Coupling damage and reliability model of low-cycle fatigue and high energy impact based on the local stress–strain approach

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Abstract Fatigue induced products generally bear fatigue loads accompanied by impact processes, which reduces their reliable life rapidly. This paper introduces a reliability assessment model based on a local stress–strain approach considering both low-cycle fatigue and high energy impact loads. Two coupling relationships between fatigue and impact are given with effects of an impact process on fatigue damage and effects of fatigue damage on impact performance. The analysis of the former modifies the fatigue parameters and the Manson–Coffin equation for fatigue life based on material theories. On the other hand, the latter proposes the coupling variables and the difference of fracture toughness caused by accumulative fatigue damage. To form an overall reliability model including both fatigue failure and impact failure, a competing risk model is developed. A case study of an actuator cylinder is given to validate this method.

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1. Introduction

Products which have fatigue failure mechanism suffer not only complex fatigue loads, but also impact loads, which can be called a fatigue damage process. Both the failure modes of fatigue and impact associated to dynamic loads can cause the initiation of cracks in products, which may propagate to a macroscopic fracture size eventually. However, the loading rate of the impact load is much higher than that of the fatigue loads, which may lead to some changes of material properties. Hence, impact damage and fatigue damage have similarities as well as distinctions.

Fatigue load may cause a certain amount of fatigue damage in each cycle, and when the total damage cumulates to a certain threshold, the failure caused by fatigue (fatigue failure for short) takes place. There are various kinds of fatigues, such as mechanical fatigue, thermal–mechanical fatigue, corrosion fatigue, etc. Except for mechanical fatigue, the rest of them are the combinative effects of environment and specific mechanical stresses, which make them more complex than sole mechanical fatigue. To simplify our assumption, this paper focuses only on mechanical fatigue. As we know, fatigue damage and life related researches generally based on different

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definitions and assumptions. Among them, the traditional nominal stress method based on the stress and stress-life (S-L) curve is the earliest and most widely used. However, plastic deformation is not considered in the traditional nominal stress method, and it cannot be applied to the analysis of low-cycle fatigue. To overcome the disadvantages of the traditional nominal stress method, a local stress strain approach is developed. Furthermore, to analyze both high-cycle fatigue and low-cycle fatigue, a number of methods have been developed by combining the Basquin model and Manson–Coffin model.

The analysis of impact damage is more difficult than that of fatigue, because of the complex properties of materials corresponding to dynamic stresses. There are two extreme conditions for impact damage. If the energy of impact is large enough, it may cause impact damage at once, such as one-impact fracture. If the impact energy is very low, the accumulated impact failure can be approximated to fatigue damage. The conditions between the two extremes are much more complex, and when fatigue damage exits at the same time, they may have a coupling relationship which leads to a more difficult issue.

Even though both fatigue and impact have been studied copiously in previous literature, the research of their relationship is rare. Refs.9–12 studied the fatigue performance with post-impact damages, but most of them focused on composite materials. Ding et al.9 performed the tensile and compressive residual-strength tests on carbon-epoxy composites to obtain the fatigue performance after drop-weight impact. The impact damage followed by tensile fatigue cycling reduced the fatigue performance of the material. The results showed that tensile fatigue strength had a linear relationship with the cycle number, and the cycle-impact sequence gave more damage than impact-cycle loading. Beheshthy et al.13 developed a fatigue life-prediction model for carbon-fibred laminates with impact damage. Tai et al.12 noted the impact energy caused the decrease of strength, and the damage zone increased with the increase in impact energy. However, due to the characteristics of composite materials, they show more susceptibility to impact load than metal materials do. Furthermore, the failure modes of composites are different from those of metals. Therefore, the analysis method of composites with fatigue and impact cannot be applied to other materials directly.

Other researchers are interested in the studies on metal materials influenced by foreign object damage.13–16 Martinez15 investigated the strength of engine blades due to foreign object damage. The results in Thompson14 indicated that residual stress relief improved the limit stress, and that dynamic impacts had less influence on the fatigue strengths than that predicted from conventional analysis. Nowell et al.15 showed that the fatigue notch factor. When the nominal stress belongs to the range of strain of the closed loop is

\[
\Delta e = |\epsilon_{i-1} - \epsilon_{i-2}|
\]

(1)

Step 2. Convert the nominal strain and stress to the local stress and strain.

The range of nominal strain \(\Delta e\) corresponding to the range of nominal stress \(\Delta S\) can be obtained from

\[
\frac{\Delta e}{2} = \frac{\Delta S}{2E} + \left(\frac{\Delta S}{2K}\right)^{1/(d')}
\]

(2)

where \(K'\) is the cyclic strength coefficient, and \(d'\) the cyclic strain hardening exponent, \(E\) the elasticity modulus.

In the plane stress condition, the range of local stress and strain can be solved through

\[
\begin{align*}
\Delta e &= \Delta \sigma /2E + (\Delta \sigma /2K')^{1/d'} \\
\Delta \sigma \Delta \sigma' &= K_3^2 \Delta e \Delta S
\end{align*}
\]

(3)

where \(\Delta \sigma\) is the local strain, \(\Delta \sigma'\) the local stress, and \(K_3\) the fatigue notch factor. When the nominal stress belongs to the
elastic range, we have $\Delta e = \Delta S/E$, then the second equation in Eq. (3) is changed into

$$\Delta e \Delta \sigma = K_i^2 \Delta S^2/E$$

(4)

**Step 3.** Calculate the fatigue life corresponding to local strain.

The life corresponding to each strain level can be obtained by referring to a $\Delta e - 2N_t$ curve or via the modified Manson–Coffin equation:

$$e_r = (\sigma'_t - \sigma_m)(2N_t)^{b} / E + \varepsilon'_b (2N_t)^c$$

(5)

where $\sigma'_t$ is the fatigue strength coefficient, $\varepsilon'_t$ the fatigue ductility coefficient, $N_t$ the fatigue life in the form of cycle times, $\sigma_m$ the average of the local stress, $e_r$ the amplitude of the local strain, and $b$ and $c$ are the material parameters.

**Step 4.** Accumulate the fatigue damage.

There are a lot of accumulative fatigue damage theories. Among them, the Miner linear cumulative law is the most widely used in engineering.

For the single full cycle:

$$D = 1/N_t$$

(6)

where $D$ is the damage caused in the cycle.

For the single half cycle:

$$D = 1/2N_t$$

(7)

The accumulative damage of $K$ cycles is

$$D = \sum_{i=1}^{K} \frac{1}{N_t}$$

(8)

The method of the finite element analysis can yield the local stress and strain through an elastic–plastic finite element analysis software, such as ANSYS and Msc. Fatigue.

2.2. Prediction of fatigue parameters

For the fatigue analysis method of Neuber equation, some parameters in Eqs. (1)–(5) have to be measured or inferred through related tests. In particular, we must obtain plentiful fatigue data to produce parameters in the Manson–Coffin equation. When the test data is difficult to get, we can employ the static tensile performance of a material, such as tensile strength $\sigma_m$, elasticity modulus $E$, fracture ductility $\varepsilon_f$ and fracture strength $\sigma_f$, to estimate the fatigue performance approximately. The common estimating methods are the general slope method and four-point correlation method.

(1) General slope method

$$b = -0.12$$

$$c = -0.6$$

$$\sigma'_t = 1.75\sigma_b$$

$$\varepsilon'_b = 0.5\varepsilon_f$$

(9)

(2) Four-point correlation method

$$b = -0.0792 + 0.179 \ln(\sigma_t/\sigma_b)$$

$$c = -0.52 - 0.25 \ln \sigma_t + \frac{1}{2} \ln \left[ 1 - 81.8 \sigma'_t E \frac{\varepsilon'_b}{\sigma_m} 0.179 \right]$$

$$\sigma'_t = 1.12 \sigma_m (\sigma_t/\sigma_b)^{0.893}$$

$$\varepsilon'_b = 0.413 \varepsilon_f \left[ 1 - 81.8 (\sigma_b/E)(\sigma_t/\sigma_b)^{0.179} \right]^{-1/3}$$

(10)

Both the general slope method and the four-point correlation method are initialized from metal materials, especially alloy steels. The four-point correlation method is more precise for other metal materials than the general slope method. In this paper, the general slope method is adopted for the sake of simplicity, since its precision for most alloy steels is satisfactory.

2.3. Impact analysis

So far, there have been a number of statistical shock models to describe shock damage. For instance, the extreme model, accumulative model, $\delta$ model and run model. These models have been applied in practical work. Of these, the extreme model and accumulative model are the two most widely used. Nevertheless, the variable of a statistical model does not have a physical meaning. It cannot be applied to the calculation of impact damage unless the specific connotation of the statistical variable is defined by analyzing the damage mechanism.

As mentioned in the introduction, there are two extreme conditions for impact damage. According to the classification of statistical shock models, one belongs to the extreme model while the other belongs to the accumulative model. Other conditions can be regarded as their mixed models. The damage of high-energy impact is also a mixed model. The corresponding impact damages can be obtained from the energy of each impact and the accumulative damages caused by history loads. The critical matter is to determine the failure criterion.

The failure mode of impact is that the crack propagates to a state of collapse, and eventually grows to the size of fracture. From the perspective of strength-stress theory, impact failure occurs due to the state that the strength of a product, which is a fixed value, is less than the instantaneous stress it bears. While from the view of fracture mechanics, impact failure occurs when the fracture toughness is lower than the stress intensity factor. While tensile strength is a static property in the strength-stress theory, fracture toughness is a dynamic property in the theory of fracture mechanics. From this opinion, the criterion based on fracture toughness is more suitable than that based on strength. Therefore, fracture toughness is chosen as the impact failure criterion variable in this paper, and the condition of no impact failure when impact load appears is

$$K_{tc}(t) \geq K_1$$

(11)

where $K_{tc}(t)$ is the dynamic fracture toughness at time $t$, and $K_1$ the threshold of fracture toughness.

3. Coupling analysis between fatigue and impact

There are few qualitative analyses of coupling relationships between fatigue and impact damage. Chen and Chen gave a coupling relationship when HCF and low-energy impacts were involved, but it is not applicable to low cycle fatigue (LCF) and high-energy impacts. This section discusses the relationship between LCF and high-energy impact.

3.1. Theories of material performance under high-energy impact

(1) Strength increases with the increase of strain rate

Micro observation shows that when a material suffers a static stress, the plastic deformation distributes uniformly, but
when it suffers a dynamic stress, i.e., the strain rate is very high, the plastic deformation concentrates in a local zone, which limits its development. Thus, the yield strength and ultimate tensile strength is improved.27–30

(2) Fracture toughness decreases with the increase of strain rate

Fracture toughness is the performance parameter to indicate the material’s resistance to crack propagation. The stress intensity factor $K_I$ increases as the stress or crack size increases. When the stress or crack size reaches a critical value, the crack goes to an instable state, and then the material will fracture. Here, $K_I$ also reaches the critical threshold, which is denoted as $K_{IC}$. The higher is the value of $K_{IC}$, the harder it is for the material to fracture.26,31

(3) Overload retardation

Plenty of tests show that, once a relatively high amplitude load (overload) appears in the load spectrum, the propagation speed of a crack will decrease apparently.32,33 After a certain cycles of loads, the propagation speed will recover to the level before the overload.

3.2. Effect of high-energy impact on fatigue performance

High-energy impact means not only high strain rate (loading rate) but also high amplitude stress. According to the theories of material performance under high-energy impact, a high strain stress will cause plastic deformation, while a high strain rate will cause the increase of tensile strength, the decrease of fracture toughness, and a retardation effect if the impact stress is much higher than the fatigue stress.

Fracture ductility has the same variation tendency as fracture toughness, while fracture strength varies in the opposite direction as it has the same variation tendency as tensile strength.

Therefore, the corresponding parameters of static performance of a material vary with the variation of the strain rate. As the strain rate increases, the tensile strength $\sigma_0$ increases. The fracture ductility $\delta_f$ increases, and the fracture strength $\sigma_f$ increases. As the interval length between two impacts is long, the effect range of the impact is to be considered. That is, the lasting time period of the effect of an impact load is $N_s$, whose length is determined by the properties (stress amplitude and strain rate) of the specific impact. When another impact arrives, the effect of a high strain rate will be renewed by the new impact. Based on the test results of various materials, it can be assumed that the relationship between tensile strength and strain rate follows a power law, which can be written as Eq. (12). In addition, the relationship between fracture toughness and strain rate can also be assumed to be a power law. As fracture ductility has a similar variation tendency with that of fracture toughness when the strain rate changes, the relationship between fracture toughness and strain rate is expressed as Eq. (13).

\[
\sigma_f(t) = \sigma_{0b} + p \epsilon_f^m
\]

\[
\delta_f(t) = \delta_{0b} - q \epsilon_f^m
\]

where $p$, $n$, $q$ and $m$ are the material constants, $\sigma_{0b}$ is the original static tensile strength, $\epsilon_{0b}$ the original static fracture ductility. The specific value of $p$ and $n$ can be derived from data fitting with material experiments between tensile strength and strain rate by a least square estimation method.

Combining with the estimation method of fatigue parameters in Section 2.2, we can get the estimation equations for the fatigue parameters with the effect of a high-energy impact load. If the general slope method is applied, the modified parameters are

\[
\left\{ \begin{array}{l}
    b = -0.12 \\
    c = -0.6
\end{array} \right.
\]

\[
\sigma_f(t) = 1.75\sigma_0 + p \epsilon_f^m
\]

\[
\delta_f(t) = 0.5\epsilon_f^m - 0.5(\epsilon_{0b} - q \epsilon_f^m)^{1/6}
\]

Then the Manson–Coffin equation is changed into

\[
\varepsilon_p = \frac{1.75(\sigma_0 + p \epsilon_f^m) - \sigma_p}{E} + 0.5(\epsilon_{0b} - q \epsilon_f^m)^{1/6} (2N_c)^{1/2}
\]

The numerical solution of impact-effect fatigue life $N_f$ can be obtained from Eq. (15) by an iteration method such as Newton’s method.

33. Effect of accumulative fatigue damage on impact performance

When an impact load is applied to a product which has already suffered fatigue loads, the probability of impact fracture will increase rapidly. This is because its accumulative fatigue damage affects its resistance performance to fracture with impact. As the fracture toughness is adopted to represent the criterion of impact failure, we consider the fracture toughness to be the effect factor of fatigue damage on impact performance. The fracture toughness degrades with the accumulation of fatigue damage, and their relationship is modeled as

\[
\Delta K_{IC} = \Delta K_{IC}(t) = \Delta K_{IC}(D(t)) = \Delta K_{IC}(t)
\]

where $\Delta K_{IC}$ is the differential value of fracture toughness caused by fatigue damage, and $D(t)$ the accumulative damage at time $t$. The analysis of fatigue damage based on toughness dissipation theory defines the fatigue damage as

\[
D = 1 - \frac{U_N}{U_0}
\]

where $U_N$ and $U_0$ are the residual and initial toughness, respectively. Then the relationship between $K_{IC}$ and $D(t)$ is

\[
D(t) = 1 - \frac{K_{IC}(t)}{K_{IC}} = \frac{\Delta K_{IC}(t)}{K_{IC}}
\]

And the transformation is

\[
\Delta K_{IC}(t) = D(t)K_{IC}
\]

Furthermore, fracture toughness itself will decrease under the effect of high strain rate. The empirical estimation equation of fracture toughness is

\[
K_{IC} = 0.32\sqrt{\pi E\sigma_{0b} \rho_c}
\]

where $\rho_c$ is the critical passive radius of the crack tip. As for the high-strength lath martensite steel, its value equals the value of strain hardening exponent. The transformation of fracture toughness is

\[
\Delta K_{IC}(t) = 0.32\sqrt{\pi E\sigma_{0b} \rho_c} D(t)
\]
The fracture toughness which is not affected by fatigue damage is

\[ K_{IC}(t) = K_{IC} - \Delta K_{IC}(t) \]

\[ = 0.32\sqrt{\pi E\sigma_{0}p_{0}\rho_{c}} - 0.32\sqrt{\pi E\sigma_{0}p_{0}\rho_{c}D(t)} \]  

(22)

As fracture strength has a similar relationship with tensile strength, we can get

\[ \sigma(t) = \sigma_{0} + \rho E \]

(23)

When considering the variation of \( \sigma_{t} \) and \( \varepsilon_{t} \), \( K_{IC} \) becomes

\[ K_{IC}(\varepsilon) = 0.32\sqrt{\pi E\sigma(\varepsilon_{0} + \rho E)(\varepsilon_{t} - \rho E)}\rho_{c} \]

(24)

Therefore, the fracture toughness which is affected by fatigue damage at time \( t \) can be written as

\[ K_{IC}(\varepsilon, t) = K_{IC}(\varepsilon) - \Delta K_{IC}(t) = 0.32\sqrt{\pi E(\sigma_{0} + \rho E)(\varepsilon_{t} - \rho E)}\rho_{c} \]

\[ - 0.32\sqrt{\pi E\sigma_{0}p_{0}\rho_{c}D(t)} \]  

(25)

4. Damage and life model based on coupling relationship

The following system is considered: a product suffers both variable amplitude fatigue loads and high-strength impact loads. The fatigue load spectrum consists of loads with \( m \) different levels. Meanwhile, the corresponding cycles of stress of the \( \text{ith} \) level \( S_{i} \) is \( N_{i} \), and the total cycles of a cyclic unit is \( L \). The impact loads arrive at random, and it follows the Poisson process whose parameter is \( \lambda \), which means the arrival times of impact load in each cycle (\( t = 1 \)) is \( \lambda \).

4.1. Fatigue damage

The fatigue spectrum is analyzed to get the nominal stress and strain amplitude. The nominal stresses and strains are \( (S_{1}, \varepsilon_{1}), (S_{2}, \varepsilon_{2}), \ldots, (S_{m}, \varepsilon_{m}) \), respectively. Then they are converted into local stress and strain \( (\sigma_{1}, \varepsilon_{1}), (\sigma_{2}, \varepsilon_{2}), \ldots, (\sigma_{m}, \varepsilon_{m}) \) by utilizing Eq. (3) or the plastic-elastic finite element method. Their corresponding fatigue life and damage can be obtained with or without the effect of impact.

1. Without the effect of impact

If the product does not suffer an impact load, or if the fatigue loads have been out of the range of the effect of impact, the fatigue life \( N_{i1}, N_{i2}, \ldots, N_{im} \) is obtained by Eq. (5).

2. With the effect of impact

If the product has suffered an impact load, and the effect remains in its effective range, the tensile strength and fracture ductility should be modified by Eqs. (12) and (13), respectively. Then the fatigue parameter is modified by Eq. (9) or Eq. (10), and fatigue life \( N_{i1}(\varepsilon), N_{i2}(\varepsilon), \ldots, N_{im}(\varepsilon) \) should be calculated by a modified Manson–Coffin Eq. (15). The lasting time of one impact effect can be assumed as \( N_{i} \).

(A) If the impact effect ends at each new stress level of fatigue cycles, the fatigue damage can be obtained from Eqs. (26)-(32). This represents the situation when the fatigue loads are not continuous.

(B) If the impact effect keeps to a new stress level of the fatigue cycles, the fatigue damage can be obtained from

The arrival times of impacts during the \( \text{ith} \) level of a fatigue cyclic unit is \( n_{i} \). According to the feature of Poisson process, the probability of event \( n_{i} = n \) is

\[ P(n_{i} = n) = e^{-\lambda N_{i}} \frac{\lambda^{n} N_{i}^{n}}{n!} \]  

(26)

Assume that the energy of each impact is similar, and then for the purpose of simplification, we can use the average value of impact. Suppose the lasting cycle of effect for one impact is \( N_{i} \), then the lasting cycles of effect for \( n_{i} \) times of impact is \( n_{i} N_{i} \), and the cycles without the effect of impact is \( N_{i} - n_{i} N_{i} \), where \( N_{i} - n_{i} N_{i} = 0 \) and \( n_{i} N_{i} \leq N_{i} \) (or else \( N_{i} - n_{i} N_{i} = 0, n_{i} N_{i} = N_{i} \)). If the Miner law is applied, the accumulative damage with the effect of \( n_{i} \) times of impact is

\[ D_{i} = \frac{N_{i} - N_{i0} + n_{i} N_{i}}{N_{i0}} \]  

(27)

Combining with Eq. (26), we can get

\[ D_{i} = \sum_{i=1}^{n_{i}} e^{-\lambda N_{i}} \frac{\lambda^{n} N_{i}^{n}}{n!} \left( \frac{N_{i} - n_{i} N_{i}}{N_{i0}} + \frac{n_{i} N_{i}}{N_{i0}} \right) \]  

(28)

Add the damage caused on a cyclic unit together, and then the unit damage \( D_{U} \) is obtained

\[ D_{U} = \sum_{i=1}^{n_{i}} D_{i} \]  

(29)

If the fatigue damage in a unit is approximated to be a linear distribution, the total damage at time \( t \) (cycles) is

\[ D(t) = \frac{t}{L} D_{U} \]  

(30)

Otherwise, determine the last level \( l \) of the last cyclic unit. \( S_{l} \) denotes its stress and \( n_{l} \) denotes the residual cycles in level \( l \) of the last incomplete cyclic unit at time \( t \). Therefore, \( n_{l} \) equals the total cycles minus the cycles of the integral number of units by time \( t \), and then minus the cycles of total integral level in the last incomplete unit, as expressed in:

\[ n_{i} = t - \left[ \frac{t}{L} \right] L - \sum_{i=1}^{l-1} N_{i} \]  

(31)

The maximum \( l \) which satisfies \( t - \left[ \frac{t}{L} \right] L - \sum_{i=1}^{l-1} N_{i} \geq 0 \) is the solution, where \( \left[ \cdot \right] \) denotes the rounding function, \( \left[ \cdot \right] L \) denotes the integral number of units by time \( t \), \( \left[ \frac{t}{L} \right] L \) means the cycles of the integral number of units by time \( t \), and \( \sum_{i=1}^{l-1} N_{i} \) means the cycles of total integral level in the last incomplete unit. Then, the total fatigue damage at time \( t \) is

\[ D(t) = \left[ \frac{t}{L} \right] D_{U} + \sum_{i=1}^{l-1} D_{i} + \sum_{i=0}^{\infty} e^{-\lambda N_{i}} \frac{\lambda^{n} N_{i}^{n}}{n!} \left( \frac{N_{i} - n_{i} N_{i}}{N_{i0}} + \frac{n_{i} N_{i}}{N_{i0}} \right) \]  

(32)

where \( \left[ \frac{t}{L} \right] D_{U} \) means the damage caused by the integral number of units by time \( t \), \( \sum_{i=1}^{l-1} D_{i} \) means the damage caused by the total integral level in the last incomplete unit, and \( \sum_{i=0}^{\infty} e^{-\lambda N_{i}} \frac{\lambda^{n} N_{i}^{n}}{n!} \left( \frac{N_{i} - n_{i} N_{i}}{N_{i0}} + \frac{n_{i} N_{i}}{N_{i0}} \right) \) means the damage caused by the residual cycles in residual level \( l \) of the last incomplete cyclic unit at time \( t \).
Eqs. (33) and (34). This situation occurs when the fatigue loads are continuous.

The probability that impact does not affect the fatigue properties at any time is $e^{-\lambda N}$, and the accumulative damage of the $l$th level of a fatigue cyclic unit is

$$D_l = e^{-\lambda N_l} \frac{N_l}{N_f} + \left(1 - e^{-\lambda N_l}\right) \frac{N_l}{N_f}$$  \hspace{1cm} (33)$$

The accumulative damage of a cyclic unit $D_l$ is the same as Eq. (29) and $n_l$ can be obtained from Eq. (31).

$$D_l = \left[ \frac{f}{L} D_0 + \sum_{i=1}^{l-1} D_i + e^{-\lambda N_i} \frac{n_i}{N_f} \right] \frac{n_i}{N_f}$$  \hspace{1cm} (34)$$

4.2. Impact damage

According to the impact failure criterion, when $K_{IC}(t) \leq K_1$, failure occurs. The stress intensity factor is

$$K_1 = \sigma \sqrt{\pi a_c}$$  \hspace{1cm} (35)$$

where $\sigma$ is the stress, and $a_c$ is the critical length of crack when fracture happens.

$R_0(t)$ denotes the probability of no impact failure, and the expression can be derived as follows:

(1) When the impact effect ends at each new stress level of fatigue cycles, then

$$R_0(t + \Delta t) = P\{K_{IC}(\tau) \geq K_1, \forall \tau \in [0, t] \cdot P\{K_{IC}(\tau) \geq K_1, \forall \tau \in [t, t + \Delta t]\}$$

$$= R_0(t) \left\{ 1 - P\{K_{IC}(\tau) \leq K_1, \exists \tau \in [t, t + \Delta t]\} \right\}$$

$$= R_0(t) - R_0(t) \left\{ P\{K_{IC}(\tau) \leq K_1, \exists \tau \in [t, t + \Delta t]|\text{effect of impact appears}\} \cdot P\{\text{effect of impact appears}\} \right\}$$

$$- R_0(t) \left\{ P\{K_{IC}(\tau) \leq K_1, \exists \tau \in [t, t + \Delta t]|\text{effect of impact doesn't appear}\} \cdot P\{\text{effect of impact doesn't appear}\} \right\}$$

$$- R_0(t) \left\{ e^{-\lambda (N_0)} (1 - \lambda) \Delta t \cdot P\{K_{IC}(\tau) \leq K_1, \exists \tau \in [t, t + \Delta t]\} + o(\Delta t) \right\}$$

where $1 - e^{-\lambda (N_0, n_0)}$ represents the probability that the impact appears in the interval $[t, t + N_0]$, $n_0$ denotes the residual cycles in the last level $l$ of time $t$ and $\Delta t$ means the probability that the impact appears in the interval $[t, t + \Delta t]$. When $\Delta t \to 0$, we have

$$\frac{dR_0(t)}{dt} = -(1 - e^{-\lambda (N_0, n_0)} + e^{-\lambda (N_0, n_0)} \lambda) P\{K_{IC}(\tau) \leq K_1 \} - e^{-\lambda (N_0, n_0)} (1 - \lambda) P\{K_{IC}(\tau) \leq K_1 \}$$  \hspace{1cm} (37)$$

The integral of the above equation is

$$R_0(t) = \exp[-(1 - e^{-\lambda (N_0, n_0)} + e^{-\lambda (N_0, n_0)} \lambda)]$$

$$- e^{-\lambda (N_0, n_0)} (1 - \lambda) \int_0^t P\{K_{IC}(\tau) \leq K_1 \} d\tau$$  \hspace{1cm} (38)$$

where

$$P\{K_{IC}(\tau) \leq K_1 \} = P\{K_{IC}(\tau) - \Delta K_{IC}(D(t)) \leq K_1 \}$$

$$= P\{K_{IC}(\tau) - K_{IC}(D(t)) \leq K_1 \}$$

$$= P\{0.92 \sqrt{\pi E\sigma(t)} (x_0 - q \theta) \rho c - K_{IC}(D(t)) \leq K_1 \}$$

where $R_0(t) = P\{D(t) \leq D_0\}$ is the probability of no fatigue failure before time $t$, and $D_0$ is the failure threshold which is usually determined by engineering demands. $R_0(t)$ is the probability of no impact failure before time $t$ which is obtained by Eq. (38) or Eq. (40).

4.3. Reliable life

The overall reliable life $R(t)$ can be achieved when both the fatigue failure and impact failure do not occur before time $t$:

$$R(t) = R_0(t) R_0(t)$$  \hspace{1cm} (41)$$

where $R_0(t)$ is the probability of no fatigue failure before time $t$, and $D_0$ is the failure threshold which is usually determined by engineering demands. $R_0(t)$ is the probability of no impact failure before time $t$ which is obtained by Eq. (38) or Eq. (40).

5. Case study

An actuator bears fatigue loads during normal operation. It also suffers impact loads at random. In order to estimate the life of the actuator when it operates in a severe environment, the fatigue loads and high-energy impact loads are analyzed and obtained with its history records. The connection component is the dangerous part of the actuator which suffers the largest stress, and this stress belongs to LCF. Hence, the above proposed method is applied to analyze the damage of the connection component. The impact loads follow the Poisson process with $\lambda = 5 \times 10^{-4}/(fatigue cycle)$ whose average strain rate is $10^4/s$, and the average stress amplitude is 1000 MPa. The fatigue load spectrum is listed in Table 1.
The material of the connection component is 30CrMnSiA. The material properties are as follows: the elastic modulus is 196 GPa, Poisson rate is 0.3, yield strength is 835 MPa, stress hardening exponent \( n = 0.063 \) and cyclic strain hardening exponent \( n' = 0.127 \), \( N_i = 1000 \) cycles.

The elastic–plastic finite element method is used to analyze the local stresses and strains of the connection component. Among them, the ANSYS result of stress distribution of the largest load, i.e., Level 3, is shown in Fig. 1. All of the average stresses and strain amplitudes are revealed in Table 2. The general slope method is utilized to estimate the fatigue parameter, and the fatigue life of every level can be obtained from Eq. (5).

The modified tensile strength, fracture ductility and fracture strength are obtained from Eqs. (12), (13), and (23), respectively. The values of parameters are: \( k = 10^3 \), \( p = 7.6 \), \( n = 0.038 \), \( q = 0.26 \), and \( m = 0.045 \). The results are \( \sigma_0(\tilde{k}) = 1090.785 \) MPa, \( \varepsilon_0(\tilde{k}) = 0.5305 \), and \( \sigma_1(\tilde{k}) = 1805.8 \) MPa.

Eq. (15) is utilized to calculate the fatigue life with the effect of impact, and the results are listed in Table 3.

Then Eq. (28) is used to get the expected modified fatigue damage with the effect of impact, and the results are shown in the last column of Table 3. As one fatigue cyclic unit suffers 2.45 times of impact for expectation, every unit suffers more than one impact at the average level. Hence, we consider the expected degradation of fracture toughness of one cyclic unit to simplify this issue.

The initial value of fracture toughness is \( K_{IC} = 0.32 \sqrt{\pi E\sigma_0(\tilde{k})} = 105.4 \) MPa/m\(^{1/2}\).

The fracture toughness affected by a high strain rate is \( K_{IC}(\tilde{k}) = 87.6 \) MPa/m\(^{1/2}\).

Therefore,

\[
\begin{align*}
K_{IC}(\tilde{k}) & = 105.4 - 10.4D(t) \\
K_{IC}(\tilde{k}, t) & = 87.6 - 10.4D(t)
\end{align*}
\] (42)

The expected fracture toughness at the end of each fatigue unit when the impact affects that point is

\[
K_{IC}(\tilde{k}, nL) = 87.6 - 10.4 \times 0.1107n
\] (43)

The expected fracture toughness at the end of each fatigue unit when the impact does not affect at that point is

\[
K_{IC}(nL) = 105.4 - 10.4 \times 0.1107n
\] (44)

The stress intensity factor of the impact stress is \( K_1 = \sigma/\sqrt{\pi a} = 56 \) MPa/m\(^{1/2}\), and \( a_c = 1 \) mm.

\[
R_k(t) = \exp \left[ \frac{-1}{Q} \right] \int_0^t P\{K_{IC}(\tilde{k}, t) \leq K_1\}d\tau
\] (46)

The value of \( K_{IC} \) at the end of each fatigue unit is shown in Table 4.
The curve of impact reliability is shown in Fig. 2. As the fatigue damage is much lower than its critical threshold and the fatigue reliability is 1 when the impact reliability tends to be 0 (within 8000 cycles), its damage can be ignored. The overall reliability is similar to what is shown in Fig. 2.

If the impact effect keeps to a new stress level of fatigue cycles, $D(t)$ should be obtained from Eqs. (33) and (34). The overall reliability is plotted in Fig. 3. Comparing Figs. 2 and 3, we can find out that the assumption of a continuous impact effect is more reasonable in this case. The connection component fails at the third unit between 6000th and 6060th cycles.

This simulation result is nearly identical with the experimental result from an accelerated life test.

If the coupling relationship between fatigue and impact is ignored, we always have $K_{IC}(t) \geq K_i$. Therefore, in this condition the impact damage can be ignored. The overall reliability when the coupling relationship is not considered is compared with the fatigue reliability alone when the fatigue is coupled with the impact damage in Fig. 4, which indicates the fatigue reliability considering the coupling relationship is a little less than the fatigue reliability (also the overall reliability) without impact. When $R = 0.7$, the overall life without considering the coupling relationship is about 20,000 cycles. But from Fig. 3, the overall life considering the coupling relationship is about 6000 cycles. Hence, the comparison with Figs. 3 and 4 shows that if the coupling relationship is ignored, the estimation...
result is much more risky, and the risk is more than 3 times as large.

6. Conclusions

Fatigue damage is a severe threat to the life of mechanical products. Moreover, the probability of failure may increase significantly by the effects of impact loads. It is very complicated to model an effective fatigue and impact system in consideration of their mutual dependence and coupling relationship. This paper attempts to analyze damage and life with a low-cycle fatigue and high-energy impact process.

The analysis of fatigue damage is based on the local stress–strain approach as the plastic deformation is caused by the high amplitude stress of LCF. Two methods to estimate the fatigue parameter in Manson–Coffin equation are presented via the material’s static performance parameters. The effect of high-energy impact on the fatigue process is performed on the static performance parameters, among which the tensile strength, fracture ductility and fracture strength are determined to be the coupling factors. The modified parameters and Manson–Coffin equation are developed.

In addition, the effect of fatigue accumulative damage on the impact process is analyzed. The criterion of impact fatigue is determined by the relationship between fracture toughness and stress intensity factor. The fracture toughness is the dependent variable affected by fatigue damage.

According to the result of a case study, impact loads have a significant effect on the fatigue damage and the reliable life of a product. The fatigue damage also has a great effect on the impact resistance performance. If the impact process and the dependence relationship with the fatigue process are ignored, there will be a higher risk to estimate its reliable life incorrectly. Based on the estimation result, appropriate maintenance can be performed.

There are still several questions to answer in future:

1. How to determine the parameters in the coupling equation if the test is not available?
2. What is $p_c$ for other materials?
3. How can we utilize the estimated results for maintenance policy?

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