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Electroweak chiral Lagrangian for left–right symmetric models: The matter sector

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Abstract

The matter sector of electroweak chiral Lagrangian up to dimension four operators for left–right symmetric models with a neutral light Higgs is provided. The connection of these operators to Yukawa couplings, anomalous gauge couplings and parameters in the matter sector of conventional electroweak chiral Lagrangian is made. It is shown that there exists proper parameter space to loosen constraint for the mass of right handed gauge boson from the mass difference of neutral K meson.

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Left–right-symmetric models (LRSMs) [1] as a viable extensions of the Standard Model (SM) provide not only a spectra of new physics at a high energy scale, but also an appealing notion of a spontaneous origin for parity and CP non-conservation. In various version of LRSMs, the most complex issue is the content of Higgs fields. Different representations of Higgs fields play different roles in physics and analysis on the minimum of the Higgs potential is complicated and difficult to handle. Since traditional model with one Higgs bi-doublet was strongly constrained [2], people are led to consider one Higgs bi-doublet LRSM with general CP violation [3] instead of spontaneous CP violation or to replace one Higgs bi-doublet by two Higgs bi-doublets in LRSM [4]. In order to avoid the complexity of Higgs sector in LRSMs and investigate the situation in which right handed gauge bosons have relative low masses, in Ref. [5], we built up an electro-weak chiral Lagrangian (EWCL) to describe LRSMs. This EWCL, which include right handed gauge bosons $W_R^{\pm}$, Z_R^0 and corresponding Goldstone bosons, is a generalization of original nonlinearly realized extended EWCL for SM fields [6]. The origin of these right handed particles in our EWCL is expected but need not to be from spontaneous parity violation (SPV) of an underlying LRSM. They can be the low energy remnants of some fundamental interactions at high energy region. We assume that beyond the particles already discovered in past high energy experiments, the lightest new particles are a light neutral Higgs and right hand $W_R^{\pm}$, Z_R^0. These right handed gauge bosons are expected to be discovered in future collider experiments. This scenario anticipate that when we approach to TeV energy region, the right hand gauge bosons will play the most important roles and display their experimental signatures earlier than most of Higgs bosons except the lightest one. The symmetry realization pattern in our EWCL is generalized from original $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}} \to U(1)_{\mathrm{em}}$ to $SU(2)_{\mathrm{L}} \otimes SU(2)_{\mathrm{R}} \otimes U(1)_{\mathrm{B-L}} \to SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}} \to U(1)_{\mathrm{em}}$. In Ref. [5], as the first step of investigation, bosonic part of interactions involving pure gauge fields and corresponding Goldstone fields of EWCL are constructed. It is purpose of this letter to extend discussions of Ref. [5] to matter part which involve fermions and for simplicity, we limit ourselves in dimension three and four operators. From phenomenological point of view, matter part of EWCL is of special importance. The most stringent

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constraints on LRSMs come from this part, such as $K^0 - \bar{K}^0$ mass difference. Without fermions, the only strong constraint for interactions among boson fields is the mixing between left and right gauge fields [7] which, as discussed in Ref. [5], can be satisfied for relative lower mass ~ 800 GeV of right handed W boson by suitably choice of parameter $\kappa \sim 10^{-2}$ and $f_L g_L \sim 10^{-1} f_R g_R$. The other constraints like those from S, T, U parameters [8] for gauge bosons are not strong enough to constrain LRSMs. Our EWCL containing matter sector provides a model independent platform to investigate phenomenological possibility evading experiment constraints. We can search for the parameter space in our EWCL satisfying these constraints. Only in the case that there is no parameter space to satisfy the constraints, our scheme is then denied in physics which imply that it is impossible in the real physical world that right handed gauge particles are lowest new particles beyond SM ones. As long as there exist some parameter space satisfying the constraints, physically our scheme is allowed and the next step of investigation will be the search for models which can generate the parameters in the allowed space. In this sense, our formulation in terms of constraints will provides hints for future model building.

Before starting to construct matter part of our EWCL, we first give a short review of its bosonic part built in Ref. [5]. Let B_μ, $W^a_{L,\mu}$, $W^a_{R,\mu}$ be electroweak gauge fields ($a = 1, 2, 3$) and two by two unitary unimodular matrices U_L and U_R be corresponding Goldstone boson fields, h be neutral Higgs field which is singlet of $SU(2)_{\rm L} \otimes SU(2)_{\rm R} \otimes U(1)_{\rm B-L}$ group. Consider covariant derivatives for Goldstone fields $D_\mu U_i = \partial_\mu U_i + i g_i \frac{\tau^a}{2} W^a_{i,\mu} U_i - i g U_i \frac{\tau_3}{2} B_\mu$ for $i = L, R$ and building blocks $X^\mu_i \equiv U^\dagger_i (D^\mu U_i)$, $\bar{W}_{i,\mu\nu} \equiv U^\dagger_i g_i W_{i,\mu\nu} U_i$ for $i = L, R$. The lowest order of chiral Lagrangian is the Higgs potential $\mathcal{L}_0 = -V(h)$ and p^2 order of Lagrangian is

$$\mathcal{L}_2 = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4} f_L^2 \operatorname{tr}\big(X_{L,\mu} X^\mu_L\big) - \frac{1}{4} f_R^2 \operatorname{tr}\big(X_{R,\mu} X^\mu_R\big) + \frac{1}{2}\kappa f_L f_R \operatorname{tr}\big(X^\mu_L X^\mu_R\big)$$
$$+ \frac{1}{4}\beta_{L,1} f_L^2 \big[\operatorname{tr}\big(\tau^3 X_{L,\mu}\big)\big]^2 + \frac{1}{4}\beta_{R,1} f_R^2 \big[\operatorname{tr}\big(\tau^3 X_{R,\mu}\big)\big]^2 + \frac{1}{2}\tilde{\beta}_1 f_L f_R \big[\operatorname{tr}\big(\tau^3 X_{L,\mu}\big)\big]\big[\operatorname{tr}\big(\tau^3 X^\mu_R\big)\big]. \tag{1}$$

p^4 order Lagrangian can be divided into six parts $\mathcal{L}_4 = \mathcal{L}_K + \mathcal{L}_L + \mathcal{L}_{HL} + \mathcal{L}_R + \mathcal{L}_{HR} + \mathcal{L}_C$ with kinetic part $\mathcal{L}_K$, left (right) part without differential of Higgs field $\mathcal{L}_L$ ($\mathcal{L}_R$), left (right) part with differential of Higgs field $\mathcal{L}_{HL}$ ($\mathcal{L}_{HR}$) and crossing part $\mathcal{L}_C$. The detail expressions are already given in Ref. [5]. These interaction terms include all possible p^4 order CP-conserving and CP-violating operators with all coefficients being functions of Higgs field h. Left–right symmetry will be explicitly realized for the theory if all coefficients with subscript L are equal to their right handed partners denoted with subscript R. If they are not equal to each other, the left–right symmetry is violated by some underlying dynamics and the differences between left and right coefficients then characterize the strength of left–right symmetry violation.

With convention $W^\pm_{i,\mu} = \frac{1}{\sqrt{2}}(W^1_{i,\mu} \mp i W^2_{i,\mu})$, $i = L, R$, the mass terms in our bosonic part EWCL is

$$\mathcal{L}_M = \frac{1}{4} f_L^2 g_L^2 W^+_{L,\mu} W^{-,\mu}_L + \frac{1}{4} f_R^2 g_R^2 W^+_{R,\mu} W^{-,\mu}_R - \frac{1}{2}\kappa f_L f_R g_L g_R \big(W^+_{L,\mu} W^{-,\mu}_R + W^+_{R,\mu} W^{-,\mu}_L\big)$$
$$+ \frac{1}{8}(1 - 2\beta_{L,1}) f_L^2 \big(g_L W^3_{L,\mu} - g B_\mu\big)^2 + \frac{1}{8}(1 - 2\beta_{R,1}) f_R^2 \big(g_R W^3_{R,\mu} - g B_\mu\big)^2$$
$$- \frac{1}{4}(\kappa + 2\tilde{\beta}_1) f_L f_R \big(g_L W^3_{L,\mu} - g B_\mu\big)\big(g_R W^{3,\mu}_R - g B^\mu\big). \tag{2}$$

The charged and neutral gauge bosons are diagonalized through rotations

$$\begin{pmatrix} W^\pm_L \\ W^\pm_R \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W^\pm_1 \\ W^\pm_2 \end{pmatrix}, \qquad \begin{pmatrix} W^3_{L,\mu} \\ W^3_{R,\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ v_1 & v_2 & v_3 \end{pmatrix} \begin{pmatrix} Z_{1,\mu} \\ Z_{2,\mu} \\ A_\mu \end{pmatrix} \tag{3}$$

with mixing parameters given by

$$\tan 2\zeta = \frac{2\kappa f_L f_R g_L g_R}{f_R^2 g_R^2 - f_L^2 g_L^2}, \qquad \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ v_1 & v_2 & v_3 \end{pmatrix} = V \Lambda \tilde{V}, \tag{4}$$

and

$$\begin{pmatrix} 1 - \alpha_{L,8} g_L^2 & -\frac{1}{2}\tilde{\alpha}_8 g_L g_R & -\alpha_{L,1} g_L g \\ -\frac{1}{2}\tilde{\alpha}_8 g_L g_R & 1 - \alpha_{R,8} g_R^2 & -\alpha_{R,1} g_R g \\ -\alpha_{L,1} g_L g & -\alpha_{R,1} g_R g & 1 \end{pmatrix} = V \begin{pmatrix} \lambda_+ & 0 & 0 \\ 0 & \lambda_- & 0 \\ 0 & 0 & 1 \end{pmatrix} V^T,$$

$$\lambda_\pm = 1 - \frac{1}{2}\alpha_{L,8} g_L^2 - \frac{1}{2}\alpha_{R,8} g_R^2 \pm \bigg[\alpha^2_{L,1} g_L^2 g^2 + \alpha^2_{R,1} g_R^2 g^2 + \frac{1}{4}\tilde{\alpha}_8^2 g_L^2 g_R^2 + \frac{1}{4}\big(\alpha_{L,8} g_L^2 - \alpha_{R,8} g_R^2\big)^2\bigg]^{1/2}.$$

$$\Lambda V^T \tilde{M}_0^2 V \Lambda = \tilde{V} \begin{pmatrix} M^2_{Z_1} & 0 & 0 \\ 0 & M^2_{Z_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tilde{V}^T, \qquad \Lambda \equiv \begin{pmatrix} \frac{1}{\sqrt{\lambda_+}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda_-}} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\tilde{M}_0^2 = \begin{pmatrix} \frac{1}{4}(1-2\beta_{L,1})f_L^2 g_L^2 & -\frac{1}{4}(\kappa+2\tilde{\beta}_1)f_L f_R g_L g_R & [(2\beta_{L,1}-1)f_L+(\kappa+2\tilde{\beta}_1)f_R]\frac{f_L g_L g}{4} \\ -\frac{1}{4}(\kappa+2\tilde{\beta}_1)f_L f_R g_L g_R & \frac{1}{4}(1-2\beta_{R,1})f_R^2 g_R^2 & [(2\beta_{R,1}-1)f_R+(\kappa+2\tilde{\beta}_1)f_L]\frac{f_R g_R g}{4} \\ [(2\beta_{L,1}-1)f_L+(\kappa+2\tilde{\beta}_1)f_R]\frac{f_L g_L g}{4} & [(2\beta_{R,1}-1)f_R+(\kappa+2\tilde{\beta}_1)f_L]\frac{f_R g_R g}{4} & [\frac{1-2\beta_{L,1}}{2}f_L^2+\frac{1-2\beta_{R,1}}{2}f_R^2-(\kappa+2\tilde{\beta}_1)f_L f_R]\frac{g^2}{2} \end{pmatrix}.$$

For above formulae in original Ref. [5], there exist some typos which are corrected now.

With above preparations, we are ready to construct matter part of EWCL. The conventional matter part of EWCL for discovered particles are already set up in Ref. [9]. Now we are going to add into it Higgs field, right handed gauge bosons and corresponding Goldstone bosons. The fermion fields involve quarks and leptons which are denoted by left and right hand doublets $q_{\alpha L,R} = \binom{u_{\alpha L,R}}{d_{\alpha L,R}}$ and $l_{\alpha L,R} = \binom{\nu_{\alpha L,R}}{e^-_{\alpha L,R}}$ with index α labeling three families. We limit in the matter part of EWCL with lowest dimensionality, i.e., dimension three Yukawa type interactions and dimension four gauge interactions.

A. Dimension three Yukawa type interactions

We first discuss lepton part. The sterile neutrino is not included in our theory. The most general dimension three Yukawa type interactions for leptons is

$$\mathcal{L}_{\mathrm{Y,lepton}} = \bar{l}_{\alpha L}\big[U_L\big(y^{\alpha\beta}+y_3^{\alpha\beta}\tau^3\big)U_R^\dagger\big]l_{\beta R} + \frac{1}{2}\big[h_L^{\alpha\beta} l_{\alpha L}^T U_L^* \mathcal{C}\big(1+\tau^3\big)U_L^\dagger l_{\beta L} + (L\to R)\big] + \text{h.c.}, \tag{5}$$

where $h^{\alpha\beta}_{L,R}$ are Hermitian functions of Higgs field h. $\mathcal{C}$ is charge conjugate matrix.

In a simplified one generation situation, $h_{L,R}$ are real functions of Higgs field. If we take unitary gauge and let Higgs field inside coefficients $y+y_3$, h_L and h_R be in its vacuum expectation value, one generation Lagrangian is reduced to

$$\mathcal{L}_{\mathrm{Y,lepton}}|_{U_L=U_R=1} = \mathcal{L}_{Me} + \mathcal{L}_{M\nu}. \tag{6}$$

Where $\mathcal{L}_{Me} = \overline{e^-}_L(y-y_3)e^-_R + \overline{e^-}_R(y-y_3)e^-_L$ is electron mass term and $\mathcal{L}_{M\nu} = (y+y_3)(\bar{\nu}_L\nu_R+\bar{\nu}_R\nu_L) + h_L(\nu_L^T\mathcal{C}\nu_L - \nu_L^\dagger\mathcal{C}\nu_L^*) + h_R(\nu_R^T\mathcal{C}\nu_R - \nu_R^\dagger\mathcal{C}\nu_R^*)$ is neutrino mass term. The parameter $y-y_3$ is electron mass, while the other three parameters $y+y_3, h_L, h_R$ are responsible for neutrino mixing and two neutrino masses. To work out their values, we introduce two self-conjugate spinors $\nu \equiv \frac{1}{\sqrt{2}}(\nu_L + \mathcal{C}\bar{\nu}_L^T)$ and $N \equiv \frac{1}{\sqrt{2}}(\nu_R+\mathcal{C}\bar{\nu}_R^T)$ which lead to $\bar{\nu} = \frac{1}{\sqrt{2}}(\nu_L^\dagger - \bar{\nu}_L^*\mathcal{C})\gamma^0$ and $\bar{N} = \frac{1}{\sqrt{2}}(\nu_R^\dagger - \bar{\nu}_R^*\mathcal{C})\gamma^0$. Then neutrino mass term can be rewritten as $(\bar{\nu}\quad \bar{N})\begin{pmatrix} 2h_L & y+y_3 \\ y+y_3 & 2h_R\end{pmatrix}\binom{\nu}{N}$ which results in mixing angle between ν, N fields $\frac{1}{2}\arctan(y+y_3)/(h_L-h_R)$ and two eigenvalues of neutrino masses $h_L+h_R \mp \sqrt{(h_L-h_R)^2+(y+y_3)^2}$. If we have parity symmetry $h_R = h_L$, mixing angle becomes $\pi/4$ and eigenvalues become $2h_L \mp (y+y_3)$. In another extreme seesaw case $h_R \gg h_L$, $y+y_3$, we have a very small mixing angle $-(y+y_3)/2h_R$, a physical light neutrino mass $m_\nu \simeq 2h_L - \frac{(y+y_3)^2}{2h_R}$ and heavy neutrino mass $m_N \simeq h_R$.

Coming back to general situation for three generations, in unitary gauge and taking Higgs field in its vacuum expectation value, (6) is still correct with

$$\mathcal{L}_{Me} = \overline{e^{-I}}_{\alpha L}\big(y^{\alpha\beta}-y_3^{\alpha\beta}\big)e^{-I}_{\beta R} + \overline{e^{-I}}_{\alpha R}\big(y^{\dagger\alpha\beta}-y_3^{\dagger\alpha\beta}\big)e^{-I}_{\beta L}, \tag{7}$$

$$\mathcal{L}_{M\nu} = \overline{\nu^I}_{\alpha L}\big(y^{\alpha\beta}+y_3^{\alpha\beta}\big)\nu^I_{\beta R} + h_L^{\alpha\beta}\nu^{IT}_{\alpha L}\mathcal{C}\nu^I_{\beta L} - h_L^{\dagger\alpha\beta}\nu^{I\dagger}_{\alpha L}\mathcal{C}\nu^{*I}_{\beta L} + L \leftrightarrow R, \tag{8}$$

which are most general mass terms for leptons. The symbol "I" indicates that they are gauge eigenstates. Since neutrino fields are anti-commuting and $\mathcal{C}$ is anti-symmetric, we have $\nu^{IT}_{\alpha L}\mathcal{C}\nu^I_{\beta L} = \nu^{IT}_{\beta L}\mathcal{C}\nu^I_{\alpha L}$ and $\nu^{IT}_{\alpha R}\mathcal{C}\nu^I_{\beta R} = \nu^{IT}_{\beta R}\mathcal{C}\nu^I_{\alpha R}$ which lead to $h^{\alpha\beta}_{L,R} = h^{\beta\alpha}_{L,R}$. Rotating the gauge eigenstates into the mass eigenstates with unitary matrices $\tilde{V}^e$ by $e^-_{L,R} = \tilde{V}^e_{L,R}e^{-I}_{L,R}$, we can reduce (7) to $\mathcal{L}_{Me} = \overline{e^-}_L\mathbf{M}^e e^-_R + \overline{e^-}_R\mathbf{M}^{e\dagger}e^-_L$ with diagonal mass matrix $\mathbf{M}^e = \tilde{V}^e_L(y-y_3)\tilde{V}^{e\dagger}_R$. Similar to that of one generation case, we can introduce $\nu^I_\alpha \equiv \frac{1}{\sqrt{2}}(\nu^I_{\alpha L} + \mathcal{C}\bar{\nu}^{IT}_{\alpha L})$ and $N^I_\alpha \equiv \frac{1}{\sqrt{2}}(\nu^I_{\alpha R}+\mathcal{C}\bar{\nu}^{IT}_{\alpha R})$ and further require $y+y_3 = y^*+y_3^*$ and $h_{L,R} = h^*_{L,R}$, (8) become

$$\mathcal{L}_{M\nu} = (\overline{\nu^I}\quad \overline{N^I})\begin{pmatrix} 2h_L & y+y_3 \\ (y+y_3)^T & 2h_R \end{pmatrix}\begin{pmatrix}\nu^I \\ N^I\end{pmatrix}. \tag{9}$$

In the case that $h_R \gg h_L$, $y+y_3$, we can diagonalize the mass matrix approximately by

$$\begin{pmatrix} 2h_L & y+y_3 \\ (y+y_3)^T & 2h_R\end{pmatrix} = U\begin{pmatrix} 2h_L - \frac{1}{2}(y+y_3)h_R^{-1}(y+y_3)^T + O(h_R^{-2}) & O(h_R^{-1}) \\ O(h_R^{-1}) & 2h_R + O(h_R^{-1})\end{pmatrix}U^T \tag{10}$$

with

$$U = \begin{pmatrix} 1-\frac{1}{8}(y+y_3)h_R^{-2}(y+y_3)^T & \frac{1}{2}(y+y_3)h_R^{-1} \\ -\frac{1}{2}h_R^{-1}(y+y_3)^T & 1-\frac{1}{8}h_R^{-1}(y+y_3)^T(y+y_3)h_R^{-1}\end{pmatrix}.$$

If $h_L = 0$, (10) leads to the standard type I seesaw mechanism, otherwise we obtain type II seesaw mechanism for neutrinos.

Now we discuss quark part. The most general Yukawa type interactions for quarks is

$$\mathcal{L}_{\mathrm{Y,quark}} = \bar{q}^I_{\alpha L}\big[U_L\big(\tau^u y_u^{\alpha\beta} + \tau^d y_d^{\alpha\beta}\big)U_R^\dagger\big]q^I_{\beta R} + \text{h.c.}, \tag{11}$$

where $\tau^u = \frac{1+\tau^3}{2}$ and $\tau^d = \frac{1-\tau^3}{2}$. Coefficients $y_u^{\alpha\beta}$, $y_d^{\alpha\beta}$ are functions of Higgs field, α, β are family indices. We can explicitly expand $y_u^{\alpha\beta}$, $y_d^{\alpha\beta}$ in terms of powers of quantum fluctuation Higgs field $\tilde{h}$

$$y_i = y_i^0 + y_i^1 \tilde{h} + O\big(\tilde{h}^2\big), \quad i = u, d, \tag{12}$$

where y_i^0, y_i^1 are matrices independent of Higgs field h. Let u (u^I) and d (d^I) be spinors representing the up- and down-type mass (gauge) eigenstates of quarks. We can rotate the gauge eigenstates into the mass eigenstates with unitary matrices $V_{L,R}^{u,d}$ by $u_{L,R} = V_{L,R}^u u_{L,R}^I$ and $d_{L,R} = V_{L,R}^d u_{L,R}^I$

$$q_{\alpha L,R} = \begin{pmatrix} u_{\alpha L,R} \\ d_{\alpha L,R} \end{pmatrix} = \big[\big(V_{L,R}^u\big)_{\alpha\beta}\tau^u + \big(V_{L,R}^d\big)_{\alpha\beta}\tau^d\big] \begin{pmatrix} u_{\beta L,R}^I \\ d_{\beta L,R}^I \end{pmatrix}. \tag{13}$$

The $y_{u,d}^0$ matrices defined in (12) are diagonalized through $M_{\text{diag}}^i = V_L^i y_i^0 V_R^{i\dagger}$ $i = u, d$. Then $(V_L^u \tau^u + V_L^d \tau^d)(\tau^u y_u^0 + \tau^d y_d^0)(V_R^{u\dagger} \tau^u + V_{L,R}^{d\dagger} \tau^d) = (\tau^u M_{\text{diag}}^u + \tau^d M_{\text{diag}}^d)$ with $M_{\text{diag}}^{u,d}$ represent the diagonal physical up- and down-quark mass matrices. The Cabibbo–Kobayashi–Maskawa (CKM) matrix in the left and right sectors are given by $V_{L,R}^{\text{CKM}} = V_{L,R}^u V_{L,R}^{d\dagger}$. If the theory are left–right symmetric, then $y_{u,d}$ matrices are Hermitian which can be diagonalized by a unitary transformation. Then we may take $V_L^u = V_R^u$ and $V_L^d = V_R^d$ respectively. This leads to $V_R^{\text{CKM}} = V_L^{\text{CKM}}$ or manifest left–right symmetry [10].

Next, we focus our attention on quark–Goldstone–boson and quark–Higgs–boson couplings. Goldstone fields can be expanded out explicitly by

$$U_{L,R} = \exp\biggl(\frac{i g_{LR}}{\sqrt{2}M_{L,R}}\phi_{L,R}\biggr), \quad \phi_{L,R} = \begin{pmatrix} \frac{\phi_{L,R}^0}{\sqrt{2}} & \phi_{L,R}^+ \\ \phi_{L,R}^- & -\frac{\phi_{L,R}^0}{\sqrt{2}} \end{pmatrix}, \quad M_{L,R} = \frac{1}{2} f_{L,R} g_{L,R}. \tag{14}$$

In terms of the masses eigenstates, Lagrangian (11) can be expanded according to the Goldstone and Higgs fields,

$$\begin{aligned}
\mathcal{L}_{\text{Y,quark}} = {} & \biggl(1 + \frac{i g_L}{2M_L}\phi_L^0 - \frac{i g_R}{2M_R}\phi_R^0\biggr)\bar{u}_L M_{\text{diag}}^u u_R + \biggl(1 - \frac{i g_L}{2M_L}\phi_L^0 + \frac{i g_R}{2M_R}\phi_R^0\biggr)\bar{d}_L M_{\text{diag}}^d d_R \\
& + \tilde{h}\big(\bar{u}_L V_L^u y_u^1 V_R^{u\dagger} u_R + \bar{d}_L V_L^d y_d^1 V_R^{d\dagger} d_R\big) - \frac{i g_R}{\sqrt{2}M_R}\bar{u}_L M_{\text{diag}}^u V_R^{\text{CKM}} \phi_R^+ d_R \\
& + \frac{i g_L}{\sqrt{2}M_L}\bar{d}_L V_L^{\text{CKM}\dagger}\phi_L^- M_{\text{diag}}^u u_R - \frac{i g_R}{\sqrt{2}M_R}\bar{d}_L M_{\text{diag}}^d V_R^{\text{CKM}\dagger}\phi_R^- u_R \\
& + \frac{i g_L}{\sqrt{2}M_L}\bar{u}_L V_L^{\text{CKM}}\phi_L^+ M_{\text{diag}}^d d_R + \text{h.c.} + O\big(\bar{q}\phi^2 q, \bar{q}\tilde{h}^2 q, \bar{q}\phi\tilde{h}q\big).
\end{aligned} \tag{15}$$

Note that for neutral Goldstones, there is no flavor-changing $\bar{q}\phi q$ coupling. However, for neutral Higgs field $\tilde{h}$, flavor-changing couplings can exist in general due to the fact that matrices $V_L^i y_i^1 V_R^{i\dagger}$ may not be diagonal. For charged Goldstone bosons, the non-diagonal CKM matrices will yields flavor-changing couplings.

B. Dimension four gauge interactions

The most general dimension four gauge interaction part Lagrangian is

$$\begin{aligned}
\mathcal{L}_{f-4} = {} & i\bar{q}_{\alpha L}\not{D}q_{\alpha L} + i\delta_{L,1}\bar{q}_{\alpha L}U_L(\not{D}U_L)^\dagger q_{\alpha L} + i\delta_{L,2}\bar{q}_{\alpha R}U_R U_L^\dagger(\not{D}U_L)U_R^\dagger q_{\alpha R} \\
& + i\delta_{L,3}\bar{q}_{\alpha L}\big[(\not{D}U_L)\tau^3 U_L^\dagger - U_L\tau^3(\not{D}U_L)^\dagger\big]q_{\alpha L} + i\delta_{L,4}\bar{q}_{\alpha L}U_L\tau^3 U_L^\dagger(\not{D}U_L)\tau^3 U_L^\dagger q_{\alpha L} \\
& + i\delta_{L,5}\bar{q}_{\alpha R}U_R\big[\tau^3 U_L^\dagger(\not{D}U_L) - (\not{D}U_L)^\dagger U_L\tau^3\big]U_R^\dagger q_{\alpha R} + i\delta_{L,6}\bar{q}_{\alpha R}U_R\tau^3 U_L^\dagger(\not{D}U_L)\tau^3 U_R^\dagger q_{\alpha R} \\
& + i\delta_{L,7}\big[\bar{q}_{\alpha L}U_L\tau^3 U_L^\dagger\not{D}q_{\alpha L} - \big(\bar{q}_{\alpha L}\not{D}^\dagger\big)U_L\tau^3 U_L^\dagger q_{\alpha L}\big] + q \to l, \delta \to \delta^l + L \leftrightarrow R,
\end{aligned} \tag{16}$$

in which $D_\mu q_{\alpha i} = (\partial_\mu + i g_i \frac{\tau^a}{2} W_{i,\mu}^a + \frac{i}{6} g B_\mu) q_{\alpha i}$ and $D_\mu l_{\alpha i} = (\partial_\mu + i g_i \frac{\tau^a}{2} W_{i,\mu}^a - \frac{i}{2} g B_\mu) l_{\alpha i}$ for $i = L, R$. $(\not{D}U_i)^\dagger \equiv \gamma^\mu (D_\mu U_i)^\dagger$, $Q = T_{3L} + T_{3R} + \frac{Y}{2}$, $Y = B - L$ with $\frac{Y}{2} = 1/6$ for quarks and $\frac{Y}{2} = -1/2$ for leptons. In unitary gauge, $U_L = U_R = 1$, the Lagrangian become

$$\mathcal{L}_{f-4}|_{\text{Unitary gauge}} = i\bar{q}_L \not{\partial} q_L - \bar{q}_L\left[\Delta_{L,1} g_L \frac{\tau^{a'}}{2}\not{W}^{a'}_L + \Delta_{L,2}\frac{\tau^{a'}}{2} g_R \not{W}^{a'}_R + \Delta^3_{L,2} g_R \not{W}^3_R + \Delta_L g \not{B}\right] q_L + \Delta^3_{L,1} g_L \not{W}^3_L$$
$$+ q \to l, \delta \to \delta^l, \Delta_{L,n} \to \Delta^l_{L,n} + L \to R, \tag{17}$$

where $a' = 1, 2$.

$$\Delta_{L,1} = 1 - \delta_{L,1} - \delta_{L,4}, \qquad \Delta_{L,2} = \delta_{R,2} - \delta_{R,6}, \qquad \Delta^3_{L,1} = (1 - \delta_{L,1} + \delta_{L,4})\frac{\tau^3}{2} + \delta_{L,3} + \delta_{L,7},$$
$$\Delta^3_{L,2} = (\delta_{R,2} + \delta_{R,6})\frac{\tau^3}{2} + \delta_{R,5}, \qquad \Delta_L = (\delta_{L,1} - \delta_{R,2} - \delta_{L,4} - \delta_{R,6} + 2Y\delta_{L,7})\frac{\tau_3}{2} + \frac{Y}{2} - \delta_{L,3} - \delta_{R,5}. \tag{18}$$

Above formulae can be used both for quarks and leptons. For quarks, $\frac{Y}{2} = 1/6$; for leptons, $\frac{Y}{2} = -1/2$ and Δ_i should be replaced by Δ^l_i. Further exchange indices $L \Leftrightarrow R$, we can obtain expressions for $\Delta_{R,1}$, $\Delta_{R,2}$, $\Delta^3_{R,1}$, $\Delta^3_{R,2}$, Δ_R and $\Delta^l_{R,1}$, $\Delta^l_{R,2}$, $\Delta^{l,3}_{R,1}$, $\Delta^{l,3}_{R,2}$, Δ^l_R. In terms of diagonal gauge bosons given by (3) and $q_\alpha = q_{\alpha L} + q_{\alpha R}$, we find

$$\mathcal{L}_{f-4}|_{\text{Unitary gauge}} = i\bar{q}_\alpha\gamma^\mu\partial_\mu q_\alpha + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{EM}} \tag{19}$$

with charge current part $\mathcal{L}_{\text{CC}}$

$$\mathcal{L}_{\text{CC}} = -\frac{1}{\sqrt{2}}\bar{q}_\alpha\big[(g_L\cos\zeta\,\Delta_{L,1} + g_R\sin\zeta\,\Delta_{L,2})\gamma^\mu P_L + (g_R\sin\zeta\,\Delta_{R,1} + g_L\cos\zeta\,\Delta_{R,2})\gamma^\mu P_R\big]$$
$$\times\big(\tau^+W^+_{1,\mu} + \tau^-W^-_{1,\mu}\big)q_\alpha - \frac{1}{\sqrt{2}}\bar{q}_\alpha\big[(-g_L\sin\zeta\,\Delta_{L,1} + g_R\cos\zeta\,\Delta_{L,2})\gamma^\mu P_L + (g_R\cos\zeta\,\Delta_{R,1}$$
$$- g_L\sin\zeta\,\Delta_{R,2})\gamma^\mu P_R\big]\big(\tau^+W^+_{2,\mu} + \tau^-W^-_{2,\mu}\big)q_\alpha + q \to l, \delta \to \delta^l, \Delta_{{}^L_R,n} \to \Delta^l_{{}^L_R,n},$$

neutral current part $\mathcal{L}_{\text{NC}}$

$$\mathcal{L}_{\text{NC}} = -\frac{1}{2}\bar{q}_\alpha\big\{\big[g_Lx_1\big(\Delta^3_{L,1} + \Delta^3_{R,2}\big) + g_Ry_1\big(\Delta^3_{R,1} + \Delta^3_{L,2}\big) + gv_1(\Delta_L + \Delta_R)\big]\gamma^\mu$$
$$- \big[g_Lx_1\big(\Delta^3_{L,1} - \Delta^3_{R,2}\big) - g_Ry_1\big(\Delta^3_{R,1} - \Delta^3_{L,2}\big) + gv_1(\Delta_L - \Delta_R)\big]\gamma^\mu\gamma^5\big\}q_\alpha Z_{1,\mu}$$
$$- \frac{1}{2}\bar{q}_\alpha\big\{\big[g_Lx_2\big(\Delta^3_{L,1} + \Delta^3_{R,2}\big) + g_Ry_2\big(\Delta^3_{R,1} + \Delta^3_{L,2}\big) + gv_2\big(\Delta_L + \Delta_R\big)\big]\gamma^\mu - \big[g_Lx_2\big(\Delta^3_{L,1} - \Delta^3_{R,2}\big)$$
$$- g_Ry_2\big(\Delta^3_{R,1} - \Delta^3_{L,2}\big) + gv_2(\Delta_L - \Delta_R)\big]\gamma^\mu\gamma^5\big\}q_\alpha Z_{2,\mu} + q \to l, \delta \to \delta^l, \Delta_{{}^L_R,n} \to \Delta^l_{{}^L_R,n},$$

electro-magnetic current part $\mathcal{L}_{\text{EM}}$

$$\mathcal{L}_{\text{EM}} = -\frac{1}{2}\bar{q}_\alpha\big\{\big[g_Lx_3\big(\Delta^3_{L,1} + \Delta^3_{R,2}\big) + g_Ry_3\big(\Delta^3_{R,1} + \Delta^3_{L,2}\big) + gv_3(\Delta_L + \Delta_R)\big]\gamma^\mu - \big[g_Lx_3\big(\Delta^3_{L,1} - \Delta^3_{R,2}\big)$$
$$- g_Ry_3\big(\Delta^3_{R,1} - \Delta^3_{L,2}\big) + gv_3(\Delta_L - \Delta_R)\big]\gamma^\mu\gamma^5\big\}q_\alpha A_\mu + q \to l, \delta \to \delta^l, \Delta_{{}^L_R,n} \to \Delta^l_{{}^L_R,n}.$$

In terms of the masses eigenstates, $\mathcal{L}_{\text{NC}}$ and $\mathcal{L}_{\text{EM}}$ keep their present form, while the charge current Lagrangian for quarks is changed to

$$\mathcal{L}_{\text{CC}} = -\frac{1}{\sqrt{2}}\bar{u}_i\gamma^\mu\big(A^{ij}_1 + B^{ij}_1\gamma^5\big)d_jW^+_{1,\mu} - \frac{1}{\sqrt{2}}\bar{u}_i\gamma^\mu\big(A^{ij}_2 + B^{ij}_2\gamma^5\big)d_jW^+_{2,\mu} + \text{h.c.}, \tag{20}$$

where

$$A^{ij}_1 = \frac{1}{2}\big[g_L\cos\zeta\big(\Delta_{L,1}V^{\text{CKM},ij}_L + \Delta_{R,2}V^{\text{CKM},ij}_R\big) + g_R\sin\zeta\big(\Delta_{L,2}V^{\text{CKM},ij}_L + \Delta_{R,1}V^{\text{CKM},ij}_R\big)\big],$$
$$B^{ij}_1 = \frac{1}{2}\big[g_L\cos\zeta\big(-\Delta_{L,1}V^{\text{CKM},ij}_L + \Delta_{R,2}V^{\text{CKM},ij}_R\big) + g_R\sin\zeta\big(-\Delta_{L,2}V^{\text{CKM},ij}_L + \Delta_{R,1}V^{\text{CKM},ij}_R\big)\big],$$
$$A^{ij}_2 = A^{ij}_1\big|_{L\leftrightarrow R,\zeta\to-\zeta}, \qquad B^{ij}_2 = -B^{ij}_1\big|_{L\leftrightarrow R,\zeta\to-\zeta}. \tag{21}$$

At low energy region, ignoring heavy fields W_2 and Z_2, our Lagrangian generate ten anomalous gauge couplings for quarks and another ten for leptons. We parameterize them as

$$\mathcal{L}_{\text{CC}} = -\frac{1}{\sqrt{2}}\bar{q}_\alpha\big(\tau^+\not{W}^+_1 + \tau^-\not{W}^-_1\big)\big[(g_L + \Delta_{\text{CL}})P_L + \Delta_{\text{CR}}P_R\big]q_\alpha + q \to l, \Delta \to \Delta^l, \tag{22}$$

$$\mathcal{L}_{\text{NC}} = -\frac{1}{2}\bar{q}_\alpha\not{Z}\left\{T_{3L}\left[\frac{g_L}{\cos\theta_W}(1-\gamma_5) + \Delta_{\text{NV}T} + \Delta_{\text{NA}T}\gamma_5\right] + Q\left(-\frac{2g_L\sin^2\theta_W}{\cos\theta_W} + \Delta_{\text{NV}Q} + \Delta_{\text{NA}Q}\gamma_5\right)\right\}q_\alpha$$
$$+ q \to l, \Delta \to \Delta^l, \tag{23}$$

$$\mathcal{L}_{\rm EM} = -\frac{1}{2}\bar{q}_\alpha \not{A}\big[Q(2e+\Delta_{{\rm EV}Q}+\Delta_{{\rm EA}Q}\gamma_5)+T_{3L}(\Delta_{{\rm EV}T}+\Delta_{{\rm EA}T}\gamma_5)\big]q_\alpha + q\to l,\ \Delta\to\Delta^l, \tag{24}$$

in which $T_{3L}-2Q\sin^2\theta_W = g_V$, $T_{3L}=g_A$ with T_{3L} the weak isospin of fermions and $Q=T_{3L}+\frac{Y}{2}$ is the charge of q_α and l_α in units of $e=g_L\sin\theta_W$. The ten anomalous couplings are connected to coefficients in our Lagrangian as

$$\begin{aligned}
\Delta_{\rm CL} &= g_L\big[\cos\zeta(1-\delta_{L,1}-\delta_{L,4})-1\big]+g_R\sin\zeta(\delta_{R,2}-\delta_{R,6}),\\
\Delta_{\rm CR} &= g_L\cos\zeta(\delta_{L,2}-\delta_{L,6})+g_R\sin\zeta(1-\delta_{R,1}-\delta_{R,4}),\\
\Delta_{{\rm NV}T} &= -\frac{g_L}{\cos\theta_W}+g_Lx_1\bigg[1-\delta_{L,1}+\delta_{L,4}+\delta_{L,2}+\delta_{L,6}-\frac{2}{Y}(\delta_{L,3}+\delta_{L,7}+\delta_{L,5})\bigg]\\
&\quad+g_Ry_1\bigg[1-\delta_{R,1}+\delta_{R,4}+\delta_{R,2}+\delta_{R,6}-\frac{2}{Y}(\delta_{R,3}+\delta_{R,7}+\delta_{R,5})\bigg]\\
&\quad+gv_1\bigg[-2+\delta_{L,1}+\delta_{R,1}-\delta_{R,2}-\delta_{L,2}-\delta_{L,4}-\delta_{R,4}-\delta_{R,6}-\delta_{L,6}+2Y\delta_{L,7}+2Y\delta_{R,7}\\
&\quad+\frac{2}{Y}(\delta_{L,3}+\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\bigg],\\
\Delta_{{\rm NA}T} &= \frac{g_L}{\cos\theta_W}-g_Lx_1\bigg[1-\delta_{L,1}+\delta_{L,4}-\delta_{L,2}-\delta_{L,6}-\frac{2}{Y}(\delta_{L,3}+\delta_{L,7}-\delta_{L,5})\bigg]\\
&\quad-g_Ry_1\bigg[1-\delta_{R,1}+\delta_{R,4}-\delta_{R,2}-\delta_{R,6}+\frac{2}{Y}(\delta_{R,3}+\delta_{R,7}-\delta_{R,5})\bigg]\\
&\quad+gv_1\bigg[\delta_{L,1}-\delta_{R,1}-\delta_{R,2}+\delta_{L,2}-\delta_{L,4}+\delta_{R,4}-\delta_{R,6}+\delta_{L,6}+2Y\delta_{L,7}-2Y\delta_{R,7})\\
&\quad+\frac{2}{Y}(-\delta_{L,3}-\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\bigg],\\
\Delta_{{\rm NV}Q} &= \frac{2g_L\sin^2\theta_W}{\cos\theta_W}+\frac{2}{Y}\big[g_Lx_1(\delta_{L,3}+\delta_{L,7}+\delta_{L,5})+g_Ry_1(\delta_{R,3}+\delta_{R,7}+\delta_{R,5})+gv_1(Y-\delta_{L,3}\\
&\quad-\delta_{R,5}-\delta_{R,3}-\delta_{L,5})\big],\\
\Delta_{{\rm NA}Q} &= -\frac{2}{Y}\big[g_Lx_1(\delta_{L,3}+\delta_{L,7}-\delta_{L,5})-g_Ry_1(\delta_{R,3}+\delta_{R,7}-\delta_{R,5})+gv_1(-\delta_{L,3}-\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\big],\\
\Delta_{{\rm EV}Q} &= -2e+\frac{2}{Y}\big[g_Lx_3(\delta_{L,3}+\delta_{L,7}+\delta_{L,5})+g_Ry_3(\delta_{R,3}+\delta_{R,7}+\delta_{R,5})+gv_3(Y-\delta_{L,3}-\delta_{R,5}-\delta_{R,3}-\delta_{L,5})\big],\\
\Delta_{{\rm EA}Q} &= -\frac{2}{Y}\big[g_Lx_3(\delta_{L,3}+\delta_{L,7}-\delta_{L,5})-g_Ry_3(\delta_{R,3}+\delta_{R,7}-\delta_{R,5})+gv_3(-\delta_{L,3}-\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\big],\\
\Delta_{{\rm EV}T} &= g_Lx_3\bigg[1-\delta_{L,1}+\delta_{L,4}+\delta_{L,2}+\delta_{L,6}-\frac{2}{Y}(\delta_{L,3}+\delta_{L,7}+\delta_{L,5})\bigg]+g_Ry_3\bigg[1-\delta_{R,1}+\delta_{R,4}+\delta_{R,2}\\
&\quad+\delta_{R,6}-\frac{2}{Y}\big((\delta_{R,3}+\delta_{R,7}+\delta_{R,5})\big)\bigg]+gv_3\bigg[-2+\delta_{L,1}+\delta_{R,1}-\delta_{R,2}-\delta_{L,2}-\delta_{L,4}-\delta_{R,4}-\delta_{R,6}\\
&\quad-\delta_{L,6}+2Y\delta_{L,7}+2Y\delta_{R,7}+\frac{2}{Y}(\delta_{L,3}+\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\bigg],\\
\Delta_{{\rm EA}T} &= -g_Lx_3\bigg[1-\delta_{L,1}+\delta_{L,4}-\delta_{L,2}-\delta_{L,6}-\frac{2}{Y}(\delta_{L,3}+\delta_{L,7}-\delta_{L,5})\bigg]\\
&\quad-g_Ry_3\bigg[1-\delta_{R,1}+\delta_{R,4}-\delta_{R,2}-\delta_{R,6}+\frac{2}{Y}\big((\delta_{R,3}+\delta_{R,7}-\delta_{R,5})\big)\bigg]\\
&\quad+gv_3\bigg[\delta_{L,1}-\delta_{R,1}-\delta_{R,2}+\delta_{L,2}-\delta_{L,4}+\delta_{R,4}-\delta_{R,6}+\delta_{L,6}+2Y\delta_{L,7}\\
&\quad-2Y\delta_{R,7})+\frac{2}{Y}(-\delta_{L,3}-\delta_{R,5}+\delta_{R,3}+\delta_{L,5})\bigg].
\end{aligned}\tag{25}$$

These anomalous couplings characterize the deviations from tree level SM gauge interaction. Neglecting them, we recover tree level SM gauge interaction. In the EWCL of SM given in Ref. [9], we can also obtain similar structures for quark and lepton gauge interactions. This can be done by identifying $\delta_{L,1-7}$ with δ_{1-7} and $\delta_{R,7}$ with δ' in Ref. [9], taking $\zeta=\delta_{R,1-6}=0$ and rotation matrix among W_L^3, B and Z_1, A by $\begin{pmatrix} W^3_{L,\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} \tilde{x}_1 & \tilde{x}_3 \\ \tilde{v}_1 & \tilde{v}_3 \end{pmatrix}\begin{pmatrix} Z_{1,\mu} \\ A_\mu \end{pmatrix}$, where

$$\begin{pmatrix} \tilde{x}_1 & \tilde{x}_3 \\ \tilde{v}_1 & \tilde{v}_3 \end{pmatrix} = \begin{pmatrix} (1+\Delta_Z)\cos\theta_W + \Delta_{AZ}\sin\theta_W & (1+\Delta_A)\sin\theta_W \\ -(1+\Delta_Z)\sin\theta_W + \Delta_{AZ}\cos\theta_W & (1+\Delta_A)\cos\theta_W \end{pmatrix} \tag{26}$$

with $\Delta_Z = -\frac{1}{2}\alpha_1 g_L g \sin 2\theta_W + \frac{1}{2}\alpha_8 g_L^2 \cos^2\theta_W$, $\Delta_A = \frac{1}{2}\alpha_1 g_L g \sin 2\theta_W + \frac{1}{2}\alpha_8 g_L^2 \sin^2\theta_W$ and $\Delta_{AZ} = \alpha_1 g_L g \cos 2\theta_W + \frac{1}{2}\alpha_8 g_L^2 \sin 2\theta_W$. Since in Ref. [9], there are eight independent parameters δ_{1-7} and δ' characterizing anomalous gauge couplings, only eight of ten anomalous couplings defined in (22)–(24) are independent. There should be two consistency constraints which from our computation happen in electro-magnetic current part(24),

$$\Delta_{EAT} = \left(Y^2 - 1\right)\Delta_{EAQ}, \qquad \Delta_{EVT} = \left(Y^2 - 1\right)\Delta_{EVQ}. \tag{27}$$

This implies that the general structure for electro-magnetic current Lagrangian only has two independent anomalous couplings. Matching our result (25) with that obtained from EWCL in Ref. [9] offers ten constrains. Solving these constraints, eight of them will allow us express eight parameters δ_{1-7}, δ' as functions of parameters in our theory and the left two gives constraints given by (27). Detail calculation shows,

$$\begin{aligned} &\delta_1 = 1 - \frac{1}{2}(a + \cos\zeta) + \bar{\delta}_1, \qquad \delta_2 = \frac{1}{2}\left(b + \frac{g_R}{g_L}\sin\zeta\right) + \bar{\delta}_2, \qquad \delta_3 = (1+n)\frac{Y}{2} + \bar{\delta}_3, \qquad \delta_4 = \frac{1}{2}(a - \cos\zeta) + \bar{\delta}_4, \\ &\delta_5 = c\frac{Y}{2} + \bar{\delta}_5, \qquad \delta_6 = \frac{1}{2}\left(b - \frac{g_R}{g_L}\sin\zeta\right) + \bar{\delta}_6, \qquad \delta_7 = (-1 + c - n)\frac{Y}{2} + \bar{\delta}_7, \qquad \delta' = (-1 + c - n)\frac{Y}{2} + \bar{\delta}', \end{aligned} \tag{28}$$

where $a = \tilde{v}_3 x_1 - \tilde{v}_1 x_3$, $b = \frac{g_R}{g_L}(\tilde{v}_3 y_1 - \tilde{v}_1 y_3)$, $c = \frac{g}{g_L}(\tilde{v}_3 v_1 - \tilde{v}_1 v_3)$, $n = \tilde{x}_3 v_1 - \tilde{x}_1 v_3$. $\bar{\delta}_{1-6}$ and $\bar{\delta}'$ are corrections from $\delta_{L,1-7}$ and $\delta_{R,1-7}$,

$$\begin{aligned} \bar{\delta}_1 &= \frac{\cos\zeta + a - c}{2}\delta_{L,1} + \frac{\cos\zeta - a + c}{2}\delta_{L,4} - Yc\delta_{L,7} - \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta + b - c\right)\delta_{R,2} + \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta - b + c\right)\delta_{R,6}, \\ \bar{\delta}_2 &= \frac{1}{2}(\cos\zeta + a - c)\delta_{L,2} - \frac{1}{2}(\cos\zeta - a + c)\delta_{L,6} - \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta + b - c\right)\delta_{R,1} - \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta - b + c\right)\delta_{R,4} + cY\delta_{R,7}, \\ \bar{\delta}_3 &= (l - n)\delta_{L,3} + l\delta_{L,7} + (m - n)\delta_{R,5}, \\ \bar{\delta}_4 &= \frac{\cos\zeta - a + c}{2}\delta_{L,1} + \frac{\cos\zeta + a - c}{2}\delta_{L,4} + Yc\delta_{L,7} - \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta - b + c\right)\delta_{R,2} + \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta + b - c\right)\delta_{R,6}, \\ \bar{\delta}_5 &= c\frac{Y}{2} + (a - c)\delta_{L,5} + (b - c)\delta_{R,3} + b\delta_{R,7}, \\ \bar{\delta}_6 &= -\frac{1}{2}(\cos\zeta - a + c)\delta_{L,2} + \frac{1}{2}(\cos\zeta + a - c)\delta_{L,6} + \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta - b + c\right)\delta_{R,1} + \frac{1}{2}\left(\frac{g_R}{g_L}\sin\zeta + b - c\right)\delta_{R,4} + cY\delta_{R,7}, \\ \bar{\delta}_7 &= -(1 - c + n)\frac{Y}{2} + (a - l - c + n)\delta_{L,3} + (a - l)\delta_{L,7} + (b - m - c + n)\delta_{R,5}, \\ \bar{\delta}' &= -(1 - c + n)\frac{Y}{2} + (a - l - c + n)\delta_{L,5} + (b - m - c + n)\delta_{R,3} + (b - m)\delta_{R,7}, \end{aligned} \tag{29}$$

where $l = \frac{g_L}{g}(\tilde{x}_3 x_1 - \tilde{x}_1 x_3)$, $m = \frac{g_R}{g}(\tilde{x}_3 y_1 - \tilde{x}_1 y_3)$. If we further ignore mixing caused by anomalous couplings in front of each $\delta_{L,1-7}$ and $\delta_{R,1-7}$ coefficients, we will find that $\bar{\delta}_i = \delta_{L,i}$ for $i = 1, \ldots, 6$ and $\bar{\delta}' = \delta_{R,7}$, i.e., right handed part $\delta_{R,1-6}$ decouple at this order of precision.

Since in LRSMs, the strongest constraint comes from neutral kaon system. We now proceed to discuss the low energy phenomenological constraints from K^0–$\bar{K}^0$ system. The neutral kaon meson mixing is described by a effective Hamiltonian $H_{\text{eff}} = H_{\text{eff}}^{W_1W_1} + H_{\text{eff}}^{W_2W_2} + H_{\text{eff}}^{W_1W_2} + H_{\text{eff}}^{h^0}$. Here $H_{\text{eff}}^{W_iW_j}$, $i, j = 1, 2$ are part of effective Hamiltonian from box diagram mediated by gauge bosons W_1, W_2 and Goldstone bosons ϕ_L, ϕ_R. $H_{\text{eff}}^{h^0}$ is the part of effective Hamiltonian arises from the flavor changing Yukawa coupling via neutral Higgs exchange at tree level.

$K^0 - \bar{K}^0$ mass difference is given by

$$\Delta m_K = 2\,\text{Re}\langle K^0|H_{\text{eff}}|\bar{K}^0\rangle = 2\,\text{Re}\langle K^0|\left(H_{\text{eff}}^{W_1W_1} + H_{\text{eff}}^{W_2W_2} + H_{\text{eff}}^{W_1W_2} + H_{\text{eff}}^{h^0}\right)|\bar{K}^0\rangle. \tag{30}$$

It is well known that contribution from SM part $H_{\text{eff}}^{W_1W_1}$ can already afford the experiment value of Δm_K. Then the total contribution of remaining parts including contributions from $H_{\text{eff}}^{W_1W_2}$, $H_{\text{eff}}^{W_2W_2}$ and $H_{\text{eff}}^{h^0}$ must be very small. Since we know that the lower bound of M_{W_2} is roughly 800 GeV [7] which is already much larger than M_{W_1}, $H_{\text{eff}}^{W_2W_2}$ must be tiny due to the suppression of factor $M_{W_1}^4/M_{W_2}^4$ and then can be ignored. For $H_{\text{eff}}^{h^0}$, from (15) it is controlled by a group of arbitrary coefficients $V_L^i y_i^1 V_R^{i\dagger}$. As discussed in Ref. [4], indirect CP violation ϵ_K will constrain these coefficients to very small numbers and therefore can also be ignored comparing its size with $H_{\rm eff}^{W_1W_1}$. Finally we must require the contribution of $H_{\rm eff}^{W_1W_2}$ to be small. Conventionally this constraint leads to very large value for M_{W_2}. An alternative way people used is to introduce in theory some other Higgs multiplets such as the second bi-doublet to cancel the contribution from $H_{\rm eff}^{W_1W_2}$ [4]. Now in terms of our EWCL (15) and (16), we propose another possibility to reduce the size of $H_{\rm eff}^{W_1W_2}$. Notice that $H_{\rm eff}^{W_1W_2} = M^{W_1W_2} + M^{W_1\phi_R} + M^{\phi_L W_2} + M^{\phi_L\phi_R} + \text{h.c.}$ When $M_{W_2}^2 \gg M_{W_1}^2$, the dominant contribution comes from the diagram mediated by exchanging W_1 and W_2 bosons as well as by exchanging ϕ_L and W_2 bosons, for the exchanging Goldstone boson ϕ_R will cause extra suppression $M_{W_1}^2/M_{W_2}^2$ due to the factor of $1/M_R \sim 1/M_{W_2}$ in front of Yukawa coupling for charged Goldstone boson ϕ_R given in (15). This shows that $H_{\rm eff}^{W_1W_2}$ should be proportional to $g_R^2\Delta_{R,1}^2$, where $g_R\Delta_{R,1}$ from (17) is the effective right handed gauge couplings. As long as $\Delta_{R,1}$ is very small, the contribution from $H_{\rm eff}^{W_1W_2}$ will be small. An extreme case is $\Delta_{R,1} = 0$ in which $H_{\rm eff}^{W_1W_2}$ vanishes completely. Taking replacement $L \to R$ in (18), we will find $\Delta_{R,1} = 1 - \delta_{R,1} - \delta_{R,4}$. In order for $\Delta_{R,1}$ to be small enough, we must have order 1 size of positive parameter $\delta_{R,1} + \delta_{R,4}$. A naive situation is to take $\delta_{R,1} \sim \delta_{R,4} \sim 1/2$. From (25), we see that the influence of this choice to anomalous couplings only happens for right handed charged current couplings Δ_{CR} due to the fact that all other anomalous couplings at most depend on $\delta_{R,1} - \delta_{R,4}$ which vanishes now. For Δ_{CR}, (25) tells us that our choice of $\delta_{R,1}$ and $\delta_{R,4}$ just cancel the contribution from bosonic part and these contributions all proportional to small mixing factor $\sin\zeta$, left contributions from other δs. Therefore our choice of $\delta_{R,1}$ and $\delta_{R,4}$ can reduce the size of effective right handed gauge coupling $g_R\Delta_{R,1}$ and do not cause large anomalous fermion gauge couplings for light gauge fields.

To summarize, we have constructed the matter sector of electroweak chiral Lagrangian up to dimension four operators for left–right symmetric models in the presence of a neutral light Higgs. In addition to generating necessary CKM matrices and masses either for leptons or quarks, dimension three Yukawa type operators can generate various seesaw mechanisms for neutrinos and effective Higgs coupling to quark pair. Dimension four operators generate gauge couplings to heavy gauge fields W_2Z_2 and anomalous gauge couplings to light gauge bosons, ten for quarks and another ten for leptons. We have shown that order 1 size of positive parameter $\delta_{R,1} + \delta_{R,4}$ will reduce the unwanted box diagram contribution from exchanging W_1W_2 bosons very much, making neutral K meson mass difference Δm_K consist with experiment data and at meantime do not cause large deviation in anomalous gauge couplings for fermions.

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