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Optimum design methodologies for pile foundations in London

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ABSTRACT

Given the importance of pile foundations in geotechnical engineering for supporting high-significance structures such as bridges, high-rise buildings, power plant stations, offshore platforms and museums, it becomes a necessity to find the best pile foundation design in terms of performance and economy. The number of piles required might exceed several hundreds or even thousands while the pile foundation cost might exceed 20% of the construction cost of the superstructure. In this work the problem of finding optimized designs of pile foundations is examined and is performed in accordance to two design code recommendations, namely Eurocode 7 and DIN 4014. The proposed structural optimization procedure is implemented in two real-world cases both located in London, UK in order to assess the efficiency of the proposed design formulation.

Introduction

Pile-supported structures are known to have existed in pre-historic times, references to cedar timber piles in Babylon can be found in the Bible. In the Middle Ages, pile foundations supported a wide assortment of structures particularly in Venice and in the Netherlands. Piled foundations are a convenient method for supporting structures built over water or where uplift loads must be resisted. Inclined or raking piles have been also used to resist lateral forces. Piles supporting retaining walls, bridge piers and abutments and machinery foundations resist both vertical and horizontal loads. The main types of piles used are driven piles, driven and cast-in-place piles, jacked piles, bored and cast-in-place piles and composite piles [1]. The first three of the above types are also called *displacement piles* since the soil is displaced as the pile is driven or jacked into the ground. In the case of bored piles, and in some forms of composite piles, the soil is first removed by boring a hole where concrete is placed or various types of precast concrete or other proprietary units are inserted.

Following the decision that piling is necessary, the engineer must make a choice from variety of types and sizes. Usually, there is only one type of pile which is satisfactory for a particular site condition [2]. In this work bearing piles will be examined although any type of piles may also be considered in the proposed formulation. *Bearing piles* are required when the soil at normal foundation level cannot support ordinary pad, strip, or raft foundations or where structures are sited on deep filling which is compressible and settling under its own weight.

The foundation cost, of real-world structural systems, can vary from 5% to 20% of the construction cost of the superstructure while the number of piles required might exceed several hundreds or even thousands. In the first part of this study the modelling of the soil-pile structure interaction using the finite element method is described while in the second part a formulation of an optimization problem is proposed, aiming at achieving the most economical-optimized design of the pile

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foundation layout. Two different design procedures are adopted and are incorporated in the optimization procedure: the German foundation code DIN 4014 [3] and the Eurocode 7 (EC7) [4] design procedures. Due to the nature of the problem, a mesh generator is used in order to create automatically the finite element mesh both for pile members and soil. Two real-world structures are considered for assessing the proposed formulation. In particular, the 16-storey and the 31-storey (Hiscocks House at Stonebridge Park and Hyde Park Cavalry Barracks both in London, UK) buildings are used as benchmark tests, for the comparative study and a significant reduction of the pile foundation cost is achieved. Although, the proposed framework is used for the design of building structures, it can also be applied with proper modifications implementing the requirements and specifications imposed for other type of structures (such as nuclear power stations, bridges etc.).

The design procedures

Two different design procedures are considered in this work in order to assess the performance of the designs obtained during the optimization process: the German foundation code DIN 4014 [3] and the Eurocode 7 [4]. Both standards are based on the following main design criteria: (i) axial bearing capacity, (ii) acceptable settlements, (iii) strength of pile as a structural element and (iv) lateral bearing capacity and acceptable horizontal displacements.

Although, both design codes provide design considerations for determining the pile resistances, comparing the two design codes it can be said that the implementation of a limit-state design procedure (suggested by the Eurocode 7) represents a significant change in the design philosophy of the DIN regulation. In particular the following limit-states should be considered and an appropriate list should be compiled (loss of overall stability, bearing resistance failure of the pile foundation, uplift or insufficient tensile resistance of the pile foundation, failure in the ground due to transverse loading of the pile foundation, structural failure of the pile in compression, tension, bending, buckling or shear, combined failure in the ground and in the pile foundation, combined failure in the ground and in the structure, excessive settlement, excessive heave, excessive lateral movement and unacceptable vibrations).

The expression used to calculate the ultimate bearing capacity of a single pile according to DIN 4014 is:

$$Q_u = Q_{su} + Q_{pu} \tag{1}$$

where Q_u is the ultimate bearing resistance of the pile, Q_{su} is the skin friction resistance load of the single pile while Q_{pu} is the point resistance load of the single pile and they are given by:

$$Q_{su} = \pi D \sum f_{su} \Delta z \tag{2}$$

$$Q_{pu} = A_p q_{pu} \tag{3}$$

where f_{su} is the ultimate skin friction resistance stress, q_{pu} is the ultimate point resistance stress. A_p is the pile base area, D is the pile shaft diameter and Δz is the effective length of the pile. The total allowable compressive load (Q_{all}) is calculated as follows:

$$Q_{all} = \frac{Q_u}{FS} \tag{4}$$

where a safety factor (FS) equal to 2 is used, according to DIN 1054 [5]. The ultimate bearing capacity of a pile group ($Q_{u,g}$) is given by the equation:

$$Q_{u,g} = N(Q_{pu} + f \cdot Q_{su}) \tag{5}$$

where $N = m \times n$ is the number of piles of the group, m is the number of rows and n is the number of columns and f is a reduction factor of the side friction resistance of the single pile, calculated from:

$$f = 1 - \frac{\theta}{90}(2 - 1/m - 1/n) \tag{6}$$

$$\theta = \arctan(D/s) \tag{7}$$

where s is the axial distance between the piles.

In the case of Eurocode 7, the design value of the ultimate pile resistance ($R_{u,d}$) is given by the following equation:

$$R_{u,d} = \frac{R_{pu,k}}{\gamma_{pR}} + \frac{R_{su,k}}{\gamma_{sR}} \tag{8}$$

where $R_{pu,k}$ and $R_{su,k}$ are the characteristic values of the base and shaft resistance, respectively, while the partial safety factors are set to $\gamma_{pR} = 1.6$ and $\gamma_{sR} = 1.3$. For the application of an axial loading V_k, the design value of an action F_d should be equal to:

$$F_d = \gamma_G \cdot P_k + \gamma_Q \cdot Q_k \tag{9}$$

where $P_k = 0.8 \cdot V_k$ and $Q_k = 0.2 \cdot V_k$ are the characteristic values of the permanent and variable actions respectively, while the corresponding partial safety factor are set to $\gamma_G = 1.0$ and $\gamma_Q = 1.3$ according to the factors R4 and formulation T4 of Eurocode 7 [4].

The soil-pile model

The main scope of the present study is to develop an automated numerical procedure that will provide optimized pile-group designs for a particular soil type and for a given axial load corresponding to the weight of the superstructure. The optimized pile-group design corresponds to the most cost-efficient foundation solution attempting a compromise between the number of piles, their diameter and length as well as the distance between them. Sand and clay are the two types of soil considered in this work discretized with four-node quadrilateral isoparametric elements are considered for the simulation of plies. In order to reduce the computational effort required during the optimization procedure 2D plane-strain analysis was performed.

For clay soil condition, an elastic–plastic material exhibiting plasticity in the deviatoric stress–strain response only is employed. The volumetric stress–strain response is linear-elastic and is independent of the deviatoric response. This material law can simulate monotonic or cyclic response of materials whose shear behaviour is insensitive to the confinement change, such as organic soils or clay under undrained loading conditions. During the application of gravity load, material behaviour is linear elastic. In the subsequent dynamic loading phase(s), the stress–strain response is considered elastic–plastic. Plasticity is formulated based on the multi-surface (nested surfaces) concept, with an associative flow rule. The yield surfaces are of the Von Mises type.

For sand soil conditions, an elastic–plastic material law is used for simulating the essential response characteristics of pressure sensitive soil materials under general loading conditions. Such characteristics include dilatancy (shear-induced volume contraction or dilation) and non-flow liquefaction, typically exhibited in sands or silts during monotonic or cyclic loading. As with the clay soil conditions, the material behaviour for the gravity loads is considered linear elastic, while for the subsequent earthquake loading phase, the stress–strain response is considered elastic–plastic. Plasticity is formulated based on the multi-surface concept, with a non-associative flow rule to reproduce dilatancy effect. The yield surfaces are of the Drucker–Prager type.

Nonlinear static or dynamic analysis needs a detailed simulation of the pile foundation in the regions where inelastic deformations are expected to develop within the pile. In order to consider the inelastic behaviour of the piles either the plastic-hinge or the fibre approach can be adopted [6]. The plastic hinge approach has limitations in terms of accuracy particularly in cyclic loading and therefore the fibre beam-column elements are preferred [7]. According to the fibre approach, each structural element is discretized into a number of integration sections restrained to the beam kinematics, and each section is divided into a number of fibres (Fig. 1) with specific material properties (A_{fib}, E_{fib}). Every fibre in the section can be assigned to different material properties, e.g. concrete, structural steel, or reinforcing bar material properties. The sections are located at the Gaussian integration points of the elements. The main advantage of the fibre approach is that every fibre has a simple uniaxial material model allowing an easy and efficient implementation of the inelastic behaviour. This approach is considered to be suitable for inelastic beam-column elements under dynamic loading and provides a reliable solution compared to other formulations.

In the numerical test examples section that follows all analyses have been performed using the OpenSees [8] platform. A bilinear material model with pure kinematic hardening is adopted for the steel reinforcement of the piles. For the simulation of the concrete the modified Kent and Park [9] model, as extended by Scott et al. in [10], is employed. This model was chosen because it allows for an accurate prediction of the demand for flexure-dominated RC members despite its relatively simple formulation. The transient behaviour of the reinforcing bars was simulated with the Menegotto–Pinto model [11]. More information about the stress–strain relations can be found in [12].

Spring elements are implemented for modelling the interaction between piles and the surrounding soil, in order to simulate the soil-pile interface. Without the use of springs the soil and pile elements move together when subjected to any loading or ground motion. With the use of these springs a more realistic model is achieved and the relative displacements between the soil and each pile can be simulated. T_z springs were used for the vertical components of the pile interface and P_y springs for the horizontal components (Fig. 2) [13]. More information about the determination of the springs' stiffness and the corresponding values can be found in [12,14]. All the nodes of the ground base are fully constrained in both x (horizontal) and y (vertical) directions, while the side boundaries are constrained in the x (horizontal) direction (Fig. 3).
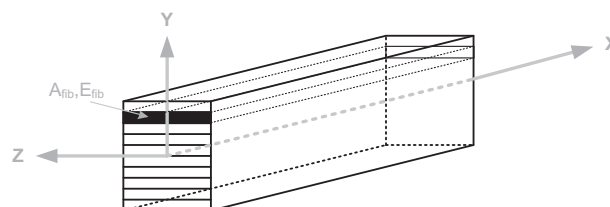

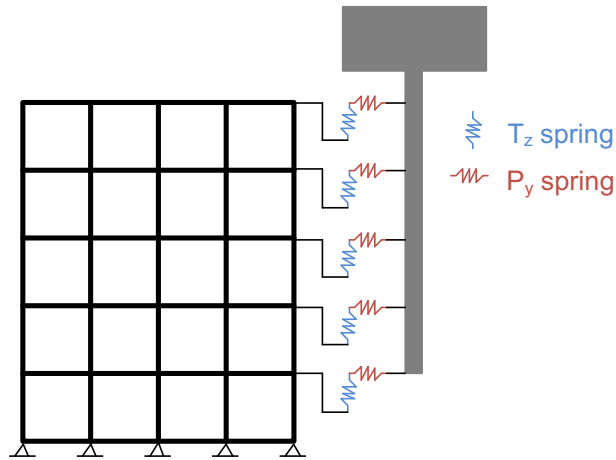**Fig. 1.** Modelling of the inelastic behaviour – the fibre approach.

Fig. 2. Components of the soil-pile interface (T_z and P_y springs).
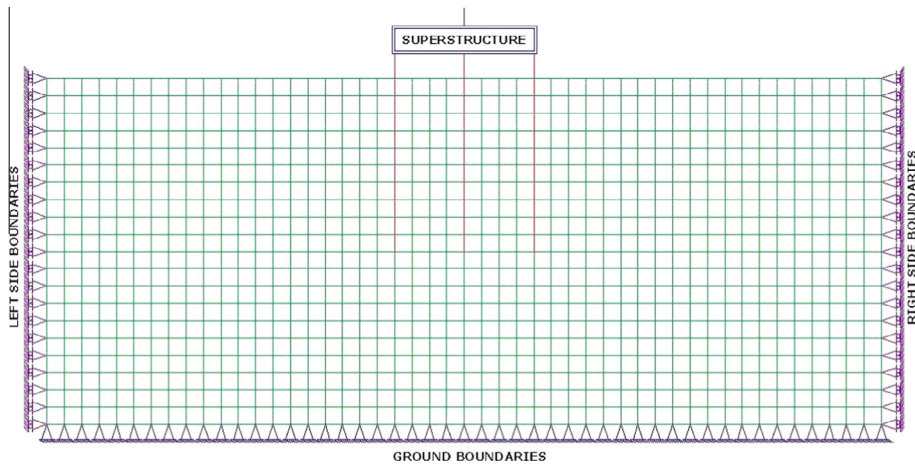

Fig. 3. Boundary constraints.

Numerical tests

Formulation of the optimization problem

In order to avoid a trial and error procedure and to obtain the best possible design under the constraints of the code requirements, a size and topology pile foundation design optimization problem is formulated as follows:

$$
\begin{aligned}
&\min_{\mathbf{s} \in F} \quad C_{pile}(\mathbf{s}) \\
&\text{where} \quad C_{pile}(\mathbf{s}) = C_{concrete}(\mathbf{s}) + C_{steel}(\mathbf{s}) + C_{labour}(\mathbf{s}) \\
&\text{subject to} \quad g_j(\mathbf{s}) \leqslant 0 \; j = 1, \ldots, k
\end{aligned} \tag{10}
$$

where $\mathbf{s}$ represents the design vector corresponding to the cross-sectional dimensions of the columns, F is the feasible region where all the constraint functions g_j are satisfied. The objective function considered is the construction cost C_{pile} of the pile foundation. $C_{pile}(\mathbf{s})$ refers of the total construction cost for the foundation, while $C_{concrete}(\mathbf{s})$ and $C_{steel}(\mathbf{s})$ refer to the total cost for the concrete and the reinforcement of the piles, respectively. The cost includes the material cost of concrete and steel reinforcement as well as the labour cost ($C_{labour}(\mathbf{s})$). The design variables of the optimization problem are the pile length (L_{pile}), the pile diameter (D) and the number of piles (N_{piles}), while the constraint functions considered are: (i) pile diameter D: $0.80 \leqslant D \leqslant 2.20$, (ii) axial distance between the piles s: $2.5D \leqslant s \leqslant 6D$, (iii) length of piles L_{pile}: $\min\{5.00 \text{ m}, 5D\} \leqslant L_{pile} \leqslant 40.00$ m and (iv) maximum settlement of the pile-head $\delta_x(\max) = 2$ cm.
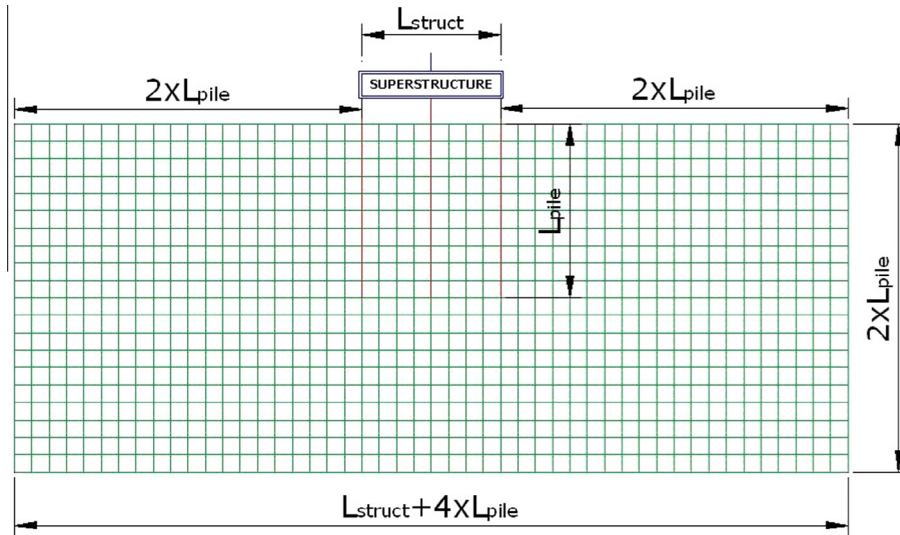
Fig. 4. Mesh dimensions.

Fig. 5. The Hiscocks House at Stonebridge Park (Tower A).

Mesh generation

Due to the nature of the problem, a dynamic mesh generator is required in order to create a finite element mesh both for pile members and soil. This is because different number of piles is assigned to each candidate optimum design that is encountered during the optimization procedure. For a given superstructure of a particular width L_{struct}, a number of piles is assigned beneath the superstructure. The length of each pile L_{pile} changes along with its diameter D and the number of piles N_{piles} until the optimization process converges to the optimal design. Thus, both horizontal (L_x) and vertical (L_y) dimensions of the mesh, change for each new design.

Fig. 6. The Hyde Park Cavalry Barracks (Tower B).

Table 1
Bored cast-in-place pile prices (in € per pile length).

| Pile diameter (m) | Project budget: cost in EURO | | |
| | $C \leqslant 5 \times 10^3$ k€ | 5×10^6 € $\leqslant C \leqslant 10 \times 10^3$ k€ | $C \geqslant 10 \times 10^3$ k€ |
	Pile price in €/m		
0.60	78.00	73.00	70.00
0.80	89.00	84.00	80.00
1.00	107.00	104.00	100.00
1.20	144.00	137.00	130.00
1.50	185.00	175.00	165.00
1.80	200.00	190.00	180.00
2.20	220.00	210.00	200.00
*Intermediate values may be obtained by linear interpolation			
	Steel price in €/kg		
Steel S500s	0.95	0.90	0.85

The total width L_x of the mesh is defined by the relations:

$$L_x = L_{struct} + 4 \cdot L_{pile}$$
$$\text{when } L_{pile} \geqslant L_{struct} \tag{11}$$

or

$$L_x = 5 \cdot L_{struct}$$
$$\text{when } L_{pile} < L_{struct} \tag{12}$$

where for each side of the structure, the mesh is extended by two times the length of the piles, and the total vertical dimension L_y of the mesh is assumed two times the length of the piles (Fig. 4):

$$L_y = 2 \cdot L_{pile} \tag{13}$$

Case studies

Two real-world buildings have been considered: The 16-storey Hiscocks House at Stonebridge Park founded on sand (Fig. 5) [15], and the 31-storey building of the Hyde Park Cavalry Barracks founded on clay (Fig. 6) [16], both in London. The overall loading of the pile group for the first test example is about 200 kN/m^2 that represents the uniformly distributed load to the foundation while at the end of the construction the piles carried 78% of the total building load and the remainder is carried by the raft. The second building is of 90 m height and its weight was calculated to be 228 MN. It is estimated that at the end of construction 60% (0.60 × 228 MN = 136.80 MN) of the building load is carried by the piles and 40% by the raft as denoted in [15,16]. A general stiff clay soil type was considered with saturated soil mass density equal to 18 kN/m^3, apparent cohesion at zero effective confinement equal to 75 kPa, reference low-strain shear modulus equal to 1.5 × 10^5 kPa and reference bulk modulus equal to 7.5 × 10^5 kPa.

The solution of the optimization problem described in Eq. (13) is performed with the evolutionary algorithms algorithm [17] described previously. The prices of the cost for a single pile are indicative, since the price of concrete, reinforcement steel and fuel as well as the labour cost varies from country to country. The comparison of the cost though, between the optimum designs and the implemented ones for the needs of this study, is conducted based on the same cost calculation procedure and pricelist. Table 1 provides indicative values for bored cast-in-place pile prices for three different categories of public works according to the total budget of the project that have been used in this study. Additional costs like VAT are not included in the costs given in Table 1 while the general expenses and the contractor's profit are defined at 18%. Furthermore a possible cost revision of the project estimated at 3% is also considered.

For the implementation of the optimization algorithm a (10 + 10) EA scheme is adopted where the number of parents and offsprings is equal to 10, while the termination criterion is 10 generations with no improvement. The iteration histories of the value of the objective function at each generation step are shown in Figs. 7 and 8 for the two test examples, respectively, for the case of the DIN design procedure. As it can be seen in both optimization runs the algorithm has converged to an optimized design in almost 5 generations of the optimization procedure. Comparing the construction cost of the initial design to the optimized design a reduction of almost three times is achieved, comparing the cost of the initial design of 7450 K€ with that of the optimized design of 501 K€ for the Tower A and the cost of the initial design of 1490 K€ with that of the optimized design of 244 K€ for the Tower B. The optimum designs obtained according to DIN and EC7 design codes are given in Tables 2
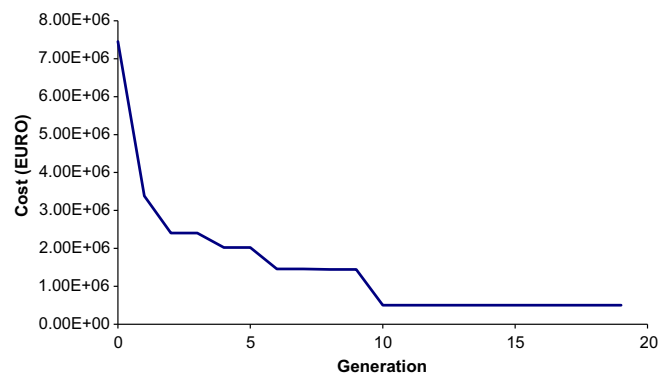

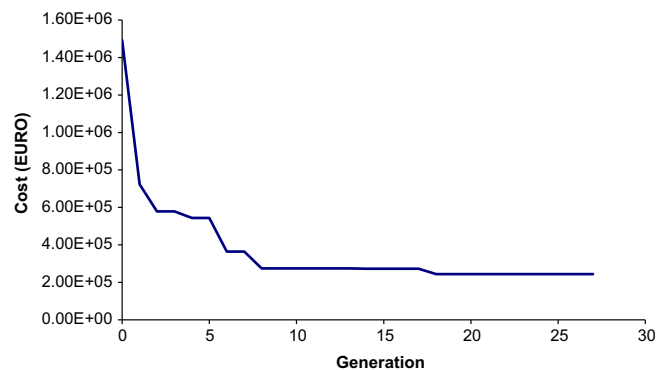**Fig. 7.** Tower A – optimization history for the DIN design procedure.

Fig. 8. Tower B – optimization history for the DIN design procedure.

Table 2
Tower A – comparison of the optimized with the standard design procedures.

Standard	Design	Cost (K€)
Implemented design [15]	D = 0.45 m, N_{piles} = 351, L = 13.0 m	500
DIN (optimized)	D = 1.5 m, N_{piles} = 64, L = 11.0 m	501
EC7 (optimized)	D = 1.6 m, N_{piles} = 64, L = 9.0 m	455

Table 3
Tower B – comparison of the optimized with the standard design procedures.

Standard	Design	Cost (K€)
Implemented design [16]	D = 0.91 m, N_{piles} = 51, L = 23.5 m	367
DIN (optimized)	D = 1.50 m, N_{piles} = 49, L = 7.0 m	244
EC7 (optimized)	D = 1.70 m, N_{piles} = 49, L = 8.0 m	342

and 3, for the two test examples respectively. For the needs of this study the implemented designs obtained from the literature [15,16] are adopted for comparison. It should be mentioned, though, that the exact soil conditions were not known in detail, since the soil data that were available to the design engineers were not accessible for our study. Therefore, the results of the comparison between the optimized designs and the implemented ones are affected by the insufficient soil data and the inconsistency of the pricelist used for the design optimization procedure and that used during the design and construction procedure of the two buildings considered. For the case of Tower A test case, the DIN design procedure leads to a design with almost the same cost with that of the original design, while EC7 leads to a reduction of 10%. On the other hand, in the case of Tower B test case, it can be seen the design obtained through EC7 standard seems to be more conservative in this test case compared to DIN. Thus the cost of the foundation corresponding to the EC7 optimum design is reduced by almost 7% compared to the original design while the DIN design is almost 33% less expensive compared to the original one. The different performance of the two design procedures is attributed to the different soil conditions for the two test examples considered, while the discrepancies noticed with reference to the implemented design it is due to the insufficient soil data that were used for the numerical investigation.

Conclusions

The main objective of our study is to present the advantages of using search algorithms into real world problems. In particular, two real world test cases found in the literature were used in order to implement a design framework formulated as an optimization problem for the design of pile foundations. In order to present the efficiency of our design framework and prove that it is not dependent on the design code used, Eurocode and DIN regulations are employed.

In this work the problem of defining the best pile foundation is formulated as a combined sizing and topology optimization problem with the aim of achieving the most economical design of the pile foundation. The design variables considered are related to both the dimensions and the number of the piles assigned to the foundation. In order to assess the efficiency of the problem formulation and the optimization algorithm adopted to deal with, two real-world problems have been considered as test cases. The main findings of this work can be summarized as follows:

- Through the implementation of the proposed formulation in two different soil conditions, it was found that depending on the soil conditions DIN design procedure can become more conservative compared to the EC7 and vice versa.
- Furthermore, through the proposed formulation pile foundation designs are achieved which are more economic compared to the foundation solutions implemented in the original design. The benefit in terms of construction cost of the optimum designs varies from 7% to 33% compared to original solutions.

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