Combined Differential Evolution Algorithm and Ant System for Optimal Reactive Power Dispatch

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Abstract

This paper proposes a hybrid approach to solve the optimal reactive power dispatch (ORPD) problem. Traditionally, ORPD is defined as the minimization of active power transmission losses by controlling a number of control variables, which is formulated as a nonlinear constrained optimization problem with continuous and discrete variables. Based on the original differential evolution (DE) algorithm, the proposed approach combines variable scaling mutation and probabilistic state transition rule used in the ant system to deal with the ORPD problem. To verify the performance of the proposed method, the similar evolution approaches such as the evolutionary programming (EP) and particle swarm optimization (PSO) are also implemented using the same study case. Testing on the IEEE 30-bus system indicates that the proposed approach can obtain better results with lower active power transmission losses and better convergence performance than the existing methods.

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1. Introduction

The ORPD is a subproblem of the optimal power flow (OPF) calculation and has a significant influence on secure and economic operation of power systems. Traditionally, ORPD is defined as the minimization of active power transmission losses by controlling the generator terminal voltages, transformer tap settings, and shunt capacitors/reactors. The purpose of ORPD is to reduce active power transmission losses and improve voltage profile in the power systems. Since the control variables such as shunt capacitors/reactors and the tap settings of transformer have the discrete nature, the ORPD is then formulated as discrete and nonlinear optimization problem.

Much research has been devoted to cope with the ORPD problem. These techniques include the nonlinear programming method [1], mixed-integer programming method [2], interior point method [3],
2. Problem Formulation

Typically, ORPD is to minimize active power transmission loss subject to a number of constraints. The objective of active power transmission loss can be expressed as follows.

\[ f_Q = \sum_{k \in \{i,j\}} P_{ik} = \sum_{i=1}^{L} \sum_{j=1}^{L} g_{ij} \left| V_i \right|^2 \left| V_j \right|^2 - 2 \left| V_i \right| \left| V_j \right| \cos(\delta_i - \delta_j) \]

where \( f_Q \) is the active power transmission loss, \( P_{ik} \) is the active power transmission loss of branch \( k \), \( L \) is the number of transmission lines, \( |V_i| \) is the voltage magnitude at bus \( i \), \( g_{ij} \) is the conductance between bus \( i \) and \( j \), and \( \delta_i \) is the voltage phase angle of bus \( i \).

The objective in (1) must satisfy the following constraints.

- **Power balance constraint.** The injected active power at each bus must be equal to active power demand plus active power transmission loss at each bus, i.e. \( P_{li} = P_{Di} + P_{li} \). Similarly, the reactive power balance constraint must be satisfied as well.

- **Reactive generation constraint.** The injected reactive power at each generation bus (PV bus) must be controlled within the lower and upper limits, i.e. \( Q_{Bi,min} \leq Q_{Bi} \leq Q_{Bi,max} \).

- **Bus voltage constraint.** Each voltage magnitude of load bus (PQ bus) must be controlled within the lower and upper limits, i.e. \( V_{i,min} \leq \left| V_i \right| \leq V_{i,max} \), in order to remain the stable operation of the power system.

- **Capacitor and transformer tap setting constraints.** The capacitor and transformer tap setting must be tuned within the lower and upper limits, i.e. \( Q_{C,min} \leq Q_{C} \leq Q_{C,max} \) for capacitor and \( T_{k,min} \leq T_k \leq T_{k,max} \) for transformer. In this paper, the capacitor and transformer tap setting are regarded as continuously adjustable variables.

3. The Proposed Approach

Based on the basic evolutionary strategies, the proposed approach achieves the fittest individual after repeated initialization, mutation, recombination, and selection operations as follows.

- **Initialization**
  
  Let \( p_0 = [p_{11}, p_{12}, \ldots, p_{1M}] \) be a trial vector representing the \( i \)th individual (\( i = 1, 2, \ldots, P \)) of the population to be evolved, where \( P \) is the population size and \( M \) is the dimension of each individual. The elements in vector \( p_i \) represent the decision variables (genes) which are randomly generated as follows.

\[ p_{ij} = p_{ij,min} + \sigma \times (p_{ij,max} - p_{ij,min}), \quad j = 1, 2, \ldots, M \]

where \( p_{ij} \) represents the \( j \)th gene of the \( i \)th individual, \( p_{ij,min} \) and \( p_{ij,max} \) mean the lower and upper bounds of \( p_{ij} \), respectively; and \( \sigma \) represents the uniform random number between 0 and 1. In this paper, the trial
vector $p_i$ represents the desired values of voltage magnitude at generation bus, transformer tap settings, and shunt capacitors.

- Variable scaling mutation
  The mutation operation of basic DE is performed by adding a differential vector to the parent individual as follows.

$$p_i' = p_i + f_m \times (p_i^a - p_i^b)$$ (3)

where $p_i^a$ and $p_i^b$ are the randomly selected individuals in the parent population, $(p_i^a - p_i^b)$ is a differential vector, and $f_m \in [0,1]$ represents the mutation factor.

To increase the diversity of the population, the variable scaling mutation (VSM) based on the 1/5 success rule \[9\] is used in this paper. VSM varies the mutation factor according to the frequency of successful mutations to avoid falling into local minima and save more computational time. The rule of updating mutation factor is as follows.

$$f_m(t + 1) = \begin{cases} k_d \times f_m(t), & \text{if } p_s(t) < 1/5 \\ k_i \times f_m(t), & \text{if } p_s(t) > 1/5 \\ f_m(t), & \text{if } p_s(t) = 1/5 \end{cases}$$ (4)

where $k_d$ and $k_i$ are the scaling factors and $p_s(t)$ is the frequency of successful mutations. The successful mutations defines the objective function of the best individual in the next generation as being better than the best individual in the current generation.

- Recombination
  In essence, the mutant individual in (3) is a noisy replica of $p_i$. To extend the local diversity of the mutant individuals, a recombination operation is introduced as follows.

$$p'_j = \begin{cases} p_j, & \text{if } \text{rand}_ij > R_r \\ p'_j, & \text{if } \text{rand}_ij \leq R_r \end{cases}$$ (5)

where $p_j$ is the $j$th gene of the $i$th individual before mutation, $p'_j$ represents the $j$th gene of the $i$th offspring individual following mutation, $\text{rand}_ij$ is a random number with normal distribution, and $R_r \in [0,1]$ is a recombination factor.

- Selection
  The basic DE used one-to-one competition to retain its offspring. This method gives rise to a rapid convergence rate. But it may lead to a higher probability of obtaining a local optimum point. To increase the global search capability, a probabilistic state transition rule used in the ant system is utilized to replace the selection operation in the basic DE algorithm as follows.

$$P_{ti} = \frac{[\tau_i(t)]^\gamma [F_i(t)]^\zeta}{\sum [\tau_i(t)]^\gamma [F_i(t)]^\zeta}$$ (6)

where $\tau_i(t)$ is the pheromone concentration of the $i$th ant at $t$th iteration, $F_i(t)$ is the objective value of the $i$th ant at $t$th iteration, $\gamma$ is the pheromone constant and $\zeta$ is the constants of the objective value.

In addition, the pheromone concentration is updated according to the following formula:

$$\tau_i(t + 1) = \rho \tau_i(t) + \Delta \tau_i$$ (7)
\[ \Delta \tau_i = \begin{cases} \frac{q}{d_i}, & \text{if } \text{ith ant is better so far} \\ 0, & \text{otherwise} \end{cases} \] 

Equations from (6) to (8) show if the objective value of the offspring individual is better than the other individuals, the pheromone concentration \( \tau \) is increased and it has more probability of surviving as a new individual in the next generation.

4. The Scheme of Proposed Approach for ORPD

The proposed approach for searching the ORPD can be described in the following step-by-step processes:

- Randomly generate the initial parent trial vector \( p_i \) as shown in (2).
- Evaluate the objective value of each parent individual using (1) subject to a number of constraints.
- Execute the variable scaling mutation and recombination operations according to the variable mutation factor \( f_m \) and recombination factor \( R_r \), as described in (4) and (5), respectively.
- Calculate the objective value of each offspring individual as described in (1).
- Utilize the probabilistic state transition rule described from (6) to (8) to select \( P \) sets of the better individuals in the population.
- Check if the objective value is converged or the maximum iteration is reached. If it is, stop the optimization process and output the dispatch results; otherwise, repeat the above processes.

5. Numerical Results

The proposed approach was verified on the IEEE 30-bus 6-generator system. For comparison, the evolutionary programming (EP) [10], PSO [5], and DE [7] methods implemented using the commercial MATLAB package were also tested using the same database.

In this case, the bus 1 is slack bus, buses 2, 5, 8, 11, and 13 are generation bus, and the others are load bus. The control variables are the desired values of voltage magnitude at the PV bus, transformer tap settings, and shunt capacitors. Table 1 shows the reactive generation constraints of each PV bus. The lower and upper limits of voltage magnitude at PV bus are 0.90 and 1.10 p.u., respectively, while the transformer tap settings are varied between 0.95 and 1.05 p.u.

Table 2 shows the parameter settings of different methods. The proposed method uses the variable scaling mutation method to vary the mutation factor between 0.05 and 0.35. For each method, the population size is set at 30. The maximum iteration of optimization process is set at 500. Due to the optimization processes of diverse methods are almost converged within 25 iterations through many trials with different random seeds, the numbers of iteration for the optimization process is then presented within 50 iterations in order to illustrate the convergence performance of diverse methods.

Fig. 1 shows the optimization process of different methods. As shown in this figure, after about 13 iterations, the proposed method converges toward the best one, while the DE, EP, and PSO methods require about 19, 18, and 15 iterations, respectively. Table 3 shows the results of reactive power dispatch for different methods, where the basic load flow solution is regarded as the benchmark. The results reveal that the proposed method can save more active power transmission losses than the other methods. The average execution times for DE, EP, PSO, and proposed methods through 30 random runs are 1.91, 1.49, 1.60 and 1.21 seconds, respectively.
Table 1. The reactive generation constraint for PV bus

<table>
<thead>
<tr>
<th>Bus</th>
<th>$Q_{Bi,\text{min}}$ (p.u.)</th>
<th>$Q_{Bi,\text{max}}$ (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.40</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>-0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>8</td>
<td>-0.10</td>
<td>0.40</td>
</tr>
<tr>
<td>11</td>
<td>-0.06</td>
<td>0.24</td>
</tr>
<tr>
<td>13</td>
<td>-0.06</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 2. Parameter settings for different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP</td>
<td>No. of competitors $P_m$</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Scaling factor $\beta$</td>
<td>0.9</td>
</tr>
<tr>
<td>PSO</td>
<td>Inertia weights $w$</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Acceleration factors $[\alpha_1 \alpha_2]$</td>
<td>[0.35 0.35]</td>
</tr>
<tr>
<td>DE</td>
<td>No. of differential vector</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Recombination factor $R_r$</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>Mutation factor $f_m$</td>
<td>0.1</td>
</tr>
<tr>
<td>Proposed method</td>
<td>Variable scaling mutation $f_m$</td>
<td>0.05–0.35</td>
</tr>
<tr>
<td></td>
<td>Pheromone decay parameter $\rho$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Fig. 1 Optimization processes of different methods.

Table 3. Results of ORPD for different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>$\Sigma P_G$ (p.u.)</th>
<th>$\Sigma Q_G$ (p.u.)</th>
<th>$Q_{\text{loss}}$ (p.u.)</th>
<th>$P_{\text{loss}}$ (p.u.)</th>
<th>$\Delta P_{\text{loss}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load flow solution</td>
<td>2.8868</td>
<td>1.0276</td>
<td>-0.2343</td>
<td>0.0528</td>
<td>0.000</td>
</tr>
<tr>
<td>DE</td>
<td>2.8631</td>
<td>0.9945</td>
<td>-0.2674</td>
<td>0.0291</td>
<td>44.92</td>
</tr>
<tr>
<td>EP</td>
<td>2.8666</td>
<td>0.9989</td>
<td>-0.2630</td>
<td>0.0326</td>
<td>38.29</td>
</tr>
<tr>
<td>PSO</td>
<td>2.8576</td>
<td>0.9649</td>
<td>-0.2970</td>
<td>0.0236</td>
<td>55.33</td>
</tr>
<tr>
<td>Proposed method</td>
<td>2.8512</td>
<td>0.9181</td>
<td>-0.3440</td>
<td>0.0172</td>
<td>67.44</td>
</tr>
</tbody>
</table>

Note: 1. The total active and reactive power demands are 2.834 p.u. and 1.262 p.u., respectively.
2. The term $\Delta P_{\text{loss}}$ (%) means the percentage that active power loss saves when compared to the load flow solution.
Conclusions

A novel approach combining the variation of basic DE and ant system has been proposed to solve the ORPD problem. The paper first introduces the formulation of ORPD problem. Then the proposed method for determining the ORPD problem is briefly reviewed. Owing to the efficient global search scheme, the proposed algorithm can offer higher probability of converging toward global solution than the other methods. Testing on the IEEE 30-bus system has shown that the proposed method can obtain better results with lower active power transmission losses and faster convergence performance than the basic DE, EP, and PSO methods. The benefit of lower active power transmission losses obtained in this paper will further provide better economic dispatch and secure operation in power systems.

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References