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Reliability Analysis of Reinforced Concrete Buildings: Comparison between FORM and ISM

El ghoulbzouri Abdelouafi[a,*],Kissi benaissa[b], Khamlichi Abdellatif[c]

[a]National School of Applied Sciences of Al Hoceima, Civil engineering Dept., Morocco
[b]National School of Arts and Trades of Casablanca, Morocco ,
[c]Modeling and Simulation of Mechanical Systems Laboratory, Faculty of Sciences at Tetouan, University Abdelmalek Essaâdi, Tetouan 93002, Morocco,

Abstract

Accounting for uncertainties that are present in geometric and material data of reinforced concrete buildings is performed in this work within the context of performance based seismic engineering design. Reliability of the expected performance state is assessed by using various methodologies based on finite element nonlinear static pushover analysis and specialized reliability software package. Reliability approaches that were considered included full coupling with an external finite element code and surface response based methods in conjunction with either first order reliability method or importance sampling method. The probability of failure according to the used reliability analysis method and to the selected distribution of probabilities was obtained. Convergence analysis of the importance sampling method was performed. The required duration of analysis as function of the used reliability method was evaluated. It was found that reliability results are sensitive to the used reliability analysis method. Durations of analysis for coupling methods were found to be higher than those associated to surface response based methods; one should however include time needed to derive these lasts. For the reinforced concrete building considered in this study, it was found that significant variations exist between all the considered reliability methodologies. The full coupled importance sampling method is recommended, but the first order reliability method applied on a surface response model can be used with good accuracy.

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Keywords: Reliability analysis, FORM, Importance sampling method, Response surface, Finite element, Seismic design.

1. Introduction

Realistic modeling based on reliability analysis of structural behavior of buildings at risk of earthquake events is the

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subject of increasing interest from the community of seismic building designers Liel *et al.*[16], Möller *et al.*[19], and Piluso et al.[22], Buratti *et al.*[2], Celik and Ellingwood[3]. Among the reasons beyond the intensive research activity in this field, one finds the huge need for diagnosis and rehabilitation of pre-code constructions, particularly in the case of historic monuments. Other reasons are associated to the emergence of new design approaches which are founded on the concept of performance-based engineering.

Performance-based engineering has gained large success in the field of earthquake engineering. Instead of the classical regulatory and non-transparent seismic code rules which were elaborated to ensure essentially a priori life safety of buildings occupants, this new approach includes additional critical states that could be important for buildings use. These performance states are associated to indicators such as the tolerable amount of damage or the accepted economic loss resulting from temporarily loss of functionality.

Predicting these performance states is considered in terms of probabilities such that occupants or owners of buildings could be aware of the risk level they are undergoing. To realize that, adequate numerical modeling of the building structural behavior and satisfactory description of uncertainty propagation are required. This is generally performed within the framework of reliability analysis. Uncertainties arising in the problem could be the result of the inherent randomness in material characteristics, geometric dimensions or applied forces. These categories of uncertainties are termed stochastic parameters. But, uncertainties could be also epistemic such as those due to lack of knowledge regarding the real values of some parameters in existing constructed buildings: reinforcement sections in structural members or junctions features that exist between columns and beams.

Predicting the real complex behavior of structures is nowadays largely performed by using the finite element method, as an example this method is used to assess crack propagation Souiyah et *al* [3], and Alshoaibi et *al* [3]. In the presence of uncertainties affecting structural model parameters, stochastic finite element has been introduced. The input data for the finite element computation are dealt with as random variables to depict the uncertain variations present in the material, geometry and loading parameters. Through uncertainty propagating modeling such as Monte Carlo process the resulting probability of response events could be computed. Finite element reliability analysis is a technique that combines stochastic finite element analysis with some performance function defining a given limit-state. The performance function depends on response quantities of the finite element analysis, and comes out to be an implicit function of the input data. The performance function separates the data space into two regions: the safe region and the failure region. The probability of failure is linked to the minimum distance separating the actual design realization from the most probable failure point laying on the limit surface, called also the design point. Since the performance function is not explicitly known and Monte Carlo process is too time consuming, search of the design point is performed habitually through various approximate reliability analysis methods.

In the first and second order reliability methods (FORM and SORM) the limit-state is approximated, at the most likely failure point in the transformed space of uncorrelated standard normal random variables, by respectively a hyper-plane and a paraboloid. A review of coupling between FORM reliability analysis and the finite element method can be found in Der Kiureghian[4]. Other significant contributions have since been presented. They include developments due to Liu and Der Kiureghian[17], Der Kiureghian and Zhang[5], Haldar and Mahadevan [11], Imai and Frangopol[14], Sudret and Der Kiureghian [25], Franchin[7], and Frier , and Sorensen[8].

Finite element reliability analysis using full coupling between a finite element code and reliability methods such as FORM or Monte Carlo tends however to be high computational time consuming for practical problems that include large number of random variables. This is so because, at any iteration, the limit-state function and its derivatives are to be evaluated through finite element computations. An effective method which combines FORM and subsequent importance sampling around the most probable failure point has been proposed by Haukaas and Der Kiureghian[12]. The Importance Sampling Method (ISM) requires only a limited number of evaluations of the limit-state function (and its gradient with respect to the random variables) to find the approximation point, followed by efficient importance sampling analysis centered at this point. Haukaas and Der Kiureghian[12] have presented numerical examples involving comprehensive nonlinear finite element models with approximately 500 random variables that state convergence of ISM.

2. Case of study

In order to evaluate the different methodologies introduced to account for uncertainties within the framework of finite element reliability analysis under OpenSees environment, a four-story reinforced concrete building structure is considered. It consists of a regular building for which the nonlinear static pushover analysis is sufficient to assess seismic performance. The inter story height is fixed at $3\,\mathrm{m}$. The bay length in both seismic directions is fixed at $4\,\mathrm{m}$. Fig. 1 gives the vertical elevation and the plane view. Fig.2 gives concrete sections of members with their reinforcements as computed by using Eurocode 2 code. Table 1the vertical load resultants at the structural nodes in kN. Table 2 the seismic lateral loads in kN.

The ultimate strengths of outer layer of concrete and of the core concrete as well as Young's modulus of the reinforcement steel are modeled as random variables. These are intended to model the inherent, irreducible uncertainty in the finite element model parameters. All the other material and geometric parameters as well as loads will be considered to be deterministically known. Table 3 gives the deterministic nominal parameters values of material data that were used during reliability analysis of the RC structure. Table 4 defines the uncertainty modeling of the random variables.

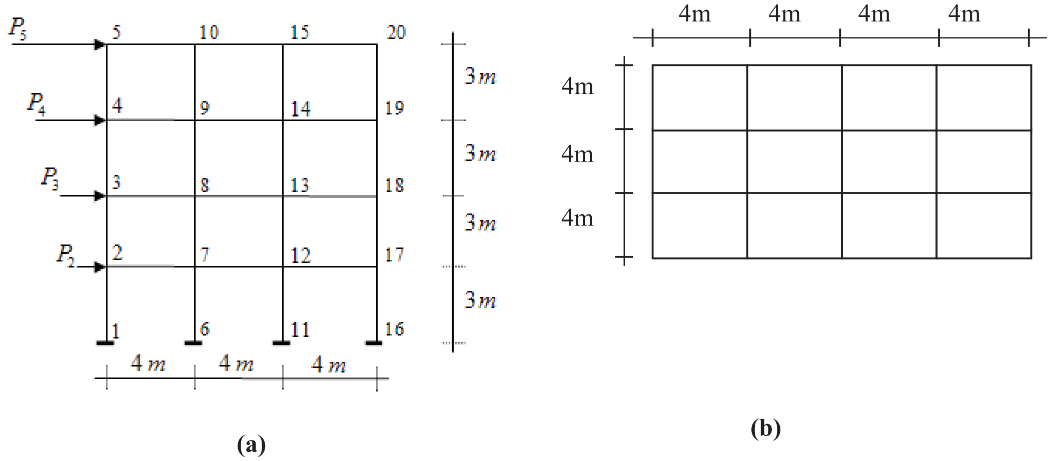


Fig.1: Four-storey two-bay reinforced concrete structure; (a) Vertical elevation, (b) Plane view.

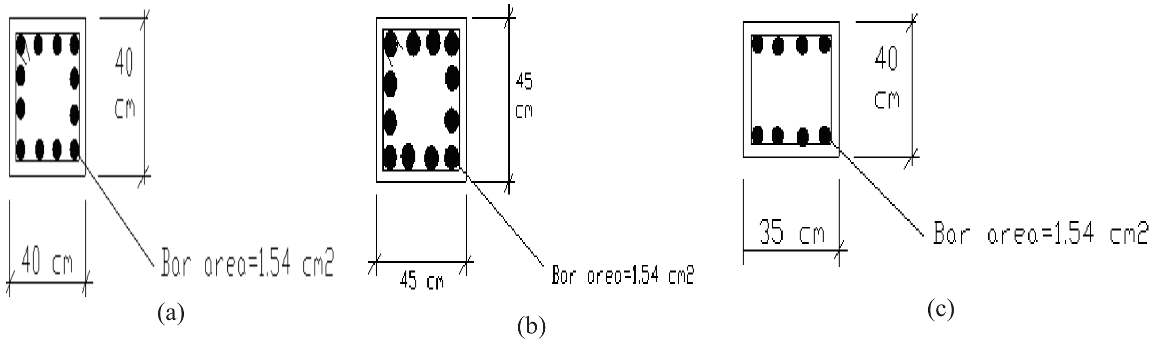


Fig.2: Members reinforcements; (a) Exterior columns, (b) Interior columns, (c) Section girders.

Table1: Vertical load resultants at the structural nodes

Node	Load	Node	Load
2	G2=188.78kN	12	G12=377.57kN
3	G3=141.6kN	13	G13=283.17kN
4	G4=94.39kN	14	G14=188.78kN
5	G5=47.20kN	15	G15=94.39kN
7	G7=377.57kN	17	G17=188.78kN
8	G8=283.17kN	18	G18=141.6kN
9	G9=188.78kN	19	G19=94.39kN
10	G10=94.39kN	20	G20=47.20kN

Table 2: Seismic lateral loads

Node	Load	Node	Load
2	P2=22.66kN	4	P4=76.95kN
3	P3=45.3kN	5	P5=90.6kN

Table 3: Deterministic nominal parameters values of the reinforced concrete building

Parameter	Value
Compressive strain of outer layer of concrete, $\varepsilon_{c,uc}$	0.002
Ultimate strength of outer layer of concrete, $f'_{cu,uc}$ (MPa)	0.0
Ultimate strain of outer layer of concrete, $\varepsilon_{cu,uc}$	0.006
Compressive strain of the core concrete, $\varepsilon_{c,cc}$	0.005
Ultimate strength of the core concrete, $f'_{cu,cc}$ (MPa)	30
Ultimate strain of the core concrete, $\varepsilon_{cu,cc}$	0.02
Tensile strength of the reinforcement steel, f_y (MPa)	500
Second slope stiffness ratio of the reinforcement steel, α	0.02
Cover (mm)	30

Table 4. Uncertainty modeling of the random variables of the reinforced concrete buildings

Variable	Mean value	Deviation ratio	Standard deviation
$f'_{c,cc}$ (MPa)	37.92	0.15	5.69
$f'_{c,uc}$ (MPa)	27	0.15	4.05
E (MPa)	200000	0.05	10000

All the random variables are assumed to be distributed according to lognormal probability distribution function. Denoting μ the mean value and σ the standard deviation, these PDF write as follows.

Lognormal distribution

$$f(x)=\frac{1}{\sqrt{2\pi}\left(\ln\left(\frac{\mu^2+\sigma^2}{\mu^2}\right)\right)^{\frac{1}{2}}x}\exp\left(-\left(\log\left(\frac{x\sqrt{\mu^2+\sigma^2}}{\mu^2}\right)\right)^2\Big/2\log\left(\frac{\mu^2+\sigma^2}{\mu^2}\right)\right) \quad (1)$$

This limit-state function seeks the probability that the horizontal displacement at the roof of the structure exceeds 0.4% of the building height, when deterministic lateral seismic load is evaluated according the Moroccan seismic code RPS2000 recommendations for a given geographic zone, soil site, ductility coefficient and building priority. The limit state function writes

$$g(x)=0.004H-d_{roof}(x) \quad (2)$$

where H is the total height of the building, in units of mm, d_{roof} is the horizontal displacement, in units of mm, of the roof node as obtained from a static nonlinear pushover-type analysis and x is the vector of random variables:

$$x=\left[f'_{c,cc} \quad f'_{c,uc} \quad E\right]^t$$

Knowing that H=12000mm, the limit-state function writes

$$g(x)=48-d_{roof}(x)\ \ (\text{mm}) \quad (3)$$

Two kinds of parametric studies were performed as indicated in the following:

- using full coupling reliability analysis, comparison between FORM and ISM methods was conducted and the influence of PDF's on results investigated;
- using response surface method and full coupling reliability analysis, comparison between FORM and ISM methods

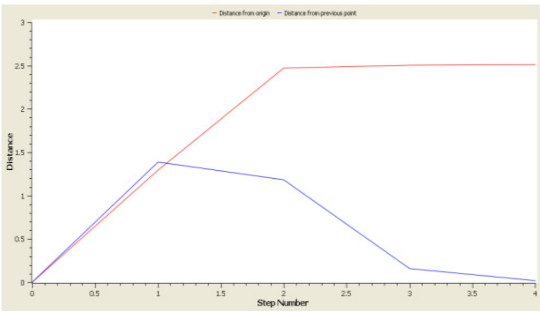
3. Results

Full reliability coupling FORM and ISM methods

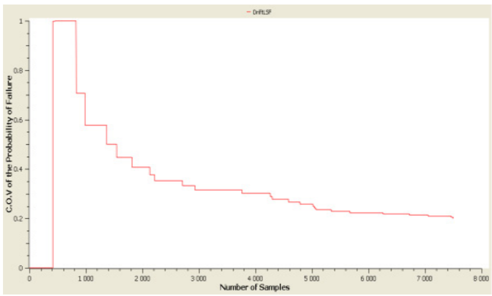
Table 5 and Fig 3 show that results obtained by Monte Carlo based Sampling Analysis method and FORM method in case of Lognormal distributions of probability.

Table 5 :Results obtained by Monte Carlo based Sampling Analysis method and FORM method

Method	Time to complete the analysis	Reliability index	Failure probability x10^{-2}
Monte Carlo(ISM)	1353.34 (sec)	2.61545	0.445555
FORM	7.438 (sec)	2.5131	0.598385



(a)



(b)

Fig.3. coupling reliability analysis; (a) Step number-FORM analysis , (b) total number of samples-ISM analysis

FORM and ISM reliability methods applied to a response surface model of the building

The response surface writes as $g(x)=48-d_{roof}(f'_{c,cc},f'_{c,uc},E)$ where $d_{roof}(f'_{c,cc},f'_{c,uc},E)$ is the interpolated displacement over the domain of variables $(f'_{c,cc}, f'_{c,uc}, E)$. Interpolation is performed according to a full factorial design of experiment table containing a total number of 27 combinations. The combinations include values of parameters corresponding to:

- lower threshold $\mu - h\sigma$;
- average value μ;
- higher threshold $\mu + h\sigma$.

Table 6 recalls the obtained results when using lognormal probability distributions and fixing the value of magnitude $h = 1$. The interpolated roof displacement, expressed in (inch), writes

$$d_{roof} = d_{roof}(f'_{c,cc}, f'_{c,uc}, E) = 5.1118 - 0.09535f'_{c,cc} - 0.48306f'_{c,uc} - 0.000096885E + 0.0071656f'_{c,cc} \times f'_{c,uc} + 0.0000039884f'_{c,uc} \times E + 0.00000061414E \times f'_{c,cc} + 0.001781f'_{c,cc} \times f'_{c,cc} + 0.021405f'_{c,uc} \times f'_{c,uc} + 0.00000000078099E \times E \text{ (inch)} \quad (4)$$

Table 6: Roof displacement as function of the considered combination

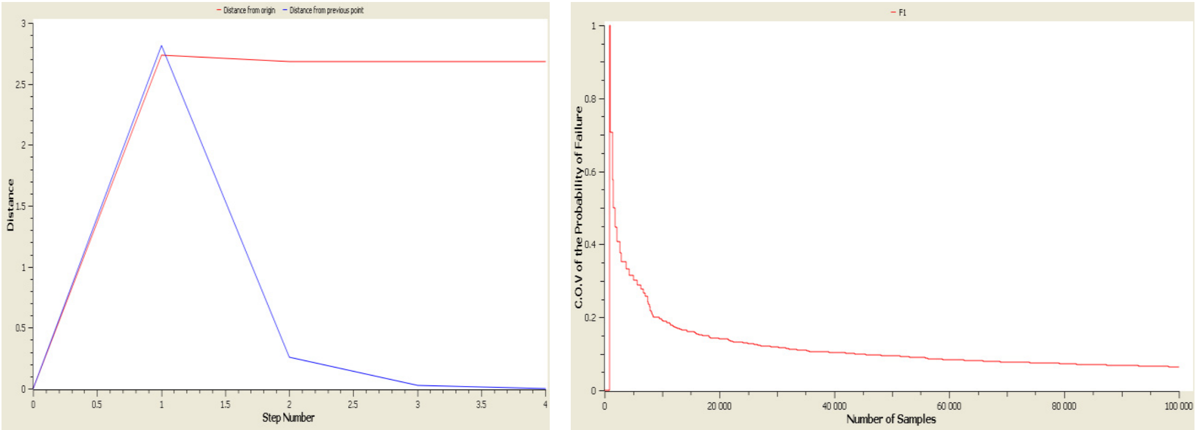
Test	$f'_{c,cc}$ (Ksi)	$f'_{c,uc}$ (KSi)	E (KSi)	droof (inch)
1	6.325	4.5034	31500	1.45347
2	6.325	4.5034	30000	1.49438
3	6.325	4.5034	28500	1.53842
4	6.325	3.916	31500	1.53183
5	6.325	3.916	30000	1.5757
6	6.325	3.916	28500	1.62299
7	6.325	3.3286	31500	1.62375
8	6.325	3.3286	30000	1.67109
9	6.325	3.3286	28500	1.7222
10	5.5	4.5034	31500	1.47263
11	5.5	4.5034	30000	1.51419
12	5.5	4.5034	28500	1.55898
13	5.5	3.916	31500	1.55374
14	5.5	3.916	30000	1.59841
15	5.5	3.916	28500	1.64656
16	5.5	3.3286	31500	1.64904
17	5.5	3.3286	30000	1.69731
18	5.5	3.3286	28500	1.74942
19	4.675	4.5034	31500	1.49385
20	4.675	4.5034	30000	1.53591
21	4.675	4.5034	28500	1.58129
22	4.675	3.916	31500	1.57818
23	4.675	3.916	30000	1.62349
24	4.675	3.916	28500	1.67234
25	4.675	3.3286	31500	1.67744
26	4.675	3.3286	30000	1.72653
27	4.675	3.3286	28500	1.77952

The response surface in terms of the performance function as derived from Eq.(3) and Eq.(4) is identified from the discrete displacement results that are listed in table 6. This is provided by using the least-square inversion technique.

Table 7 and Fig 4 show that results obtained by RSM/ISM method and RSM/ FORM method in case of Lognormal distributions of probability.

Table 7. Results obtained by FORM and ISM reliability methods applied to a response surface

Method	Time to complete the analysis	Reliability index	Failure probability x10^{-2}
Monte Carlo(ISM)	1353.34 (sec)	2.73674	0.310261
FORM	0.297 (sec)	2.68424	0.363478



(a) (b)

Fig.4. Response surface (RSM) reliability analysis; (a) Step number-FORM analysis , (b) total number of samples-ISM analysis.

4. Conclusions

It has been shown that the full coupling reliability analysis does not predict the same results than the approximate response surface based reliability method. This last underestimates in general the probability of failure. The maximum relative difference between these various methodologies results has reached 41.5%. This occurs between the Coupling/FORM and the SRM/ISM methods. Within the framework of the same methodology of reliability analysis (either full coupling or response surface), the approximate method FORM does not give the same results than the more precise modified Monte Carlo ISM Method. In general, FORM overestimates the probability of failure. The obtained results have shown that full coupling reliability analysis conducted with ISM is recommended because FORM analysis could exaggerate sometimes the probability of failure. It is remarkable to observe the antagonist effect resulting from the association of SRM/FORM because this method gives results that are closer to the more exact Coupling/ISM. The ISM method is always more time consuming than FORM approximation. To make an objective comparison regarding computational cost, one should recall also that response surfaces must be identified and that additional labor is required for that. Influence on reliability data of the chosen PDF's to model uncertainties is very significant when considering full coupling reliability analysis. This effect reduces however when surface response based reliability analysis is performed. In order to perform reliability analysis of seismic performance based design, huge care should be given to selection of PDF's that model parameters uncertainties (these should be is general identified through experiments). It is not sufficient to give only means and standard deviations, the PDF's must also be specified. When applying surface response based reliability analysis, it was shown that FORM approximation overestimates here again the probability of failure in comparison with the accurate

ISM. These conclusions could not be generalized without precautions to other problems dealing with reliability analysis and thorough analysis is needed to assess performance of the various methodologies.

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