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On the Conductors of mod l Galois Representations Coming from Modular Forms

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0. INTRODUCTION

Let $\mathbb{F}$ be a finite field of characteristic l and let $\rho: \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to GL_2(\mathbb{F})$ be a continuous representation. Serre [S3] has defined for ρ a weight $k(\rho)$, a character $\varepsilon(\rho)$, and a conductor $N(\rho) = \prod_{p \neq l} p^{n_p(\rho)}$. When ρ is odd and absolutely irreducible he conjectured the existence of a newform $f \in S_{k(\rho)}(N(\rho), \varepsilon(\rho))$ whose associated l-adic representation σ_f (Eichler-Shimura, Deligne [Del]) gives ρ by reduction mod l (see [S3] for details). This conjecture is interesting even when ρ arises in this way from *some* newform g (see [S3, R] for applications to the Fermat conjecture). In this case our first observation is

0.1. PROPOSITION. *$N(\rho)$ divides the level $N(g)$ of g.*

This is an immediate consequence of known facts about l-adic and automorphic representations, and answers affirmatively a question of Serre[1] (see [S3, 3.2, Remark 5]).

We next show that $N(g)$ cannot be big and different from $N(\rho)$ except for twisting. More precisely, let Ξ' be the set of Dirichlet characters of order a power of l and of conductor dividing the product of primes $p \neq l$, $p \mid N(g)$. We will prove

0.2. THEOREM. *There exists $\chi'_0 \in \Xi'$ such that $N(g \otimes \chi'_0)$ divides $N(g \otimes \chi')$ for any $\chi' \in \Xi'$. If $p^s \| N(g \otimes \chi'_0)$ for a prime $p \neq l$ then $p^s \| N(\rho)$ unless $s = 1, 2$.*

In fact we can get more precise results: let $\pi = \otimes_p \pi_p$ be the automorphic representation of $GL_2(\mathbb{A})$ associated with g.

[1] September 87—Serre told me he is aware that this follows from Carayol's work.

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0.3. Proposition. *The local component π_p is ordinary cuspidal in the case $s=2$ of Theorem 0.2. The case $s=1$ breaks into a principal series subcase and a special series subcase.*

Remarks. 1. It is well known that π_p is special if and only if the image of inertia under the corresponding local Galois representation is infinite. Equivalently this happens if and only if there is a unique Dirichlet character χ ramified only at p such that pth Fourier coefficient of (the newform corresponding to) $f \otimes \chi$ is non-zero. In the principal series there are two such χ's and in the cupsidal case, none.

2. Our theorem says that the problem of reducing the level in the Serre conjecture at a prime $p \neq l$ is trivial, namely reduces to local considerations, except in the cases given in Theorem 0.2. In some of these remaining cases the problem is solved: Ribet [R] treats the special case, when the weight is two, the character is trivial, and under some additional technical assumptions (but he allows $p=l$). Ribet's methods are global and highly non-trivial. A key ingredient in his approach is certain combinatorial relations between Hecke modules over $\mathbb{Z}$ coming from Shimura and modular curves. Such relations were in fact first observed in joint work with Jordan [Jo-Li1] and conversations with Jordan and Ribet. To obtain them one goes through the quaternion algebra B over $\mathbb{Q}$ of discriminant $p\infty$, and they can be viewed as a refinement of the Langlands' correspondence between various twisted forms of $GL_2(\mathbb{Q})$, which only relates Hecke modules over $\mathbb{Q}$. In joint work with Jordan analogous combinatorial relations are developed for the special case, arbitrary weight, and more general quaternion algebras [Jo-Li2]. Moreover, as the image of the correspondence above consists of the holomorphic forms which are special or cuspidal at p, one may hope to deal with the cuspidal case $s=2$ above using B as well.

The organization of the paper is as follows. In Section 1 we sketch an elementary proof for the equality of the Swan conductor of an l-adic representation and its reduction. The local results we need are given in Section 2, and the (easy) globalization occupies Section 3. In Section 4 we briefly recall the dictionary between Galois and automorphic representations and deduce 0.1, 0.2, and 0.3 from results on Galois representations.

1. Local Theory: Generalities

Let p and l be distinct primes and K a finite extension of $\mathbb{Q}_p$. Put $D = \mathrm{Gal}(\bar{K}/K)$ and let $I \subset D$ be the inertia subgroup. Let E be a finite extension of $\mathbb{Q}_l$ with ring of integers $\mathcal{O}$, maximal ideal $\mathcal{M}$, and residue field $\mathbb{F}$. Let

W be a d-dimensional vector space over E and $\omega: D \to GL(W)$ a continuous representation. Fix an $\mathcal{O}$-lattice $L \subset W$ preserved by $\omega(D)$. Such an L always exists since D is compact. Let $W_0 = L/\mathcal{M}L$ and let $\rho: D \to GL(W_0)$ the corresponding reduction of ω. The conductor exponents of ω and ρ are defined in [S2, S3]:

$$n(\omega) = n_p(\omega) = \dim\ W/W^{\omega(I)} + \mathrm{sw}(\omega^{ss})$$

$$n(\rho) = n_p(\rho) = \dim\ W_0/W_0^{\rho(I)} + \mathrm{sw}(\rho).$$

Here $\omega^{ss}: D \to GL(\mathrm{Gr}\ W)$ is the semi-simplification of ω with respect to the filtration defined by Grothendieck's nilpotent endomorphism [S2, S-T]. Put $\Omega = \omega^{ss}(I)$. As Ω is finite the Swan conductor is defined as

$$\mathrm{sw}(\omega^{ss}) = \sum_{k=1}^{\infty} \frac{\dim(\mathrm{Gr}\ W/(\mathrm{Gr}\ W)^{\Omega_k})}{[\Omega : \Omega_k]},$$

where Gr W is the graded space associated to the filtration, and Ω_k is the kth inertia subgroup of Ω. Likewise,

$$\mathrm{sw}(\rho) = \sum_{k=1}^{\infty} \frac{\dim(W_0/W_0^{R_k})}{[R : R_k]}$$

with $R = \rho(I)$ and R_k its higher inertia subgroups.

1.1. PROPOSITION. $\mathrm{sw}(\rho) = \mathrm{sw}(w^{ss})$.

For convenience, we sketch the proof of this well-known fact (see, e.g., [S2, 2.1]). We will need the equally well-known

1.2. LEMMA. *The Swan conductor is invariant under tame extensions.*

More precisely, let E/K be a finite Galois extension of local fields with residue characteristic p and E' an intermediate extension, Galois over K with E/E' tame. Let V be a finite dimensional vector space and ρ': $\mathrm{Gal}(E'/K) \to GL(V)$ *a representation. Then ρ' defines a representation ρ:* $\mathrm{Gal}(E/K) \to GL(V)$ *trivial on* $\mathrm{Gal}(E/E')$, *and we claim that* $\mathrm{sw}(\rho) = \mathrm{sw}(\rho')$.

Proof. For $i \geq -1$ let $G_i = \mathrm{Gal}(E/K)_i$, $G_i' = \mathrm{Gal}(E'/K)_i$, and $G_i'' = \mathrm{Gal}(E/E')_i$ be the higher inertia groups. As E/E' is tamely ramified, the ramification index

$$e' = e(K/E) = |G_0''| = |\mathrm{Ker}(G_0 \to G_0')|$$

is prime to p, so that $G_1 \simeq G_1'$. For $s \in G_{-1}$ and $\sigma \in G_{-1}'$ its restriction to E' put

$$i(s) = 1 + \sup\{k \mid s \in G_k\},$$

$$i'(\sigma) = 1 + \sup\{k \mid \sigma \in G_k'\}.$$

By [S1, IV 1, Proposition 3],

$$i'(\sigma) = \frac{1}{e'} \sum_{\alpha \in G_{-1}''} i(s\alpha) = \frac{1}{e'} \sum_{\alpha \in G_0''} i(s\alpha) = \frac{1}{e'} (i(s) + e' - 1),$$

since in the sum above $s\alpha \in G_0$ iff $\alpha \in G_0''$ and $s\alpha \in G_1$ iff $\alpha = 1$. Equivalently $G_{e'i+j} \simeq G_{i+1}'$ for $i \geqslant 0$, $1 \leqslant j \leqslant e'$. Thus,

$$\operatorname{sw}(\rho) = \sum_{i=0}^{\infty} \sum_{j=1}^{e'} \frac{\dim(V/V^{\rho(G_{e'i+j})})}{[G_0 : G_{e'i+j}]} = \sum_{i=0}^{\infty} \frac{e'}{e'} \frac{\dim(V/V^{\rho(G_{i+1}')})}{[G_0' : G_{i+1}']} = \operatorname{sw}(\rho').$$

Proof of 1.1. Grothendieck's filtration induces on L an ω-invariant filtration with $\mathcal{O}$-free quotients $(\operatorname{Gr} L)_i$ and by reduction we get a representation $\rho^{ss} : D \to GL(\operatorname{Gr} W_0)$. Since ρ^{ss} is the semi-simplification of ρ with respect to the reduction of Grothendieck's nilpotent homomorphism, $\rho^{ss}(I)$ is a quotient of R. As the kernel $R \to \rho^{ss}(I)$ is upper triangular with respect to the filtration it has an l-power order, so that the extension $\bar{K}^R/\bar{K}^{\rho^{ss}(I)}$ is tame. By the lemma $\operatorname{sw}(\rho_1) = \operatorname{sw}(\rho^{ss})$, where ρ_1 is ρ^{ss} composed with the projection $\rho(D) \to \rho^{ss}(D)$.

Likewise, $\rho^{ss}(I)$ is a quotient (by reduction) of Ω, and the kernel of this quotient map is a finite subgroup of the pro-l group $\operatorname{Ker}(GL(\operatorname{Gr} L) \to GL(\operatorname{Gr} W_0))$. It is therefore tame, and we can use the lemma to get $\operatorname{sw}(\rho^{ss}) = \operatorname{sw}(\tilde{\rho})$, where ρ_2 is ρ^{ss} composed with the projection $\omega^{ss}(D) \to \rho^{ss}(D)$.

Now let P be the wild inertia subgroup of D, namely the pro-p Sylow subgroup of I. As $p \neq l$ it follows that $\omega(P), \ldots, \rho_2(P)$ are all isomorphic, and that the spaces of invariants of corresponding subgroups of these groups have equal dimensions. We get that corresponding terms in the sums defining $\operatorname{sw}(\rho_2)$ and $\operatorname{sw}(\omega^{ss})$ are equal. Hence $\operatorname{sw}(\rho_2) = \operatorname{sw}(\omega^{ss})$. Likewise $\operatorname{sw}(\rho_1) = \operatorname{sw}(\rho)$. The proposition follows.

1.3. *Remark.* One can give a slightly different proof of 1.1 and 1.2 using the upper numbering of the inertia groups. One then has to show that $\operatorname{sw}(\omega)$ is invariant under semi-simplification.

2. The Local Theory in a Special Case

Keeping the notation of Section 1 we now suppose that $K=\mathbb{Q}_p$ and $d=\dim V=2$. We may suppose $W=E^2$, $L=\mathcal{O}^2$. Let $P\subset I$ be the wild inertia subgroup. We can and will identify $I/P\cong\prod_{q\neq p}\mathbb{Z}_q(1)$ and denote by Q the subgroup of I containing P such that $Q/P\cong\prod_{q\neq p,l}\mathbb{Z}_q(1)$. Let $\mu_p\subset\bar{\mathbb{Q}}_p$ be the group of pth roots of 1.

2.1. Proposition. $n(\omega)\geqslant n(\rho)$.

Proof. Clearly $\dim W^{\omega(I)}\leqslant\dim W_0^{\rho(I)}$. Now use the definitions of $n(\rho)$, $n(\omega)$, and Proposition 1.1.

2.2. Proposition. *If $\rho(Q)\neq\{1\}$ there exists a character $\chi\colon D\to \mathrm{Gal}(\mathbb{Q}_p(\mu_p)/\mathbb{Q}_p)\to\mathcal{O}^\times$ with trivial reduction such that*

$$n(\omega)\geqslant n(\omega\otimes\chi)=n(\rho\otimes\chi)=n(\rho).$$

Proof. As before $\rho(Q)\cong\omega(Q)$, since $\mathrm{Ker}(GL_2(\mathcal{O})\to GL_2(\mathbb{F}))$ is a pro-l group, and the spaces of invariants of corresponding subgroups have equal dimensions. Moreover $\mathrm{sw}(w^{ss})=\mathrm{sw}(\rho)$ by 1.1. If $W_0^{\rho(Q)}=0$ then $n(\rho)=2+\mathrm{sw}(\rho)=n(w)$ and we take $\chi=1$. If not, necessarily $W^{\rho(Q)}$ is one dimensional ($W^{\rho(Q)}=W\Rightarrow\rho(Q)=1$). As Q is normal in D and $\rho(Q)$ has order prime to l we may write in terms of a basis for L

$$\omega=\begin{pmatrix}\chi_1 & 0\\ 0 & \chi_2\end{pmatrix},$$

where $\chi_1,\chi_2\colon D\to\mathcal{O}^\times$ are suitable characters with $\chi_1(Q)=\{1\}$ and $\chi_2(Q)\neq\{1\}$. If $n(\omega)\neq n(\rho)$ then $\chi_1(I)\neq\{1\}$, but its reduction $\bar{\chi}_1\colon D\to\mathbb{F}^\times$ is unramified. Hence $\chi_1=\chi_u\chi_r$ where χ_u is unramified whereas χ_r factors through $\mathrm{Gal}(\mathbb{Q}_p(\mu_p)/\mathbb{Q}_p)$ and has trivial reduction. Take $\chi=\chi_r^{-1}$. Then $\dim W^{(\omega\otimes\chi)(I)}=1=\dim W_0^{\rho(I)}$, so $n(\omega)\geqslant n(\omega\otimes\chi)=n(\rho)$.

From now until the end of the section we suppose that $\rho(Q)=\{1\}$. Let K be the quadratic unramified extension of $\mathbb{Q}_p$, and put $\mathrm{Gal}(K/\mathbb{Q}_p)=\{1,\sigma\}$. Let Ξ_p be the set of characters $\chi\colon D\to\mathcal{O}^\times$ which factor through $\mathrm{Gal}(\mathbb{Q}_p(\mu_p)/\mathbb{Q}_p)$ and have trivial reduction.

2.3. Proposition. *With the above notation assume*

a. $\rho(Q)=\{1\}$,

b. $n(\omega)\leqslant n(\omega\otimes\chi)$ *for any* $\chi\in\Xi_p$,

c. ω *is ramified.*

Then exactly one of the following cases occurs:

1. $p \equiv 1 \pmod{l}$ *and in a suitable basis for* W, $\omega = \begin{pmatrix} \chi_1 & 0 \\ 0 & \chi_2 \end{pmatrix}$, *where* $\chi_1, \chi_2 : D \to \mathcal{O}^\times$ *are characters,* χ_1 *is unramified and* $\chi_2 | I$ *has a finite, l-power order. In this case* $n(\omega) = 1$ *and* $n(\rho) = 0$.

2. *In a suitable basis of* W, $\omega = \begin{pmatrix} \chi_1 & \alpha \\ 0 & \chi_2 \end{pmatrix}$, *where* $\alpha | I \neq 0$ *and* χ_1, χ_2 *are unramified characters such that* $\chi_1 \chi_2^{-1}$ *maps a Frobenius element to p. Then* $n(\omega) = 1$, $n(\rho) = 0$.

3. $p \equiv -1 \pmod{l}$. *There exists a tamely ramified character* χ *of* $\mathrm{Gal}(\bar{\mathbb{Q}}_p/K)$ *whose restriction to inertia has a finite, l-power order. Here* $\omega \cong \mathrm{ind}(\mathrm{Gal}(\bar{\mathbb{Q}}_p/K), D, \chi)$ *and* $n(w) = 2 > n(\rho)$.

Under the Langlands correspondence (see Section 4) *Cases* 1, 2 *and* 3 *correspond to the principal, special, and cuspidal cases, respectively.*

2.4. Example (for Case 3). Let ω be the 3-adic representation of $G = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ associated with the elliptic curve 75A in the Antwerp tables [A IV]. Let ω_5 be the restriction of ω to a decomposition group $D \simeq \mathrm{Gal}(\bar{\mathbb{Q}}_5/\mathbb{Q}_5)$ for 5 in G. Let $\rho : G \to GL_2(\mathbb{F}_3)$ and $\rho_5 : D \to GL_2(\mathbb{F}_3)$ be the reductions of ω and ω_5, respectively. Then ρ_5 is at most tamely ramified, since $|GL_2(\mathbb{F}_3)| = 48$ is prime to 5; if ρ_5 were unramified, $E[3]$ and hence E would have good reduction at 5, since E has potential good reduction (see [S-T, Corollary 2]). Hence ρ_5 is actually ramified. The Kodaira type of the special fiber $E \times \bar{\mathbb{F}}_5$ is IV, so that the Néron model at 5 is $\mathbb{G}_a \times \mathbb{Z}/3\mathbb{Z}$. Hence $E \times \bar{\mathbb{F}}_5[3]^I \cong \mathbb{Z}/3\mathbb{Z}$, where I is the inertia subgroup of D. It follows that $\rho(I) = \{\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}\}$ in a suitable basis, and we see that $n(\rho_5) = 1 + \mathrm{sw}(\rho_5) = 1 < 2 = n(w_5)$.

2.5. *Remark.* Note that $|E(\mathbb{F}_2)| = 1$, so that the characteristic polynomial of Frob_2 is $t^2 - 2t + 2$. Hence $\rho(\mathrm{Frob}_2)$ had order 8 and determinant -1. The image $\rho(G)$ must therefore be all of $GL_2(\mathbb{F}_3)$; in particular it is geometrically irreducible; Serre's conjecture indicates that ρ comes from a modular form in $S_2(\Gamma_0(15))$, and this is indeed the case—the representation $\omega' : G \to GL_2(\mathbb{Q}_3)$ given by the Tate module of $X_0(15)$ has the same reduction ρ. Note however that $\omega(I)$ is finite, whereas $\omega'(I)$ is not!

Proof of 2.3. Let $\tilde{u} \in I$ be a topological generator for $I/\mathbb{Q} \cong \mathbb{Z}_l(1)$ and $\tilde{F} \in D$ a Frobenius element. Put $u = \omega(\tilde{u})$, $F = \omega(\tilde{F})$. Then $FuF^{-1} = u^p$. The set $\{\lambda_1, \lambda_2\}$ of the eigenvalues of u is therefore preserved under $\lambda \mapsto \lambda^p$. If $\lambda_1 = \lambda_2$ then $\lambda_1 = 1$. Otherwise we could twist by an appropriate $\chi \in \Xi_p$ to decrease $n(\omega)$, so that by b and c, $\lambda_1 = \lambda_2 = 1$, u is unipotent $\neq 1$, and we find Case 2. If $\lambda_1 \neq \lambda_2$ but $\lambda_1^p = \lambda_1$ (so that $\lambda_2^p = \lambda_2$) we can twist again by a suitable $\chi \in \Xi_p$ to decrease $n(\omega)$ unless λ_1 or λ_2 are 1. We may assume $\lambda_1 = 1$. Then $\lambda_2^p = \lambda_2 \neq 1$ and λ_2 has an l power order, so $l | (p-1)$. This is Case 1. If $\lambda_1^p = \lambda_2 \neq \lambda_1$ we get Case 3 similarly.

3. 2-Dimensional Representations of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$

Let l, E, and $\mathcal{O}$ be as before and let W be a 2-dimensional space over E. Consider a continuous representation

$$\omega\colon G = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to GL(W).$$

Let $L \subset W$ be an $\mathcal{O}$-lattice preserved by $\omega(G)$ and let $\rho\colon G \to GL(W_0)$ be the corresponding reduction. Let M be the product of the primes dividing $N(\omega)$, μ_M the group of Mth roots of 1 in $\bar{\mathbb{Q}}$, and put

$$\Xi = \{\chi\colon G \to \mathcal{O}^\times \mid \chi \text{ factors through } \mathrm{Gal}(\mathbb{Q}(\mu_M)/\mathbb{Q}) \text{ and has trivial reduction}\}.$$

3.1. Proposition. *There exists a character $\chi_0 \in \Xi$ such that $N(\omega \otimes \chi_0)$ divides $N(\omega \otimes \chi)$ for any $\chi \in \Xi$. We have $N(\rho) \mid N(\omega \otimes \chi_0)$. If $n_p(\omega \otimes \chi_0) \neq n_p(\rho)$ for such a χ_0 then the restriction $\omega_p = \omega\,|_{D_p}$ of ω to a decomposition group $D_p \cong \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ is one of Cases* 1, 2, *or* 3 *of Proposition* 2.3.

Proof. This is an evident globalization of 2.3. The only point concerning the twist by χ_0 is to observe that $\Xi \cong \bigoplus_{p \mid M} \Xi_p$ canonically, with Ξ_p as in Section 2.

4. Proof of 0.1, 0.2, and 0.3

Let $g \in S_k(N, \varepsilon)$ be a newform. Jacquet and Langlands associated to g an automorphic representation $\pi_g = \bigotimes_p \pi_{g,p}$, where $\pi_{g,p}$ is an admissible, irreducible representation of $GL_2(\mathbb{Q}_p)$. They defined, via equality of L and ε factors, a correspondence between such local representations $\pi_{g,p}$ and F-semisimple representations of the Weil-Deligne group $WD(\mathbb{Q}_p)$ of $\mathbb{Q}_p$, and started a proof, finished by Tunnell, that this correspondence is bijective [De2, Ja-La, Tu]. Moreover, representations of $WD(\mathbb{Q}_p)$ are in bijection with l-adic representations of $\mathrm{Gal}(\mathbb{Q}_p/\mathbb{Q}_p)$ for any $l \neq p$. On the other hand we also have the l-adic representation σ_g of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ associated to g by Deligne. Carayol proved [Car], by completing the previous work of Deligne, that the F-semisimplification $\sigma_{g,p}$ of the restriction of σ_g to a decomposition group $D_p \cong \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ corresponds as above $(l \neq p)$ to $\pi_{g,p}$.

The resulting correspondence between irreducible admissible representations of $GL_2(\mathbb{Q}_p)$ and F-semisimple l-adic 2-dimensional representations of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ indeed relates to Cases 1, 2, and 3 of Proposition 2.3 principal, special, and cusidal representations, respectively (loc. cit.). Moreover the sets Ξ of Section 4 and Ξ' of Section 0 correspond under class field theory. As this correspondence is compatible with twists corresponding under class field theory, Theorem 0.2 and Propositions 0.1 and 0.3 follow immediately from Proposition 3.1 and the following

4.1. LEMMA. *Let* $N=\prod_p p^{n_p(g)}$ *be the factorization of the level of* g. *Then for* $p \neq l$, $n_p(g)=n_p(\sigma_{g,p})$.

Proof. This is well known. It follows from the theory of the new vector that $n_p(g)$ is the conductor exponent of $\pi_{g,p}$ (see, e.g., [Cas] or [De2]). On the other hand the formalism of the Weil-Deligne group [De3, Ta] shows that the ε-factor of $\sigma_{g,p}$ has the form $\varepsilon(\sigma_{g,p}, s) = (\text{const} \neq 0) \cdot p^{-sn_p(\sigma_{g,p})}$. By the defining property of the Langlands correspondence, we are reduced to a problem in the representation theory of $GL_2(\mathbb{Q}_p)$: to show that the exponent of the conductor in the sense of the new vector of $\pi_{g,p}$ is the exponent occurring in the ε factor $\varepsilon(\pi_{g,p}, s)$. This equality is frequently described as well known, but the author was initially unable to find a proof in the literature. However, a very elegant proof of a more general result is given in [J-PS-S].

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