LETTER TO THE EDITOR

One-Term Lorentzian and Sharpening of the Noise Power Spectrum

Dear Sir:

Yi-der Chen (1) has shown that the auto noise power spectrum for an arbitrary chemical system at equilibrium never exceeds the corresponding one-term Lorentzian. In this letter a sufficient condition for this to be the case for any spectrum is given. The condition is easily verified for a rather broad class of functions which includes the chemical equilibrium case.

Let $G(t)$ be the spectrum, where $t = \omega^2$. Assume $G \geq 0$ to be differentiable and the limit $G_\infty = \lim tG(t)$, as $t \to \infty$ to exist. The one-term Lorentzian is $G_L(t) = G_\infty/(G_\infty/G(0) + t)$. We will have $G(t) \leq G_L(t)$ if and only if $F(t) \geq F(0) + t$, where $F(t) = G_\infty/G(t)$. By the mean-value theorem of differential calculus $F(t) = F(0) + F'(\xi)t$, with $0 \leq \xi \leq t$. Thus, if $F'(t) \geq 1$ for all t, then $G(t) \leq G_L(t)$. This simple statement of the problem demands nothing of G but differentiability and the existence of G_∞.

Now suppose $G(t) = \Sigma a_i/f_i(t)$, where the $a_i > 0$ are constants and the f_i have the properties $f_i(t) > 0, f_i'(t) \geq 1$ and $\lim t^{-1}f_i(t) = 1$, as $t \to \infty$. Then $F(t) = \Sigma a_i/\Sigma a_i/f_i(t)$ and $F' = (\Sigma a_i)(\Sigma a_i f_i'/f_i^2)/(\Sigma a_i/f_i)^2 \geq (\Sigma a_i)(\Sigma a_i/f_i^2)/(\Sigma a_i/f_i)^2$. If $x_i = \sqrt{a_i}$, $y_i = \sqrt{a_i}/f_i$, then $F' \geq (\Sigma x_i^2)(\Sigma y_i^2)/(\Sigma x_i y_i)^2 \geq 1$, by the Cauchy inequality. The case treated in (1) corresponds to $f_i(t) = b_i + t$, with $b_i > 0$ constant.

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REFERENCE

1. Chen, Y. 1978. Differentiation between equilibrium and nonequilibrium kinetic systems by noise analysis. *Biophys. J.* **21**:279.

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