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DGLAP versus perturbative Pomeron in large momentum transfer hard diffractive processes at HERA and LHC

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ABSTRACT

We evaluate within the LO DGLAP approximation the dependence on energy of the cross section of the photo(electro)production of vector meson V ($V = J/\psi, \ldots$) in the hard elastic processes off a parton $\gamma^* + g \to V + g$ as the function of momentum transfer $t = (q_\gamma - p_V)^2$. We demonstrate that in the limit $-t \geqslant m_V^2 + Q^2$ the cross section does not contain double logarithmic terms in any order of the DGLAP approximation leading to the energy independent cross section. Thus the energy dependence of cross section $\gamma^* + p \to J/\psi + \text{rapidity gap} + X$ is governed at large t by the gluon distribution within a proton, i.e. it is unambiguously predicted within the DGLAP approximation including the stronger $W_{\gamma N}$ dependence at larger $-t$. This prediction explains recent HERA data. The calculations which follow perturbative Pomeron logic predict opposite trend of a weaker $W_{\gamma N}$ dependence at larger t. We explain that at the HERA energies double logarithmic terms characteristic for DGLAP approximation dominate in the hard processes as the consequence of the constraints due to the energy–momentum conservation. We give predictions for the ultraperipheral hard diffractive processes at the LHC and show that these processes are well suited for looking for the contribution of the single logarithmic terms due to the gluon emission in the multi-Regge kinematics. We also comment on the interrelation between energy and t dependence of the cross sections of the hard exclusive processes.

1. Introduction

The expansion of the pQCD into the domain of the small x processes requires accounting for the large double logarithmic terms $\alpha_s \log(x_0/x) \log(Q^2/Q_0^2)$. This can be done within the double logarithmic approximation (DLA) which follows from the DGLAP evolution dynamics. Single logarithmic terms $\alpha_s \log(x_0/x)$ arising from the gluon radiation in the multi-Regge kinematics are accounted for in the alternative perturbative Pomeron approximation. The latter approximation includes the LO double logarithmic terms also [1] but neglects the running of the coupling constant. The interrelation between these approximations has been understood recently and led to the resummation models [2–5]. In the resummation models the onset of the perturbative Pomeron occurs at extremely high energies.

The DGLAP evolution equation gives a good description of the HERA data on the structure function $F_{2p}(x, Q^2)$ [6] and the cross sections of hard exclusive processes observed at HERA [7–9]. In this Letter we consider the hard inelastic diffractive (HID) processes with large momentum transfer $-t$ and large rapidity gap, like $\gamma + p \to J/\psi + \text{rapidity gap} + X$ (Fig. 1), that were studied at HERA recently [10–12]. We show that the specific model independent properties of the DGLAP approximation which are absent in the pQCD calculations of Pomeron exchange at large $-t$ (cf. Refs. [13,14]) allow to describe the HERA data.

In the DGLAP approximation the amplitudes are rapidly increasing with the incident energy since the $\log(x_0/x)$ terms that define the energy dependence of the amplitudes, are multiplied by large logarithms that arise from the integration over parton transverse momenta. Consider now the diffraction processes with large momentum transfer defined above. It was understood recently [9] that the cross section for diffractive charmonium photoproduction off a parton does not increase with energy for $-t \geqslant M_{J/\psi}^2$ in striking contrast with a rapid increase of the exclusive charmonium photoproduction at $t = 0$ since logs arising from the integration over

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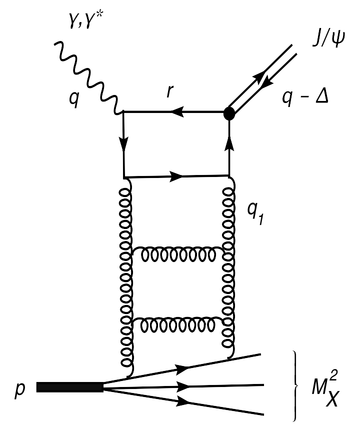

Fig. 1. The Feynman diagram describing the double diffractive process in the triple "Pomeron" limit in pQCD (there is also a cross diagram, not depicted explicitly.)

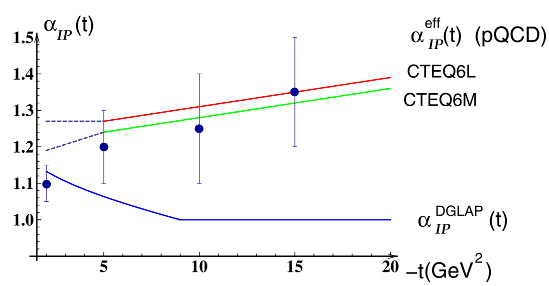

Fig. 2. The comparison between the experimental data and theoretical prediction for the HID cross section at HERA for the "effective Pomeron" $\alpha_{\mathbb{P}}^{\mathrm{eff}}(t)$, i.e. (1/2) logarithmic derivative of the cross section $d\sigma/dt$, obtained after integrating between the energy dependent cuts, as given in the text. The dashed curve means large theoretical uncertainties in the corresponding kinematic region. The values are given at for $W_{\gamma p} = 150$ GeV. In the same figure we depict also "true (DGLAP) Pomeron", i.e. logarithmic derivative $\alpha_{\mathbb{P}}(t)^{\mathrm{DGLAP}} = \frac{1}{2}\frac{d(\log(d^2\sigma/(dt\,dx_J)))}{d\log(x/x_J)}$ at this energy. $\Lambda_{\mathrm{QCD}} = 300$ MeV.

parton transverse momenta are $\log(M_V^2/(Q_0^2 - t))$, and thus disappear at $-t \sim M_V^2$. The quantitative theory of such phenomena was developed in Ref. [9], see also brief discussion below. This result is valid in all orders of DGLAP approximation and thus the cross section of diffractive charmonium production off a parton is energy independent at large $-t$.

The cross section of the HID processes depends on center of mass energy $W_{\gamma N}$ at large $-t$, since the kinematic range of the allowed invariant masses M_X^2 is increasing with energy due to the larger phase space available for a fragmentation of the knocked out parton. Parton fragmentation is described by the parton distribution function in a proton, see Fig. 1. The more rapid increase of the cross section with $W_{\gamma N}$ at large $-t$ is due to the more rapid increase of the gluon density, $xG(x, Q^2)$, at small x at larger Q^2. This behavior of $xG(x, Q^2)$ been predicted within the DGLAP approximation, and observed at HERA. Indeed, if we parametrize $xG_p(x, Q^2) \propto (x_0/x)^{\kappa(Q^2)}$, the exponent $\kappa(Q^2)$ is increasing with virtuality $\kappa(Q^2) = 0.048\log(Q^2/\Lambda^2)$ for $x \leqslant 0.01$ [15]. This property is absent within the concept of the Pomeron exchange. We find that it is this property that explains the recent HERA data on large $-t$ diffraction – see Fig. 2.

The dominance of the double logarithmic terms in a wide kinematic region in the two scale processes, shows that the multi-Regge dynamics could be revealed in the very special one scale processes where the Q^2 evolution is suppressed. The large $-t$ ultraperipheral processes at LHC represent an example of such phenomena. We explain that in the kinematical range $-t \geqslant M_{J/\psi}^2$ the double logarithmic (DL) terms are absent. So the HID phenomena represent a golden plated process for uncovering the onset of the pQCD Pomeron. Switching from HERA to ultraperipheral processes at LHC significantly increases the kinematical window allowed for the multi-Regge kinematics. In this case it is possible to select kinematics where a small dipole scatters off partons with rather large x, with up to 9–10 units in rapidity available for the gluons within the multi-Regge kinematics. An unambiguous signature of these gluons will be a rapid increase of the diffractive cross section with energy in the region where DGLAP predicts the energy independent cross section.

The amplitude for the exclusive process: $\gamma + p \to Z + p$ at nonzero t was studied in Ref. [16] a long time ago. Recent derivation of the amplitudes of the hard diffractive processes within the DGLAP approximation in Ref. [9] led to the significantly different dependence of these amplitudes on the $\alpha_s(M_{J/\psi}^2 - t)$ and on the collision energy, cf. the discussion in the end of the Letter.

The formulae for the cross sections of diffractive photoproduction of J/ψ off proton target have been suggested also in Ref. [17] within the dipole model generalized to include screening corrections and the processes of proton dissociation. These formulae differ from the ones which follow from the QCD factorization theorem and the DGLAP approximation [7,9]. They do not include photon scattering off a parton and its further fragmentation, and do not correspond to the triple "Pomeron" limit. As a result, the obtained dependence on the collision energy and on the hadron mass produced in the proton fragmentation significantly differs from the one obtained in Ref. [9]. The dependence of the QCD evolution on t is in variance with the dependence of amplitudes on the running coupling constant within the DGLAP approximation derived in Ref. [9]. The dependence of cross sections on the incident energy suggested in the model of [17] contradicts to experimental data, cf. discussion in Ref. [12].

The Letter is organized as follows. In Section 2 we compare the predictions of the DGLAP approximation for HID processes with the experimental data at HERA and show that the rate of increase of the cross section with $W_{\gamma p}$ predicted within DGLAP approximation including a more rapid $W_{\gamma p}$ dependence at larger $-t$ is in a good agreement with the experimental data. Note, that the indications of the slowing down of the energy dependence of the cross section of HID off gluon were first found in the analysis of experimental data in Ref. [18] where dependence on energy of elastic amplitude for the photo(electro)production processes off gluon was considered as a fitting parameter. In Section 3 we present illustrative estimate of the possible effects of multi-Regge dynamics in the ultraperipheral processes at LHC in the regime $-t \geqslant M_V^2$. In Section 4 we discuss the implications of our results for the study of the generalized parton distributions (GPDs).

2. DGLAP description of HID processes: Theory versus experiment

The pQCD description of HID processes was recently developed in Ref. [9]. The differential cross section in the kinematic range $-t \leqslant Q^2 + M_V^2$ was derived in Ref. [9] and is given by the following formula[1]:

$$\frac{d\sigma}{dt\,dx_J} = \Phi\left(t, Q^2, M_V^2\right)^2 \frac{(4N_c^2 I_1(u))^2}{\pi u^2} G(x_J, t). \qquad (1)$$

[1] At moderately large t we perform our calculation with double logarithmic accuracy only. Hence the scale associated with the $\gamma \to J/\psi$ vertex is not unambiguously defined. The analysis in Ref. [8] of the exclusive reaction $\gamma + p \to J/\psi + p$ found that the scale is smaller than $m_{J/\psi}^2$ – closer to $Q_{\mathrm{eff}}^2 \sim 3$ GeV^2. Changing $m_{J/\psi}^2 \to 3$ GeV^2 in the above formulae would result in the applicability of the obtained curves at smaller t.

Here

$$u=\sqrt{16N_c\log(x/x_J)\chi'},\quad \chi'=\frac{1}{b}\log\left(\frac{\log((Q^2+M_V^2)/\Lambda^2)}{\log(-t+Q_0^2)/\Lambda^2}\right),$$
$$x_J=-t/\left(M_X^2-m_p^2-t\right),\quad x\sim 3\left(Q^2+M_V^2\right)/(2s),$$
$$b=11-2/3N_f,\quad N_c=3,\quad s=W_{\gamma p}^2 \tag{2}$$

The factor $\Phi(t,Q^2,M_V^2)$ in Eq. (1) is the energy independent function, which depends on the details of the wave functions of the produced quarkonium and the photon. The second factor corresponds to the distribution of gluons in a parton, calculated in the DL approximation, The last factor in Eq. (1) is the gluon structure function of the nucleon that can be calculated using e.g. CTEQ6 data (we neglect small contribution of the quark sea). The function I_1 is the modified Bessel function. In this equation M_X^2 is the invariant mass of the hadronic system produced due to the diffractive dissociation of a proton (see Fig. 1), $\Lambda=300$ MeV, and $-t$ is the transverse momentum transfer. For the photoproduction processes we are going to discuss in this Letter the external photon is real: $Q^2=0$ and $V=J/\Psi$ to ensure presence of hard scale already for small t. In this case the energy dependence is the same for the production of the longitudinally and transversely polarized onium states. The analysis of light vector meson production requires a separate consideration because of the important role of configurations where one of the quarks carries nearly all vector meson momentum (the end point contribution). This contribution maybe significant at intermediate t though it is suppressed at large $-t$ by the Sudakov form factor, cf. [19].

Let us stress two characteristic features of Eq. (1). One is that at small $-t$ close to zero and fixed M_X^2 the rate of the increase of the cross section with energy for fixed M_X^2 is approximately double of the rate of the increase of the gluon density. Second is the absence of the energy dependence at $-t\geqslant M_V^2$. Indeed, for $t=0$ we obtain $\chi'=\chi$, i.e. the energy dependence is proportional to $I_1(u)^2/u^2$, instead $I_1(u)/u$ for the case of the total cross section, with the same u. On the other hand for $-t\geqslant M_V^2$ and fixed x_J cross section is independent on energy in the arbitrary order of the DGLAP approximation:

$$\frac{d^2\sigma}{dx_J\,dt}=\text{const.} \tag{3}$$

Let us compare now the theoretical prediction with the recent experimental data. This comparison, as it was first noted in Ref. [18], is not straightforward, since the HERA experiments, see e.g. [12], report the integral over invariant masses:

$$\frac{d\sigma(s,t)}{dt}=\int_{B(s)}^{A(s)}\frac{dM_X^2}{(M_X^2-t)^2}\frac{d^2\sigma}{dt\,dx_J}(x_J,s,t),$$
$$A(s)=0.05s-t,\quad B(s)=1\ \text{GeV}^2, \tag{4}$$

where we changed the integration variable from x_J to M_X^2. The characteristic feature of Eq. (4) is that the dependence of the cross section on $W_{\gamma p}$ comes from two sources: the energy dependence of the differential cross section for photon scattering off gluon, Eq. (1), and the $W_{\gamma N}$ dependence of the limits of integration over M_X^2. In particular, in the $-t\geqslant M_{J/\psi}^2$ kinematics where the cross section of photo(electro)production of vector meson from gluon target is energy independent, the dependence on $W_{\gamma p}$ of the experimentally measured quantity is given by

$$\frac{d\sigma(s,t)}{dt}=U(t)\int_{B(s)}^{A(s)}G\left(\frac{-t}{M_X^2-t},t\right)dM_X^2. \tag{5}$$

Here $U(t)$ is an energy independent function that can be expressed through the function Φ in Eq. (1). Thus the kinematic restrictions depend on the collision energy leading to the energy dependent cross section. As we already mentioned in the Introduction, both the HERA data and the DGLAP evolution equations show that the gluon pdf increases with the increase of the gluon virtuality Q^2 for $x\leqslant 0.5$ [15]. This property of the structure functions translates into a steeper energy dependence of $d\sigma/dt$ given by Eq. (5) with the increase of $-t$. In order to compare the theoretical prediction with the experimental data we calculated the integral (4) numerically for all $-t$. We present the main results of our calculations in Fig. 2. In order to compare with the experimental data we calculate the logarithmic derivative

$$I(s,t)=\frac{1}{2}\frac{d\log(d\sigma/dt)}{d\log(s)}, \tag{6}$$

for $s=2\cdot10^4$ GeV^2 (we denote this quantity as $\alpha_{\mathbb{P}}^{\text{eff}}(t)$), and compare it with the data for $I(s,t)$ presented in Fig. 9 in Ref. [12]. It is referred to in Ref. [12] as the "Pomeron" trajectory $\alpha_{\mathbb{P}}(t)-1$. We present our results calculated using Eqs. (1), (5) in Fig. 2 and compare them with the experimental data as reported in Ref. [12]. In the calculation we use CTEQ6M and CTEQ6L gluon parton distribution functions (pdf) $G(x_J,t)$, and neglect small contribution of the quark sea. For small $-t$ ($-t\sim 2$ GeV^2) the curves for such "effective Pomeron" are given by the dashed lines, since for these $-t$ the integration region includes the range of $x/x_J\sim 0.1$–1, where nondiagonal (GPD) effects not included in our treatment may be important. In addition, the gluon pdf's for moderate Q^2 are subject to significant uncertainties. The results are clearly within the experimental errors. For comparison we also depict in Fig. 2 the logarithmic derivative of the double differential cross section, given by Eq. (1), at $\tilde{W}^2=x_J\cdot W_{\gamma p}^2$, which corresponds to the "true" Pomeron in the triple Pomeron limit as given by Eq. (1).

Existing calculations of cross sections of large t diffractive processes within perturbative Pomeron hypothesis, cf. [14] predict qualitatively different interplay of t and W dependence. In particular, in difference from the DGLAP approximation they predicted that dependence on energy of the gluon pdf of the roton should be independent of Q^2. Also, perturbative Pomeron trajectory only weakly depends on $-t$, this dependence given by an expression $\alpha_{\mathbb{P}}(t)\sim(\alpha_s(M_V^2-t)/\alpha_s(M_V^2))\alpha_{\mathbb{P}}(t=0)$, cf. [13]. As a result perturbative Pomeron calculations predict the decrease with $-t$ of the rate of the increase of the cross section of $\gamma+p\to J/\psi+\text{rapidity gap}+X$ with energy. This prediction is in variance with the HERA data [12].

Observation that the pQCD Pomeron regime does not set in for the HERA kinematics can be understood as the consequence of the constraints due to the energy–momentum conservation [20–22]. Indeed, the concept of the pQCD Pomeron is based on the assumption of the dominance of the gluon radiation in the multi-Regge kinematics at high energies. However there must be at least 2–2.5 units in rapidity for each gluon radiation. This means that for the radiation of even one gluon in the multi-Regge kinematics a rapidity window of at least 4–5 units in rapidity is required. One should add to this interval the interval in rapidity characteristic for the fragmentation regions. At least 2–2.5 units due to the photon fragmentation plus region in rapidity occupied by DL terms plus a larger than 2–2.5 units interval for the proton fragmentation (proton fragmentation into small masses is suppressed by the power of t). Thus it seems that there is no room in the HERA kinematics for the gluon radiation in the multi-Regge kinematics. On the contrary the DGLAP approximation conserves longitudinal momentum, and the DL approximation is a good approximation to the DGLAP formulae at small x. The analysis carried in Ref. [2]

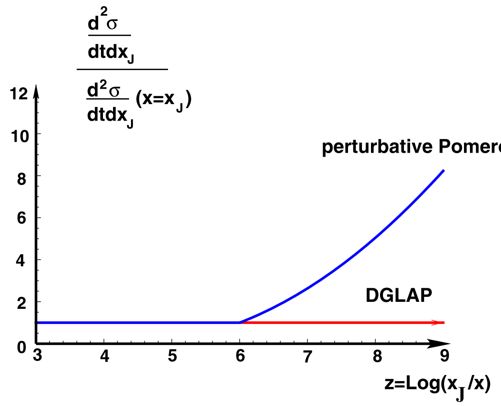

Fig. 3. The increase of the cross section $d^2\sigma/(dt\,dx_J)$ with energy ($z = \mathrm{Log}(x/x_J)$) at LHC in DGLAP and perturbative "Pomeron" scenarios (for fixed x_J).

within the resummation models has found also that the kernel (the amplitude for scattering of the gluon off a parton) is well described within the DGLAP approximation for reasonable values of $Q^2 \sim 20$–$50\ \mathrm{GeV}^2$, down to $x \sim 10^{-4}$–10^{-5}. In fact such a good agreement between theory and experiment can be considered as an experimental verification of the energy–momentum constraints on the gluon radiation within the multi-Regge kinematics.

3. Multi-Regge gluons in HID processes with large momentum transfer $-t$ at LHC

Consider now the ultraperipheral processes at the LHC (for a review see [24]). In this case one may have up to 14 units in rapidity, i.e. up to 9 units in rapidity may be available for a ladder describing gluon-parton scattering. As we have discussed above, for moderate $-t$ a significant part of this kinematic window will be filled by gluon radiation leading to the double logarithms. Different steps of the diffractive process occupy different regions of rapidity: first interval is fragmentation of virtual photon or heavy quarkonium which is dominated by gluon radiation within the DL kinematics. This range corresponds to the effective change of transverse momenta from $\sim (M_V^2 - t)/4$ in the impact factor to $\sim -t/4$. The rest of the evolution with energy will be determined by the radiation of gluons within the multi-Regge kinematics, corresponding to a single-scale process, whose amplitude does not contain double logarithms. A simple estimate suggests that for small $-t$ most of the kinematic range (even at the LHC) will be dominated by double logarithms. For larger $-t$ the precise kinematic range dominated by double logarithms would be smaller and can be estimated. However, we shall not need it, if we are interested in a model independent signature of the gluon radiation in the multi-Regge kinematics. Indeed, as it is clear from the previous subsection, for $-t \geqslant M_V^2$ the double logarithmic terms are absent, and the entire increase of the double differential cross section $d^2\sigma/(dt\,dx_J)$ will be due to multi-Regge gluons. In Fig. 3 we show (for illustrative purposes only), the behavior of the $d^2\sigma/(dt\,dx_J)$ as a function of energy using currently popular pQCD Pomeron models with intercept $\alpha_{\mathbb{P}}^{\mathrm{BFKL/resummed}}(t) \sim 1.25$ (note that this intercept value is only weakly dependent on the value of α_s for small $\alpha_s \sim 0.2$–0.25). One expects to see a rapid increase of cross section starting from $\log(x_J/x) \geqslant 5$. The absence of such a rise will be a sign that multi-Regge dynamics does not appear before the onset of the black limit, when the pQCD is not applicable any more.

Note, that comparing a rate of a rise of cross section in the ultraperipheral processes on nucleon and nucleus, it is possible to find the onset of the black disk regime [23].

4. Implications for the studies of GPDs in small x exclusive processes

The necessity to account for the DL terms in the QCD evolution at small x has important implications for the interpretation of the energy dependence of the $-t$ slope for exclusive vector meson production, DVCS studied at HERA.

Indeed the factorization theorem [25] which allows to express the amplitude through the convolution of the photon and the vector meson wave functions and the GPD was proven in the limit $(Q^2 + M_V^2) \gg -t$. In the limit when $-t$ is comparable to $(Q^2 + M_V^2)$, it requires modifications because of the disappearance of the difference in the scales.

For $-t$ close to zero the DGLAP evolution of α' which is present at the boundary condition leads to a rather slow variation of α' with Q^2 [26].

To be able to compare with experimental data for α'_{eff} we approximate the amplitude as $A(t = 0, s_0)\exp(\alpha'_{\mathrm{eff}} t \ln(s/s_0))$ and estimate:

$$\alpha'_{\mathrm{eff}} \sim \big(\alpha(0) - 1\big)/Q^2. \tag{7}$$

Note that the analysis of the J/ψ exclusive photoproduction suggests the effective scale of the order $Q^2_{\mathrm{eff}} \sim 3\ \mathrm{GeV}^2$ leading to

$$\alpha'_{\mathrm{eff}} \sim 0.07\ \mathrm{GeV}^{-2}. \tag{8}$$

The estimate of Eq. (8) should be compared to the experimental numbers measured at HERA by ZEUS [27] and H1 [28] Collaborations which agree well with each other. For example, in case of photoproduction ZEUS reported $\alpha' \sim 0.115 \pm 0.018\ \mathrm{GeV}^{-2}$ with $\alpha_{\mathbb{P}}$ for the last two points at $-t = 1.0, 1.3\ \mathrm{GeV}^2$ consistent with one. It is of interest also that the ZEUS and H1 experimental data do not contradict to a smaller value of α' for electroproduction of J/ψ. In particular ZEUS reports for $Q^2 = 6.8\ \mathrm{GeV}^2$, $\alpha' \sim 0.07 \pm 0.05\ \mathrm{GeV}^{-2}$ which is consistent with the pattern expected for the discussed effect.

The studies of 3D image of the nucleon at small x require reaching $-t \sim 2\ \mathrm{GeV}^2$ in the J/Ψ production in order to probe the gluon density at small impact parameters. It follows from our discussion that reaching the range of applicability of the factorization theorem [25] for such $-t$ will require Q^2 of the order of $10\ \mathrm{GeV}^2$.

5. Conclusion

We have shown that DGLAP predictions are in a good agreement with the behavior of HID processes observed at HERA. We found that the ultraperipheral collisions at LHC are a unique place where the onset of gluon radiation in the multi-Regge kinematics may be observed in the near future. The phenomenon of a strong suppression of the QCD evolution with increase of $-t$ (effectively presence of a kind of a wall between hard and soft regimes) has broader implications. It may result in the flattening of the effective Pomeron trajectory $\alpha_{\mathbb{P}}(t)$ at large $-t$ at a value close to one. The data on the process $\gamma + p \to \rho + p$ at $-t \geqslant 0.8\ \mathrm{GeV}^2$ are consistent with such a trend [29].

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