distribution planning. European Journal of Operational Research 1994;72:503–17.
[12] Fumero F, Vercellis C. Synchronized development of production, inventory, and distribution schedules. Transportation Science 1999;33:330–40.
[13] Boudia M, Louly MAO, Prins C. A reactive GRASP and path relinking for a combined production–distribution problem. Computers and Operations Research 2007;34:3402–19.
[14] Boudia M, Prins C. A memetic algorithm with dynamic population management for an integrated production–distribution problem. European Journal of Operational Research 2009;195:703–15.
[15] Bard JF, Nananukul N. The integrated production-inventory-distribution-routing problem. Journal of Scheduling 2009;12:257–80.
[16] Lei L, Liu S, Ruszczynski A, Park S. On the integrated production, inventory, and distribution routing problem. IIE Transactions 2006;38:955–70.
[17] Toth P, Vigo D. The vehicle routing problem. Philadelphia: SIAM; 2002.
[18] Karimi B, Fatemi SMT, Wilson JM. The capacitated lot sizing problem: a review of models and algorithms. Omega 2003;31:365–78.
[19] Glover F, Laguna M. Tabu Search. Kluwer: Boston, MA; 1997.
[20] Gendreau M. On the importance of allowing infeasible moves in tabu search heuristics, presented at the INFORMS National Meeting, Denver, October 2004.
[21] Clarke G, Wright JW. Scheduling of vehicles from a central depot to a number of delivery points. Operations Research 1964;12:568–81.
[22] Evans JR. An efficient implementation of the Wagner–Whitin algorithm for dynamic lot-sizing. Journal of Operational Management 1985;5:229–35.
[23] Wagner HM, Whitin TM. Dynamic version of the economic lot size model. Management Science 1958;5:89–96.
[24] Glover F. Tabu Search and Adaptive Memory Programming—Advances, Applications and Challenges. In: Barr RS, Helgason RV, Kennington JL, editors. Computing Tools for Modeling. Optimization and Simulation: Interfaces in Computer Science and Operations Research,. Boston, MA: Kluwer; 1996. p. 1–75.
[25] Glover F. A Template for Scatter Search and Path Relinking. In: Hao JK, Luton E, Ronald E, Schoenauer M, Snyers D, editors. Artificial Evolution Lecture Notes in Computer Science, vol. 1363. Berlin: Springer; 1998. p. 13–54.
[26] Martí R, Laguna M, Glover F. Principles of scatter search. European Journal of Operational Research 2006;169:359–72.
[27] Yamashita DS, Armentano VA, Laguna M. Scatter search for project scheduling with resource availability cost. European Journal of Operational Research 2006;169:623–37.
[28] Resende MGC, Ribeiro CC, Glover F, Martí R. Scatter search and path-relinking: Fundamentals, advances, and applications. In: Gendreau M, Potvin J-Y, editors. Handbook of Metaheuristics. 2nd Ed.. Springer; 2010.
[29] Bertazzi L, Paletta G, Speranza MG. Minimizing the total cost in an integrated vendor-managed inventory ystem. Journal of Heuristics 2005;11:393–419.
[30] Trigeiro WW, Thomas LJ, McClain JO. Capacitated lot sizing with setup times. Management Science 1989;35:353–66.
[31] Toledo FMB, Armentano VA. A Lagrangian-based heuristic for the capacitated lot-sizing problem in parallel machines. European Journal of Operational Research 2006;175:1070–83.