Numerical simulation of thermal stratification in an elbow branch pipe of a tee junction with and without leakage

T. Lu, H.T. Li, X.G. Zhu

School of Mechanical and Electrical Engineering, Beijing University of Chemical Technology, Beijing 100029, China
China Nuclear Power Engineering Co., Ltd., Beijing 100840, China

Abstract
Thermal stratification can cause thermal fatigue of the piping system of a nuclear power plant. One of the regions most at risk of suffering from thermal fatigue is a small elbow pipe branching off from the main pipe of the coolant loop for the drain or letdown system in the chemical and volume control system (CVCS). This work focuses on a fundamental description of the thermal stratification caused by turbulent penetration and buoyancy effects in the elbow branch pipe using large-eddy simulations (LESs). The LES results for the normalized temperature, mean temperature, and root-mean square (RMS) temperature were found to be in good agreement with the available experimental data which confirms that LES can predict the thermal stratification in a closed elbow branch pipe where cold fluids are stagnant. Subsequently, the flow and heat transfer were numerically predicted using LES when leakage occurred in the elbow branch pipe. The numerical results show that the thermal stratification region is pushed towards the horizontal part and may remain there for a long time for a leakage ratio of 1%. However, thermal stratification is quickly eliminated for a leakage ratio of 5%, although there is a higher power spectrum density (PSD) of the temperature in the early stages of the leakage. It may be concluded that a small leakage ratio can result in the elbow branch pipe being at high risk of thermal fatigue caused by thermal stratification.

1. Introduction
Thermal stratification or striping is of significant importance in nuclear power plants because it can lead to transient temperature fluctuations at the adjacent pipe walls which may result in thermal fatigue and failure of the pipe. In a previous review, Iida (1992) noted that serious fatigue failures have occasionally been experienced in piping systems, pumps, and valves. The causes of fatigue failures can be divided into two categories: mechanical-vibration-induced fatigue and thermal-fluctuation-induced fatigue. By comparison of the stresses due to thermal shock and thermal stratification, Miksch et al. (1985) concluded that the cracks observed were in essence caused by thermal stratification. Tenchine (2010) suggested that piping thermal stratification or mixing is one of several thermal hydraulic challenges which progressively increase with the power and the size of a sodium cooled nuclear reactor. Thermal fatigue incidents occurring in a tee piping system are susceptible to turbulent temperature mixing effects. A European study of the thermal fatigue evaluation of piping systems has been initiated, including assessment of the fatigue significance of turbulent thermal mixing effects in piping systems and identification of the significant fatigue parameters (Metzner and Wilke, 2005).

Thermal stratification or striping can be predicted using computational fluid dynamics (CFD) simulations as well as measured experimentally. Turbulent isothermal and thermal mixing phenomena have been numerically and experimentally investigated by Frank et al. (2010). The isothermal case involved turbulent mixing of two water streams at the same temperature in a tee junction in the horizontal plane to exclude any buoyancy effects, while the thermal case had a temperature difference of $15\,^\circ C$ between the hot and cold water in a tee junction in the vertical plane in order to induce thermal stripe phenomena. Experimental temperature and velocity measurements and numerical simulations were carried out by Kamide et al. (2009) in order to study the thermal hydraulic aspects of thermal striping in a mixing tee. Depending on the momentum ratio of the main pipe to the branch pipe ($M_b$), flow patterns in the tee were classified into three groups: wall jet ($M_b > 1.35$), deflecting jet ($0.35 < M_b < 1.35$), and impinging jet ($M_b < 0.35$). Tests to investigate the interaction between the main coolant piping and the stagnant attached lines by turbulence.
penetration have been carried by Kim et al. (1993), and a loading
definition for thermal striping was proposed.

Large-eddy simulations (LESs) are a very popular way of predicting flow details such as velocity, temperature, and vortex. Both collision and co-current thermal striping in a mixing tee have been numerically simulated by Hu and Kazimi (2006) using LES solved by the commercial CFD code FLUENT. Numerical results of normalized mean and fluctuation temperatures were compared with experimental measurements. The mixing in tee junctions made of different materials with different pipe wall thicknesses were investigated by Kuhn et al. (2010) using different LES subgrid scale (SGS) models in order to identify the influence of the SGS on the simulation results, and the results were also compared with available experimental data. Their study showed the ability of LES to accurately predict thermal fluctuations in turbulent mixing. Temperature fluctuations in a mixing tee were simulated by Lu et al. (2010) in FLUENT using the LES with the Smagorinsky–Lilly (SL) SGS model with consideration of buoyancy effects. Their numerical results were in good agreement with the available experimental data, showing the validity of the LES model for predicting the mixing of hot and cold fluids in a mixing tee junction. The temperature fluctuations and structural response of coolant piping at a mixing tee were investigated using LES by Lee et al. (2009). Their study showed that the temperature difference between the hot and cold fluids in a tee junction and the enhanced heat transfer coefficient due to turbulent mixing are the dominant factors affecting the thermal fatigue failure of a tee junction.

In a nuclear power plant, several small pipes branch off from the main pipe of the coolant loop, with a temperature of 300 °C and a velocity of 10 m/s in the drain or letdown system in the chemical and volume control system (CVCS). These pipes are often bent and connected to a closed valve (Ourmaya et al., 2006). The turbulence of the main pipe flow usually initiates a perturbation flow in the elbow branch pipe and hot water penetrates into it. In addition, the fluid in the elbow is relatively stagnant and at a low temperature. Thermal stratification may arise if the perturbation momentum and buoyancy remain in dynamic balance in a region where the hot fluid is above the cold fluid.

This work focuses on a fundamental description of the thermal stratification caused by turbulent penetration and buoyancy effects using large-eddy simulations (LES), for the two cases where leakage of the elbow branch pipe does and does not occur. Firstly, based on the experimental model and flow parameters in the literature (Ourmaya et al., 2006), the LES for the case without leakage of the closed elbow branch pipe was completed. Then the numerical results of the transient temperature were compared with the experimental data to confirm that LES has the capability to predict the temperature fluctuations. After validation of the numerical simulations, LES for other cases with leakage of the elbow branch pipe were performed in order to predict extent of penetration of hot water into the horizontal part of the elbow branch pipe.

2. Mathematical model and numerical simulations

As shown for the model in Fig. 1 (Ourmaya et al., 2006), at the beginning hot water at temperature of 65 °C and velocity of 6 m/s flows in the top main rectangular channel, with cross-section of 60 mm x 10 mm, which is connected to an elbow branch pipe initially filled with stagnant cold water at temperature of 15 °C. The bore of the elbow pipe is 43 mm, which is similar to that of the 2 in branch pipe in a nuclear power plant. The details of geometry and flow parameters can be found in reference (Ourmaya et al., 2006). As a consequence of turbulence at the tee junction, mixing between hot and cold water occurs and may propagate downwards in the vertical – and even in the horizontal – part of the elbow branch pipe. Consequently, thermal stratification occurs with the fluctuating interface between hot and cold water, which results in the elbow branch pipe being at risk of thermal fatigue.

For incompressible flow, the filtered Navier–Stokes and energy equations can be written as (Ndombo and Howard, 2011)

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho \mathbf{u})}{\partial x_j} = 0 \]  

\[ \frac{\partial \rho \mathbf{u}_i}{\partial t} + \frac{\partial (\rho \mathbf{u}_i \mathbf{u}_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} - \rho \beta (T - T_0) g_j \]

\[ + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \mathbf{u}_i}{\partial x_j} + \frac{\partial \mathbf{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \mathbf{u}}{\partial x_k} \delta_{ij} \right) \right] \]  

\[ \frac{\partial \rho \mathbf{T}_i}{\partial t} + \frac{\partial (\rho \mathbf{T}_i \mathbf{u}_j)}{\partial x_j} = \frac{\partial}{\partial x_k} \left( \lambda \frac{\partial \mathbf{T}}{\partial x_k} - \rho \mathbf{u}_i \mathbf{u}_j \right) \]

Here \( \mathbf{\bar{v}} \) represents the implicit filtering operation on the flow variable, to the grid and \( \mathbf{\bar{v}}^{(0)} \) is the subgrid part of the flow variable, and \( \rho, \beta, \mu, \mu_t, \lambda, \) and \( c_p \) represent the density, thermal expansion coefficient, molecular viscosity, turbulent viscosity, thermal conductivity, and specific heat capacity, respectively. The effect of the unresolved scale on the resolved scale in the above equations is represented by the subgrid scale (SGS) stress, which is modeled using the eddy viscosity hypothesis. In this work, the Smagorinsky–Lilly model is used for the turbulent viscosity, and is given by

\[ \mu_t = \rho L_s^2 |\mathbf{S}|^2 \]

where \( L_s \) is the mixing length for the subgrid scales, \( L_s \) and \( |\mathbf{S}| \) are computed using

\[ L_s = \min(kd, C_f V^{1/3}) \]

\[ |\mathbf{S}| = \sqrt{2 \mathbf{S}_n \mathbf{S}_n} \]

\[ \mathbf{S}_n = \frac{1}{2} \left( \frac{\partial \mathbf{u}_i}{\partial x_j} + \frac{\partial \mathbf{u}_j}{\partial x_i} \right) \]

where \( k \) is the von Karman constant of 0.42, \( d \) is the distance to the closest wall, \( C_f \) is the Smagorinsky constant, and \( V \) is the volume of the computational cell.
Fully-developed velocity profiles in both the main rectangular channel and the elbow branch pipe calculated using the steady Reynolds stress model (RSM) were used as the inlet velocity conditions. The steady fields of the RSM were set as the initial condition of the rectangular channel for the LES calculations. A stagnant velocity field and temperature of 15 °C were initially set for the elbow branch pipe at the beginning of the LES.

The LES simulations were performed with different meshes and finally the case with about 1.8 million cells was selected as the grid independent model. A constant time step of 5 ms was used for the simulations. The total simulation time for the mixing process was 300 s. Taking a leakage ratio of either 1% or 5% of the main channel flow rate, LES was conducted for a leakage beginning at the 300th s and lasting 50 s. To correspond to the experimental measurements, the temperatures close to the wall (1 mm) at angles from 43° to 65° in the plane of \( y = 0 \) mm were recorded at time intervals of 5 ms.

### 3. Results and discussion

The normalized mean temperature and normalized temperature fluctuation intensity are used to describe the time-averaged temperature and time-averaged temperature fluctuation intensity. The normalized temperature is defined as

\[
T' = \frac{T - T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}
\]

where \( T \) is the instantaneous temperature, \( T_{\text{cold}} \) is the temperature of the cold fluid, and \( T_{\text{hot}} \) is the temperature of the hot fluid. The normalized mean temperature is

\[
\overline{T'} = \frac{1}{N} \sum_{n=1}^{N} T'
\]

where \( N \) is the total number of sampling times.

The normalized temperature fluctuation intensity is given by the root-mean-square (RMS) of the temperature fluctuations.
\[ T_{RMV} = \sqrt{\frac{1}{N} \sum_{n=1}^{N}(T_i - \bar{T})^2} \]  

(10)

3.1. Validation of numerical results for the case without leakage

Fig. 2 shows a comparison between numerical and experimental results for the normalized temperature (Ourmaya et al., 2006) at different angles in the plane of \( y = 0 \) mm when the elbow branch pipe is stagnant. Although the numerical results are not always consistent in time with the experimental data, most of the temperature fluctuation of the numerical results is very close to that of the experimental data.

The normalized mean temperatures and normalized RMS temperatures at different angles are shown in Fig. 3. The normalized mean temperature decreases with increasing angle, which

![Velocity vector contoured by temperature at different leakage times in the plane of y = 0 mm](image)

Fig. 5. Velocity vector contoured by temperature at different leakage times in the plane of \( y = 0 \) mm: (a–c) \( l = 1\% \); (a’–c’) \( l = 5\% \).
means the average temperature in the top of the bent part is higher than that in the bottom. However, the normalized RMS temperature at an angle of 47° is the largest among the three points, which means that the point at an angle of 47° has the largest temperature fluctuation intensity. For the normalized mean temperature, the absolute error of the numerical result is in the range of ∼5% of the experimental data. For the normalized RMS temperature, the absolute error of the numerical result is in the range of ∼10% of the experimental data except for the point at an angle of 51° where the absolute error is ∼33%. To large extent, the numerical results for both the normalized mean and RMS temperatures are in good agreement with the experimental data, which validates LES as a means of predicting the temperature fluctuation in an elbow branch pipe due to turbulent penetration and buoyancy effects. Since the LES were successfully validated, the flow and heat transfer were numerically simulated for the case of leakage from the elbow branch pipe.

3.2. With leakage from the branch pipe

The horizontal part of the branch elbow pipe connects a valve in a nuclear power plant, leakage might occur in the valve. When leakage occurred, the leakage ratio of the elbow branch pipe to the main pipe is defined by

$$l = \frac{Q_{\text{branch delve}}}{Q_{\text{main inlet}}}$$ (11)

where $Q_{\text{branch delve}}$ is the outlet flow rate of the elbow branch pipe and $Q_{\text{main inlet}}$ is the inlet flow rate of the main pipe.

3.2.1. Velocity vectors and temperature contours

Fig. 4 shows the velocity vector contoured by the temperature at the 300th s for the case without leakage. At the 300th s, hot water penetrates into the vertical part of the elbow branch pipe due to the turbulence and can even reach the bent part. In the bent part thermal stratification occurs as a consequence of the dynamic interface of the hot fluid over the cold fluid because the turbulence momentum decreases with the penetration depth and it remains in balance with the buoyancy. If the thermal stratification phenomenon continues, the bent part is at risk of thermal fatigue. Although there is zero flow rate at the end of the elbow branch pipe, there is some circulation in the horizontal part of the elbow branch pipe due to the turbulence penetrating from the main pipe. If leakage from the elbow branch pipe occurs, the thermal stratification interface will shift down the horizontal part. In our study of elbow branch pipe leakage ratios of $l = 1\%$ or $5\%$ beginning at the 300th s, we take the velocity and temperature as shown in Fig. 4 as the initial conditions.

Fig. 5 shows the velocity vector contoured by temperature with different leakage ratios of the elbow branch pipe at different leakage times. In the initial stage of the leakage, as shown in Fig. 5a and a′, the thermal stratification interface goes ahead from the bent part to the horizontal part. The advancing speed for the case of $l = 5\%$ is faster than that for $l = 1\%$. With the progress of the leakage, as shown in Fig. 5b and b′, there is a slight difference in the distributions of the thermal stratification interface for the two different leakage ratios. The thermal stratification interface of the case with a leakage ratio of $l = 1\%$ looks more horizontal than that for $l = 5\%$. The mixing of cold and hot fluids of the case of a leakage ratio of $l = 5\%$ is stronger than that for $l = 1\%$ because in the former case the fluid produces a stronger secondary flow when passing the bent part. The larger leakage ratio has a bigger turbulence momentum which overcomes the buoyancy. At the 50th s, as shown in Fig. 5c, the temperature distributions are obviously different. For a leakage ratio of $l = 5\%$, the fast fluid is distributed in the bottom of the horizontal part while the hot fluid occupies the top due to the buoyancy as shown in Fig. 5c. However, for a leakage ratio of $l = 1\%$, the situation is the opposite of that for $l = 1\%$. Although the cold fluids occupy the top of the horizontal part, the temperature difference between the top and the bottom is very small due to the stronger turbulence for a leakage ratio of $l = 5\%$, as shown in Fig. 5c′.

3.2.2. Normalized temperature changes

Fig. 6 shows the normalized temperature changes with different leakage ratios in the bent part of the elbow. For the case with smaller leakage ratio of $l = 1\%$ as shown in Fig. 6a, the normalized temperatures at angles of $0°–55°$ increase to almost unity, which is the normalized temperature of the hot fluid in the main pipe in the first 10 s of leakage, while the normalized temperature at an angle of $60°$ increases unevenly to 1 at around the 47th s. Moreover, the normalized temperature at an angle of $65°$ almost retains the temperature of the cold fluid in the first 45 s and then increases slowly. For a larger leakage ratio of $l = 1\%$, as shown in Fig. 6a′, the normalized temperatures in the bent part quickly and smoothly increase to 1 in the first 5 s and the bent part becomes full of the hot fluid from the main pipe. A comparison of the results for the two different leakage ratios suggests that lower leakage ratios more readily give rise to thermal stratification.

Fig. 7 shows the normalized temperature changes at different locations of the horizontal part for different leakage ratios. The situation for the horizontal part is similar to that for the bent part.
For the case with a smaller leakage ratio of $l = 1\%$, as shown in Fig. 7a and b, the normalized temperature in the top of the horizontal part increases more rapidly than in the bottom. Moreover, the normalized temperature of the point of $z = -0.45$ is equal to the temperature of the cold water in the first 50 s of leakage due to the buoyancy. As shown in Fig. 7a’ and b’, the normalized temperature in the case of the larger leakage ratio of $l = 5\%$ increases to almost unity in the first 16 s of leakage, which means that at this leakage ratio the horizontal part is full of hot water from the main pipe after leakage for 16 s and the original cold water in the elbow branch pipe has been lost. Comparison of the different leakage ratios indicates that thermal stratification occurs more readily, and lasts longer, for smaller the leakage ratios.

### 3.2.3. The power spectrum density of the temperature fluctuation

Fig. 8 shows a comparison of the normalized temperature changes with leakage time in the horizontal part of the elbow branch pipe in the plane of $y = 0$ mm: (a) and (b) $l = 1\%$; (a’) and (b’) $l = 5\%$.

![Fig. 7. Normalized temperature changes with leakage time in the horizontal part of the elbow branch pipe in the plane of $y = 0$ mm: (a) and (b) $l = 1\%$; (a’) and (b’) $l = 5\%$.](image)

For the case with a smaller leakage ratio of $l = 1\%$, as shown in Fig. 7a and b, the normalized temperature in the top of the horizontal part increases more rapidly than in the bottom. Moreover, the normalized temperature of the point of $z = -0.45$ is equal to the temperature of the cold water in the first 50 s of leakage due to the buoyancy. As shown in Fig. 7a’ and b’, the normalized temperature in the case of the larger leakage ratio of $l = 5\%$ increases to almost unity in the first 16 s of leakage, which means that at this leakage ratio the horizontal part is full of hot water from the main pipe after leakage for 16 s and the original cold water in the elbow branch pipe has been lost. Comparison of the different leakage ratios indicates that thermal stratification occurs more readily, and lasts longer, for smaller the leakage ratios.

### 3.2.3. The power spectrum density of the temperature fluctuation

Fig. 8 shows a comparison of the normalized temperature changes for different leakage ratios at the different locations with the strongest temperature fluctuation intensity. The normalized temperature for the larger leakage ratio of $l = 5\%$ increases more rapidly than for the smaller one. The normalized temperature for the smaller leakage ratio of $l = 1\%$ fluctuates during the leakage. Fig. 9 shows a comparison of the power spectrum density (PSD) of the temperature with the strongest fluctuation intensity for different leakage ratios. In the first 5 s of the leakage, increasing leakage ratios result in an increase in the PSD, but after 5 s, smaller leakage ratios lead to higher values of PSD. Although larger leakage ratios can induce a higher PSD, the period of thermal stratification is shorter. It can be concluded that with decreasing leakage ratios,
the elbow branch pipe suffers from increasing risk of thermal fatigue caused by thermal stratification.

4. Conclusions

As thermal stratification can result in thermal fatigue in the piping system of a nuclear power plant, safety and integrity evaluations of the piping system have become an important issue. Because of the complication of the thermal stratification itself, the flow and heat transfer should be fundamentally understood. One of the regions most at risk of suffering from thermal fatigue is a small elbow pipe branched off from the main pipe of the coolant loop for the drain or letdown system in the chemical and volume control system (CVCS). This work focuses on a fundamental description of the thermal stratification caused by turbulent penetration and buoyancy effects using LES. In the absence of leakage, the LES results are in good agreement with the available experimental data, which validates the LES as a means of predicting the thermal stratification occurring in the elbow branch pipe. Given this validation of LES, the flow and heat transfer were numerically predicted when leakage occurred in the elbow branch pipe. The numerical results show that, for a leakage ratio of 1%, the thermal stratification region is pushed down the horizontal part and may remain for a long time in the horizontal part. The thermal stratification dissipates rapidly at a leakage ratio of 5%, although in this case there is a larger power spectrum density of the temperature in the early stage of the leakage. It can be concluded that with decreasing leakage ratios, the elbow branch pipe suffers from increasing risk of thermal fatigue caused by thermal stratification.

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References