Stress-strain characteristics as a source of information on the destruction of rocks under the influence of load

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Abstract
This paper presents the problems associated with the complete stress-strain characteristics of the destruction of rocks under load, in the full range of their strain. The most important factors influencing the course of a stress-strain curve are discussed, i.e.: the shape and size of samples and the conditions experienced when conducting the experiment in a testing machine. Presented mechanical and energy parameters were obtained in laboratory tests carried out by loading rock samples in a servo controlled testing machine as well as some indices devised on the basis of these parameters. The interpretation of complete stress-strain characteristics allow solve various issues connected with underground mining, especially assessing natural hazards which are common in underground mining for minerals, as well as designing and managing mining works within underground workings.

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1. Introduction

In natural conditions a rock is subjected to load, resulting from the influence of the other rocks surrounding it. Due to load displacements and strains in their structure, the destruction of rocks takes place. Using modern research methodology and appropriate equipment, it is possible to create a model showing the destruction of rocks in laboratory conditions and therefore conduct tests on the destruction of rocks whilst experiencing a range of different forms of stress, i.e. compressive, tensile, bending, and shear stress. Using a testing machine and destroying rocks in different load systems provides insight into the processes that lead to their destruction. When testing under compressive stress states, the system of ‘plates of a testing machine – compressed rock sample’ is the simplest model of rock mass, where the sample is a model of a pillar, for example, and the plates of the testing machine represent layers of rocks above (roof) and below (floor). By inputting a rock sample in to a testing machine we get a representation of its destruction in the form of a stress-strain curve. The shape of the curve depends on the petrography of the rock and for samples of the same type of rock it depends on a range of factors. The most important of these being: the size and shape of samples, the method used to control the stiff testing machine, the strain rate applied during tests and the moisture content of the rock tested. Using the stress-strain curve values of mechanical parameters, including those of stress-strain and energy, are determined. The information, obtained from analysing the course of destruction samples of rocks in different loading conditions,
used to solve problems associated with geoengineering and mining. The example presented in this paper is: the use of a stress-strain curve interpretation in mining practices to determine rock and rock mass susceptibility to rock bursts.

2. Stress-strain characteristics of rocks – selected problems

We obtain the stress-strain curves of rocks in tests conducted with testing machines. Until the 1970s research was conducted in so-called ‘soft’ testing machines. The tests represented the increasing of stress-strain characteristics, eventually reaching the value of maximal stress (where the destruction of rocks takes place) (Wawersik & Fairhurst, 1970).

Bieniawski’s research (1970) enabled the stress-strain characteristics of a compressed rock sample in the full course of its strain to be obtained. The full characteristics were obtained through testing rock samples in servo controlled testing machines. To obtain the full course of destruction of a sample in a testing machine a number of conditions must be met. The machine has to be sufficiently stiff in relation to the rock sample, and the hydraulic system supplying the machine has to be sufficiently efficient.

Stress-strain characteristics obtained during tests on a rock sample in a servo controlled testing machine have a non-linear course and consist of an increasing and decreasing part. The rising part of the stress-strain characteristics reflects the pre-critical phase, when there is an increase both in stress and strain until the compressed sample reaches the value of maximal stress. In the decreasing part of stress-strain characteristics, which we call the post-critical phase, there is a decrease in stress until it reaches the value of residual stress. A decrease in stress in the post-critical part is accompanied with an increase in strain until the value of residual strain is reached.

Stress-strain characteristics of rocks are not linear, as rocks are not linearly deformable. To determine the values of some of the mechanical parameters of the stress-strain curve, in its pre-critical phase (Ulusay & Hudson, 2007) and post-critical phase (Bukowska, 2013), are represented by straight lines.

The course of stress-strain characteristics is different for different rocks and depends on numerous factors including the conditions in which the experiments were conducted and the size of the samples. Numerous researchers have described stress-strain characteristics of rocks by using a compression test in a stiff testing machine since the 1960s (Cook, 1965; Wawersik & Fairhurst, 1970; Peterson, 1978; Okubo & Nishimatsu, 1990; Bezat, 1987; Pella, 1993).

Many years of research into the strength and deformability of rocks under uniaxial compression and conventional triaxial compression shows that there are multiple stages of strain in rocks across their full stress-strain characteristics.

In the pre-critical part of a stress-strain curve we can distinguish three main stages of the destruction of a rock sample (Fig. 1):

- Stage I – the clamping and sealing phase. The plates of the stiff machine adjust to the surface of a sample, and the rock stiffens as pores and micro-cracks close.

- Stage II (elastic phase) – the course of the curve is linear.

- Stage III – starting with the threshold of dilation and finishing when the sample reaches its maximal strength. The course of a stress-strain curve in this phase is non-linear. The angle of inclination of the tangent to the stress-strain curve decreases, illustrating a more dynamic increase in strain in relation to stress. Additionally we can observe here a significant increase in transverse strain, and a rapid increase in volumetric strain. The phenomenon in Stage III is associated with an anomalous, in elastic increase in the volume of rocks resulting from compressive load. Moreover, Stage III is the beginning of the process of destruction the structure of the rock.

Based on tests conducted under axisymmetric compressive stress we devised a more complex description of strain in a sample in the pre-critical phase, and more stadia of the strain. The stadia of rock strain under axisymmetric compressive stress, including longitudinal strains, both transverse and volumetric are presented in Fig. 1:

- Stadium I – the non-linear strain of rock. The influence of differential strain causes the closing of micro-defects and micro-cracks. Initially, all the characteristics are non-linear. In later stadia of loading a rock sample there may be a linear dependence between stress and transverse strain.

- Stadium II – the linear strain of rock. Moduli of longitudinal strain and transverse strain have a constant value.

- Stadium III – linear longitudinal strain, non-linear transverse strain and volumetric strain. In the initial phase of the rock destruction process a propagation of primary micro-cracks, present in the rock, occurs. The relative dilation process begins. The threshold of relative dilation occurs at a value of between 0.3 and 0.6 of critical stress.
Stadium IV – non-linear strain of rocks, which characterise an advanced process of stable crack propagation. In the stadium the value of modulus of longitudinal strain decreases while the value of the modulus of transverse strain increases.

Stadium V – non-linear strain of rocks, accompanied by an increase in the volume of rock (absolute dilatancy). Crack propagation is uncontrolled, the value of critical stress is reached and the rock is destroyed.

In the post-critical part of stress-strain characteristics Wawersik and Fairhurst (1970) obtained two curves reflecting two different ways of rock destruction caused by loading (Fig. 1). The post-critical curve resembling the curve of class I refers to the behaviour of rocks after exceeding critical stress, when static crack propagation occurs (Fig. 2). Then, a further decrease in the carried load is possible if additional work is carried out. This decrease occurs as a result of further loading and supplying the compressed sample with additional energy. In rocks in which the post-critical curve has a course similar to the curve of class II the destruction process is maintained spontaneously until the sample loses its strength. After initiating the process of destruction it is continued uncontrollably and to stop the process it is necessary to withdraw the piston of a testing machine. The course of the destruction process, according to a class II curve, is characteristic for rocks of very high brittleness.

One of the factors influencing the pattern of the characteristics of a stress-strain curve is the steering of a testing machine. Testing machines equipped with servo-control systems allow for the programming of any pattern of loading a sample. As a parameter controlling the test we can use:

- signals of longitudinal strains of a sample expressed with a steady piston stroke speed,
- the value of increase in loading force (axial),
- the steady rate of axial strain, then the controlling signal is the partial longitudinal strain of a sample, measured and transferred to the steering system with a sensor fixed in mid-height of the sample,
- the steady rate of the circumferential strain of a rock sample, then the signal controlling the test is the signal sent from a strain gauge measuring changes to the circumference of a cylindrical sample.

To conduct the tests of the destruction of rock samples according to Class II by Wawersik and Fairhurst (1970) the tests have to be controlled with a circumferential strain rate.

Finisńska (1994), based on the uniaxial compression tests of rocks in a stiff testing machine, conducted at a steady rate of circumferential strain, mentioned four patterns of strain course in a rock sample in the post-critical state (Fig. 3):

- model I – destruction in one strain phase,
- model II – destruction in a few successive main phases (macro-cracks),
- model III – destruction in a few successive phases of a single crack (micro-cracking), without a distinctive main phase,
- model IV – steady destruction with weakly distinguished phases of cracking.

Bukowska (2005a, 2005b), based on tests conducted in a servo controlled testing machine steered with the signal of total sample strain, reflecting the piston stroke, identified six types of stress-strain curves for sedimentary rocks (sandstones, mudstones, claystones and hard coal) (Fig. 4). Differences in the shapes of stress-strain curves under uniaxial compression are clearly visible in the pre-critical and the post-critical phases. The differences are, to a large extent, associated with:

- the length of the phase of closing micro-cracks in the pre-critical phase,
the occurrence of local decreases in stresses representing the length of cracks in the pre-critical and the post-critical phase,

• a ‘smooth’ or ‘staircase’ post-critical curve,

• the possibility of approximating the post-failure phase with a linear or hyperbolic dependence heading to residual strain.

3. The influence of certain factors on the shape of stress-strain characteristics

Dimensions of samples resulting from their slenderness and diameter (base) influence the general shape of a stress-strain curve. The process of destruction is closely associated with the size of the object to be destroyed. When the rock is highly fragmented and we have to deal with particles measuring only hundredths of millimetres, then a significant portion of time is spent on destruction mineral grains. When it is an object of a few centimetres (rock sample) then its destruction occurs mainly along the inter-granular surfaces. When we refer to the destruction of a part of a rock massif where workings are driven, then the magnitude is of between several metres and hundreds of metres. We then analyse the structure of rock-mass, considering structural defects occurring there. They result from the fact that surfaces of weakening and discontinuity occur in the rockmass. When applying a stress-strain curve of a certain volume of rockmass to research tasks of geotechnology and mining, we have to be aware of the effect of scale.

Lowering the slenderness of a sample results in decreasing the inclination of the post-critical curve to the strain axis (Hudson, Brown, & Fairhurst, 1971; Labuz & Biolzi, 1991; Merwe, 2003). Increasing the size of a sample results in an increase in the inclination of the post-critical curve.

Natural phenomena occurring in the rockmass as well as processes induced by engineering activities depend on the strain rate of rocks. A lot of unfavourable phenomena in the rockmass occur at different strain rates. The influence of strain rates on the course of stress-strain characteristics has been confirmed by numerous tests. Results of this research were published abroad as early as the 1930s. More recent research dates back to the 1960s and later (Bieniawski, 1970; Peng, 1973; Blanton, 1981; Lis & Kijewski, 1987; Chong, Turner, & Boresi, 1989; Krzysnst, 1990; Olsson, 1991; Lajtai, Duncan, & Carter, 1991). The research shows that with an increase in strain rate, the pre-critical curve has a steeper inclination in relation to the strain axis. Bukowska’s (2005) research shows that the inclination of the pre-critical curve for sedimentary rocks, within the range of strain rate used in strain rate tests, generally, shows a slight increase or remains the same.

As research conducted by Bieniawski (1970), Peng (1973) and more recent research by Bukowska (2005) shows, the inclination of the post-critical curve increases or decreases with an increase in strain rate (Fig. 5).

The influence of temperature on the course of rock destruction is not clear. The strength of rocks usually decreases as the temperature increases. In most rocks an increase in temperature causes an increase in their deformability. It means that at room temperature they behave as though they have an elastic—brittle body, and at a high temperature like an elastic—plastic one.

Tests of rocks under triaxial stress showed that an increase in the value of the confining pressure causes an increase in the strength of rocks and an increase in the value of residual stress (Fig. 6). At high values of confining pressure there is a transition from the brittle state to the ductile state and a change in the model of rock destruction.

An analysis of the influence of strain rate and confining pressure on the values of post-critical parameters, e.g. of Carboniferous rocks of GZW, showed that a confining pressure of 0–70 MPa has a greater influence on post-failure parameters than a strain rate of $10^{-4}$–$10^{-1}$ s$^{-1}$ (Bukowska, in press).
4. Geomechanical properties of rocks determined by using full stress-strain characteristics and the dynamics of rock destruction

The stress-strain curve is an image of the destruction of a rock sample. By analysing the course of the curve we determine values of both mechanical parameters and energy parameters. It refers to the uniaxial tests conducted in different load systems and tests under a complex stress state. Using a servo controlled testing machine the values of mechanical parameters and energy parameters of the rocks, in the pre-critical and the post-critical part, are determined. The main parameters determined by analysing stress-strain characteristics are presented in Figs. 7 and 8.

Based on the values determined using the course of destruction a rock sample we can determine the values of various parameters and indices which provide information on the dynamics of destruction a rock medium and can be used to solve tasks e.g. in geotechnology and mining. The most important of these parameters are:

- critical differential stress (so called differential strength limit) $\sigma_1 - \sigma_3$,
- residual differential stress $\sigma_{\text{res}} - \sigma_3$,
- specific energy of pre-critical destruction $A_{\text{pre}} = A_n + A_{\text{el}}$,
- dissipated strain energy $A_{\text{dys}} = A_{\text{el}} + A_{\text{post}}$,
- kinetic energy $E_k = E_n - A_{\text{el}} - A_{\text{post}} - A_{\text{kin}}$.

Using the obtained shape of stress-strain characteristics, we obtain information concerning the dynamics of the destruction of a rock sample. The shape of the post-critical part of the characteristics and its inclination show the differentiated dissipation of elastic energy. The fast course of this process, when the dissipation rate increases, results in an increase in the value of kinetic energy. This is attributed to the occurrence of geodynamic phenomena in the rockmass. When the process of dissipating elastic energy is slow, the dissipation of energy can be significant and the value of kinetic energy will decrease to zero. Organoenic rocks (hard coals) of the Upper Silesia Coal Basin, in comparison with clastic rocks (sandstones, mudstones) and clay rocks (claystones), reach a smaller inclination of the post-critical curve (higher values of post-peak failure modulus) at lower values of compression strength. It means that when being destroyed (exceeding critical stress) the dynamics of the destruction of coal is higher than the dynamics of the destruction of clastic rocks and clay rocks. To illustrate this, in Fig. 9, we present the values of post-peak failure modulus for coal, clastic rocks and clay rocks (Bukowska, 2005),

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**Fig. 5** — The course of force-displacement (time) curves for fine-grained sandstone at different values of strain rate used in tests; a) rate 0.008 mm/s; b) rate 0.08 mm/s; c) rate 0.8 mm/s; d) rate 8 mm/s.
at a comparable value of compression strength of 25 MPa, calculated according to the devised dependences.

5. Use of stress-strain curve in mining practice

The image of the course of destruction rocks resulting from loading them in their full range of strain, in the pre-critical and the post-critical phase, became a new quality; and the post-peak failure modulus determined on the course of the post-critical curve together with the residual strength and the residual strain are parameters, which can be used to solve various problems concerning mainly mining geomechanics and geoengineering (Fig. 10).

For the needs of the Polish hard coal mining industry (underground mining) various indices are devised based on a stress-strain curve of rock samples compressed in a testing...

Fig. 6 – The course of force-displacement (time) curves for fine-grained sandstone at confining pressure of: a) 0 MPa; b) 20 MPa; c) 30 MPa; d) 50 MPa.

Fig. 7 – The main mechanical parameters and energy parameters determined with a stress-strain curve of loaded rock.
machine. Some of them have found their practical application. E.g.: the energy index of coal’s natural susceptibility to rockbursts $W_{ET}$ (Szecowka, 1972). It is determined with pre-critical stress-strain characteristics. Using the course of the curve in its post-critical part the ODR index, called the ‘period of dynamic destruction of coal ODR’, was devised in order to be used to determine the ‘sensitivity’ of coal to dynamic load concerning tremor and rockburst occurrence in hard coal mining (Kidbyński & Smolka, 1988). An example of an index considering a full range of the strain of rock samples, and the pre-critical and the post-critical properties are: the index of rockmass susceptibility to rockbursts $W_{TG}$ and the index of rockmass kinetic energy $W_{Ek}$ (Bukowska, 2012).

It is important that rockmass susceptibility to rockbursts is its natural feature and depends on, among other things, the geomechanical properties of a coal seam and layers of rocks surrounding it (for Carboniferous rockmass), the lithology and petrography of rocks, the depth of exploitation, the geological history of the rockmass, coal seam thickness and water-bearing of rockmass.

Rockbursts occur in the exploited rockmass susceptible to them. Important factors, influencing the occurrence of geodynamic phenomena such as rockbursts, are mining factors (technical factors) associated with current mining activities.

To evaluate Carboniferous rockmass susceptibility to rockbursts we can use the index of rockmass susceptibility to rockbursts $W_{TG}$ and the index of rockmass kinetic energy $W_{Ek}$. The first one uses the full stress-strain characteristics and on its basis, with the pre-critical part, we determine Young’s modulus of rocks in a depth interval of between up to 100 m above the seam and 30 m below the seam; and with the post-critical part we determine the value of post-peak failure modulus within the same depth interval. The idea of construction of the index originates from work by Petukhov and Linkov (1979), where they considered the post-critical properties of coal and presented the influence of post-critical strain on the stability of workings. However observations and experiences of Carboniferous rockmass in the Upper Silesian Coal Basin (GZW) in Poland show, that in the evaluation of Carboniferous rockmass susceptibility to rockbursts it is important to examine geomechanical properties of rocks up to the height of 100 m above and 30 m below a given seam, as the source of over 95% of rockbursts is located in the said depth interval. Taking into consideration the information and values of the index of rockmass susceptibility to rockbursts $W_{TG}$, a classification of Carboniferous rockmass susceptibility to rockbursts was devised. The kinetic energy index $W_{Ek}$ was devised based on the value of kinetic energy. Its value was

![Fig. 8](image1) - Image of distribution of total strain energy: $\sigma$ – stress; $\varepsilon$ – strain; pre-critical: $A_{el}$ – elastic energy; $A_n$ – non-recoverable energy; post-critical: $A_{post}$ – energy of post-critical destruction; $A_{el \ post}$ – recoverable energy.

![Fig. 9](image2) - Post-peak failure modulus of coal and waste rock calculated according to functional dependences (Bukowska, 2005) for a compression strength of 25 MPa.
calculated using stress-strain characteristics and the value of normal energy from the pre-critical and the post-critical part of the characteristics. To construct the index we used a coefficient of structural weakening of the rockmass, according to the classification of Geological Strength Index (GSI) and the equation devised by Hoek (1994). Showing that the kinetic energy of a seam and the surrounding rockmass is a function of their strength index, \( W_{Tg} \) was defined and a classification of Carboniferous rockmass susceptibility to rockbursts up to 100 metres above a seam and 30 metres below a seam was devised. It is important that both indices of rockmass susceptibility to rockbursts \( W_{Tg} \) and \( W_{e} \) (together with other natural conditions of Carboniferous rockmass (seam thickness and the depth of a seam, rockmass structure, the distance from a potentially seismogenic layer, and seismicity of the rockmass) are the basis to develop GEO, a system of geological–geomechanical assessment of rockmass susceptibility to rockbursts (Bukowska, 2013). The GEO system for assessing susceptibility to rockbursts, together with the classification of assessed susceptibility to rockbursts for Carboniferous rockmass, and a classification of rockburst hazard to mine workings was verified in a few dozen tests sites in GZW.

### 6. Conclusions

The conditions of loading rocks in situ are, in laboratory conditions, modelled through loading rocks in a testing machine. Stress-strain characteristics are an image of the course of destruction rock loaded in a stiff testing machine. The pre-critical part of the curve represents the destruction of rock until it reaches the value of critical (destructive) stress. In the pre-critical part an increase in stress is accompanied by an increase in rock strain. The post-critical part shows rock destruction after the value of maximal stress is reached until residual stress is reached. In the post-critical part a decrease in stress is accompanied by an increase in strain.

The shape of stress-strain characteristics is different for different rocks and depends on numerous factors. The most important ones influencing the course of the destruction curve are: shape and size of the sample and the conditions of the experiment in the testing machine.

Analysing the course of the curve of rock destruction under load, we obtained values of geometrical and energy parameters and drew conclusions concerning the dynamics of rock destruction. Thus, the stress-strain curve gives information about the destruction of the loaded rocks.

Based on the analysis of the courses of the curves and calculations of the values of mechanical parameters and energy parameters, and various indices devised based on them, it is possible to utilize the data when solving various problems of underground mining. Particularly it refers to the assessment of natural hazards and the designing of mining activities in underground workings e.g. exploitation and liquidation.

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