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Preserving the lepton asymmetry in the brane world

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Abstract

In models where the Standard Model spectrum is localized on a brane embedded in a higher-dimensional spacetime, we discuss the lepton number violation induced by the emission of right-handed neutrinos from the brane. We show that the presence of the right-handed neutrinos in the bulk may lead to rapid lepton number violating processes which above the electroweak scale would wash away any prior lepton or baryon asymmetry. We derive constraints on the Yukawa couplings of these states in order to preserve the lepton asymmetry. We show that this has a natural interpretation in the brane world.

1. Introduction

The generation and survival of a baryon asymmetry is a requirement of any realistic cosmology and hence enables one to put strong constraints on phenomenological models beyond the standard model. While all three ingredients [1] necessary for the generation of a baryon asymmetry are contained within the standard model, it is well established that electroweak baryogenesis cannot provide an asymmetry of sufficient magnitude [2]. Even in the context of the supersymmetric standard model, baryon number generation is difficult [3] and requires a particular supersymmetric mass spectrum. Therefore, the observed baryon asymmetry of the Universe (BAU) is likely to arise from short-distance physics above the TeV scale. While sphaleron effects [4] may not be primarily responsible for the generation of the BAU, they certainly do modify any pre-existing asymmetry. For example, a baryon asymmetry with $B-L=0$, will be washed away by sphaleron interactions [5]. On the other hand, any primordial $B-L$ asymmetry will be reshuffled by the rapid anomalous $(B+L)$-violating sphaleron transitions above the electroweak phase transition temperature, T_c. Thus a pure lepton asymmetry can be converted into a baryon asymmetry [6]. However, any baryon/lepton number violating interaction (other than purely $B+L$ violating interactions) in equilibrium above T_c, will once again lead to the total erasure of both B and L. Thus, in order for successful leptogenesis to occur [6], the right-handed Majorana neutrino, whose out-of-equilibrium decays generate the requisite $B-L$ excess, must be massive enough to suppress lepton number violating interactions above the weak scale in order to preserve the asymmetry against erasure [7].

Going beyond the standard model, however, introduces the possibility of a hierarchy of mass scales

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rendering the weak scale unstable to radiative corrections [8]. In addition to attempts at solving the hierarchy problem [9–11], extended objects arising in certain string vacua suggest that our universe is a (3 + 1)-dimensional brane embedded in a higher-dimensional spacetime. While all fields charged under the gauge group are generally supposed to be confined to a brane due to the conservation of gauge flux, gauge singlets can in principle propagate freely in the bulk with far-reaching consequences [12,13]. In the case of the graviton, one can lower the fundamental scale of gravity down to the TeV scale thereby providing a solution to the hierarchy problem [9]. In the case of right-handed neutrinos, one can naturally derive small Dirac masses for neutrinos [12,14–16].

Gauge singlets can be produced through the scatterings of brane-localized particles. Once they are produced, they propagate into the bulk with almost no probability of further interacting with brane fields since the brane occupies only a tiny volume of the total space. These singlet-emission processes, therefore, show up as missing energy signals in colliders. There is, however, an important difference between the emission of gravitons and right-handed neutrinos in that the latter may appear not only as missing energy but also as a source of lepton number violation on the brane. Given our discussion above, it is clear that right-handed neutrino emission into the bulk can have important consequences regarding the BAU.

In this Letter, we will show that brane-world models with bulk right-handed neutrinos can lead to a devastatingly small BAU. We further derive conditions on the neutrino sector in order to protect the lepton and baryon asymmetries. We apply these results to a number of specific brane-world scenarios. We show that these conditions are a natural consequence of the brane world.

2. Lepton number violation in the brane world

We begin by discussing the phenomenon of L violation and its implications for the BAU for a generic configuration where a three brane with coordinates x_μ ($\mu = 0, \ldots, 3$) is embedded in a higher-dimensional space with extra dimensions y_a ($a = 1, \ldots, \delta$). We introduce bulk fermions $\Psi^\ell(x, \vec{y})$ ($\ell = 1, 2, \ldots, n_s$) which possess lepton number with the decomposition

$$\Psi^\ell(x, \vec{y}) = \begin{pmatrix} \psi_L^\ell(x, \vec{y}) \\ \psi_R^\ell(x, \vec{y}) \end{pmatrix}, \tag{1}$$

where n_s is the number of fermion species. The component fields $\Psi_{L,R}^\ell$ can be expanded in a complete set of functions $f_{L,R}^{\ell\,\vec{n}}(\vec{y})$ with respective coefficients $\psi_{L,R}^{\ell\,\vec{n}}(x)$, where $\vec{n}$ is a set of integers; one for each extra dimension. Dirac neutrino masses arise from the coupling of $\psi_R^{\ell\vec{n}}$ to the composite SM singlet $\bar{\mathbf{L}}^\alpha \mathbf{H}^c$ [12]

$$S = \sum_{\vec{n}} \int d^4x\, h_{\vec{n}}^{\alpha\ell} \bar{\mathbf{L}}^\alpha(x) \mathbf{H}^c(x) \psi_R^{\ell\,\vec{n}}(x), \tag{2}$$

where $\mathbf{L}^\alpha = \mathbf{L}^{e,\mu,\tau}$ are the lepton doublets, $\mathbf{H}$ is the Higgs field, and

$$h_{\vec{n}}^{\alpha\ell} \equiv \lambda^{\alpha\ell} f_R^{\ell\,\vec{n}}(\vec{y}_0), \tag{3}$$

where $\vec{y}_0$ is the position of the brane, and λ is a $3 \times n_s$ Yukawa matrix. From the experimental perspective, the solar and atmospheric neutrino anomalies can, respectively, be explained by $\nu_e \leftrightarrow \nu_\mu$ [17] and $\nu_\mu \leftrightarrow \nu_\tau$ [18] oscillations with small contributions from the sterile states $\psi_{L,R}^{\ell\,\vec{n}}(x)$. It is thus reasonable to assume that the couplings to the zero modes alone already provide a good fit to experiment with appropriate textures [15]. That is, the unitary matrix U which performs the diagonalization $U(h_{\vec{0}} h_{\vec{0}}^\dagger)U^\dagger = \mathrm{diag}\big((h_{\vec{0}}^1)^2, (h_{\vec{0}}^2)^2, (h_{\vec{0}}^3)\big)^2$ explains the neutrino oscillation data to a good approximation. Neutrino masses would then be given by $m_\alpha = h_{\vec{0}}^\alpha \langle \mathbf{H} \rangle$ if there are no mixings with the higher KK modes. However, each left-handed neutrino ν_α (the neutral component of $\mathbf{L}^\alpha$) mixes with the higher KK states via an angle θ_α with

$$\tan^2\theta_\alpha = \sum_{\vec{n} \neq \vec{0}} h_{\vec{n}}^{\alpha\ell} \frac{1}{m_{\ell\vec{n}}^2} h_{\vec{n}}^{\dagger\ell\alpha} \langle \mathbf{H} \rangle^2, \tag{4}$$

where $m_{\ell\vec{n}}$ is the KK mass of $\psi_{L,R}^{\ell\,\vec{n}}(x)$. This angle must be small, $\tan\theta_\alpha \lesssim 0.1$, according to the experimental constraints from reactor, accelerator, solar and atmospheric neutrino data [15]. The solar and atmospheric anomalies, respectively, imply $\delta m_{\mathrm{sol}}^2 = m_2^2 - m_1^2 = 3.7 \times 10^{-5}\ \mathrm{eV}^2$ and $\delta m_{\mathrm{atm}}^2 = m_3^2 - m_2^2 = 3.0 \times 10^{-3}\ \mathrm{eV}^2$ assuming large mixings in both cases. We note that although we have generated Dirac masses in this model, the cosmological constraint on additional neutrino states [19] is not violated since the right-handed states are necessarily much more weakly coupled than their left-handed counterparts. In what follows, all discussions will be based on the effective action (2).

For now, we assume that a primordial $B-L$ asymmetry has been generated at some high temperature $T_{B-L} \gg T_c$. Below T_{B-L}, brane matter and sphalerons are in thermal and chemical equilibrium. By assigning each particle species a chemical potential, and using gauge and Higgs interactions as conditions on these potentials (with generation indices suppressed), one obtains

$$\mu_{d_L} - \mu_{u_L} = 0, \qquad \mu_{l_L} - \mu_\nu = 0,$$
$$\mu_{u_R} - \mu_{u_L} = -\mu_{d_R} + \mu_{d_L} - \mu_{l_R} + \mu_{l_L} = \mu_H, \tag{5}$$

where the constraint on the weak isospin charge, $Q_3 \propto \mu_W = 0$ has been employed. From (5), one can write down a simple set of equations for the baryon and lepton numbers and electric charge which reduce to:

$$B \propto 4N\mu_{u_L}, \qquad L \propto 3\mu - N\mu_H,$$
$$Q \propto 2N\mu_{u_L} - 2\mu + (4N+2)\mu_H, \tag{6}$$

where $\mu = \sum \mu_{\nu_\alpha}$ and N is the number of generations. Equilibrium sphaleron transitions further restrict

$$3N\mu_{u_L} + \mu = 0. \tag{7}$$

In the absence of any other $B-L$ violating interactions (in equilibrium), these conditions ultimately give

$$B = \frac{28}{79}(B-L), \tag{8}$$

for $N=3$ generations. Thus, in the absence of a primordial $B-L$ asymmetry, the baryon number is erased by equilibrium processes [20]. Note that barring new interactions (in an extended model) the quantities $\frac{1}{3}B - L_e$, $\frac{1}{3}B - L_\mu$, and $\frac{1}{3}B - L_\tau$ remain conserved.

It is straightforward to see that the emission of right-handed neutrinos from the brane into the bulk is a source of lepton number violation which leads to the erasure of any $B-L$ asymmetry. At temperatures above T_c, the dominant lepton number violating interactions are the scattering of top quarks into right-handed neutrinos and a lepton doublet, $Q^c t_R \to L^c \psi_R^{\vec{n}}$, $Q^c L \to t_R^c \psi_R^{\vec{n}}$, and $t_R L \to Q\psi_R^{\vec{n}}$, as depicted in Fig. 1. At high temperatures, the production rates for a given neutrino flavor and its $SU(2)$ charged lepton partner are identical as the mixing angle (4) vanishes. Therefore, the thermal and chemical balance imposed by fast electroweak interactions between neutrinos and their $SU(2)$ partners still holds. If the rates of these processes are faster than the expansion rate of the Universe, while the reverse process (being suppressed by the number of kinematically accessible KK states) is not, they lead to an additional condition on the chemical potentials, namely $\mu = 0$. In this case, one can easily see that all of the chemical potentials are forced to zero as is the BAU.

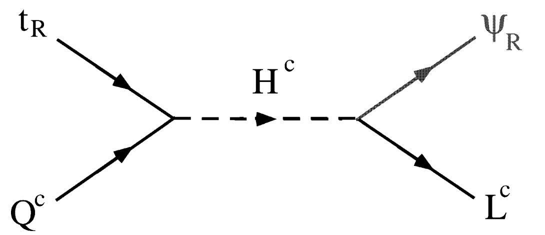

Fig. 1. The scattering of third generation quarks into $\psi_R \mathbf{L}^c$. The right-handed neutrino, with non-zero momentum along the extra dimension, induces lepton number nonconservation on the brane.

We next consider this process in more detail. Given the collision processes in Fig. 1 then the difference between the number densities of ν_L and ν_L^c of a given generation evolves as

$$(\partial_t + 3H)(n_\nu - n_{\nu^c}) = -\Gamma_F + \Gamma_R + \frac{1}{6}(\partial_t - 3H)(n_b - n_{b^c}), \tag{9}$$

where the last term comes from the sphaleron contribution. The forward process is given by

$$\Gamma_F = \sum_{\vec{n}} \int \frac{d^3\vec{p}_Q}{2(2\pi)^3|\vec{p}_Q|} \frac{d^3\vec{p}_R}{2(2\pi)^3|\vec{p}_R|} \frac{d^3\vec{p}_L}{2(2\pi)^3|\vec{p}_L|} \times \frac{d^3\vec{p}_{\vec{n}}}{2(2\pi)^3 E_{\vec{n}}} \Big\{ f_{Q^c} f_{t_R}(1-f_{L^c}) \times \big|\mathcal{A}\big(Q^c t_R \to L^c \psi_R^{\vec{n}}\big)\big|^2 + f_{Q^c} f_L(1-f_{t_R^c})\big|\mathcal{A}\big(Q^c L \to t_R^c \psi_R^{\vec{n}}\big)\big|^2 + f_{t_R} f_L(1-f_Q)\big|\mathcal{A}\big(t_R L \to Q\psi_R^{\vec{n}}\big)\big|^2 \Big\} + \text{c.c.} \tag{10}$$

Here, we follow the procedure and notation described in [21]. In (9), H is the Hubble expansion parameter and $f^{-1} = e^{(E-\mu)/T} + 1$ is the phase space distribution for fermions. For the latter, $f \approx (1 + \mu/T)/(e^{E/T} + 1)$ is an excellent approximation since the chemical potential consistent with the BAU is small, $|\mu| \sim 10^{-10}$ T. The reverse process, Γ_R is phase space suppressed relative to the forward process. The suppression is related to the relative thickness of the brane, Δ, and the size of the compact bulk space, R, namely, $\Gamma_R \simeq \max((\Delta/R)^\delta, 1/(RT)^\delta)\Gamma_F$. Since the number of kinematically accessible KK states is given by $N_{\rm KK} = (RT)^\delta$, and we consider $T \lesssim \Delta^{-1}$, we see that the suppression is given simply by $\Gamma_R \simeq (1/N_{\rm KK})\Gamma_F$. In a standard 4D picture with a single right-handed state, we clearly have $\Gamma_R = \Gamma_F$ and since ν_R can be assigned a lepton number, there is no lepton number violation in the theory. However, if $\Gamma_R \neq \Gamma_F$, there is an effective violation of lepton number if the forward rate is fast compared with the expansion rate, H, while the reverse rate is not.

In evaluating (9), we approximate f by the Maxwell–Boltzmann form, and take $1 - f \approx 1$. The lepton asymmetry in ν_L is defined by

$$L_\nu = \frac{n_\nu - n_{\nu^c}}{s} = \frac{15\mu}{4\pi^2 g_* T} + \mathcal{O}\left(\frac{\mu}{T}\right)^3, \qquad (11)$$

where the entropy density, $s \approx \frac{2\pi^2}{45} g_* T^3$, obeys $\frac{ds}{dt} = -3sH$ in an adiabatically expanding universe and g_* is the number of relativistic degrees of freedom; equal to about 10^2 for the standard model spectrum. Then direct evaluation of (9) gives

$$\frac{d}{dt}(B-L) = \sum_{\vec{n}} \frac{3h_t^2}{4(2\pi)^5} \frac{T^3}{s} \sum_\alpha \left(h_{\vec{n}} F\left(\frac{m_{\vec{n}}}{T}\right) h_{\vec{n}}^\dagger\right)_{\alpha\alpha} \times (\mu_H + \mu_{\nu_\alpha}), \qquad (12)$$

where the sum over $\vec{n}$ extends up to the heaviest kinematically accessible KK mode; $m_{\vec{n}}^{\max} \sim 2\pi T$. Here the function $F(x)$ varies slowly with x; it is ≈ 2 for $x \ll 1$ and ≈ 1 for $x \approx 1$.

We next discuss the implications of (12) which depend on the scheme used to generate neutrino masses and oscillations. We distinguish two broad classes: (i) a hierarchy in the overlap of the wavefunctions, $f_R^{\ell\vec{0}}(\vec{y}_0)$, or (ii) a hierarchy in the Yukawa couplings, $\lambda^{\alpha\ell}$.

2.1. Hierarchy from wavefunction overlap

Here, we assume that the hierarchy among the neutrino masses as well as the requisite textures for neutrino oscillations are both generated by the zero mode wave functions $f_R^{\ell\vec{0}}(\vec{y}_0)$. Namely, we take the entries of the effective Yukawa coupling matrix $\lambda^{\alpha\ell}$ to be of order one. This implies that $(h_{\vec{n}}h_{\vec{n}}^\dagger)_{11} \approx (h_{\vec{n}}h_{\vec{n}}^\dagger)_{22} \approx (h_{\vec{n}}h_{\vec{n}}^\dagger)_{33} \equiv h_{\vec{n}}^2$. Consequently (12) reduces to

$$\frac{d}{dt}(B-L) = -\Gamma(T)(B-L), \qquad (13)$$

with

$$\Gamma(T) = \sum_{\vec{n}} \frac{h_t^2 h_{\vec{n}}^2}{20(2\pi)^3} \frac{10N+3}{22N+13} \frac{g_* T^4}{s} F, \qquad (14)$$

where F as noted above is $O(1)$, and we have expressed $N\mu_H + \mu$ in terms of $B-L$ using Eqs. (6) and (7). This equation implies that the emission of right-handed neutrinos from the brane drives the system towards $\mu = -N\mu_H$, so long as the reverse process is sufficiently suppressed. Thus if the rate of L violation is also fast compared to the expansion rate of the Universe (at the same time sphaleron processes are in equilibrium), this constraint, when combined with other equilibrium conditions, drives *all* asymmetries in the system to zero. Indeed, the equilibrium relation $\mu = -N\mu_H$ is the same condition which arises in the presence of heavy right-handed neutrinos in four dimensions, and avoiding it places a bound on the combination h^2/M [7,20,22], where h is the right-handed Yukawa coupling and M is the Majorana mass of the right-handed neutrino.

A more quantitative determination of the damping of $B-L$ via (13) as the temperature falls to $T \sim T_c$ requires some knowledge of the cosmological evolution at higher temperatures. We assume that there is a critical temperature $T_{\rm brane} \geqslant T_{B-L}$, below which the standard FRW evolution holds. T_{B-L} is the temperature at which a $B-L$ asymmetry is produced and must be greater than T_c (unless a mechanism for producing a net baryon asymmetry below T_c is provided). Thus our initial conditions are such that at $T = T_{\rm brane}$, the bulk is empty, and the brane contains hot SM matter with an energy density

$$\rho = \frac{\pi^2}{30} g_* T^4, \qquad (15)$$

where the expansion rate of the Universe is simply $H^2 = \rho/3M_{\rm Pl}^2$. The emptiness of the bulk is a reasonable assumption if $T_{\rm brane}$ is the reheat temperature, and the inflaton is a brane-localized field (See, e.g., [23]). The emission of gravitons and right-handed neutrinos depletes the energy density on the brane beyond the dilution due to expansion. However, this energy exchange between the brane and the bulk does not affect H which depends only on the sum of the brane and bulk energy densities (see, e.g., [24]).

We next must determine if either the forward or backward rates are fast compared with the expansion rate, H. Using Eq. (14), we see that

$$\frac{\Gamma_F}{H} \simeq 10^{-3} h^2 \frac{N_{\rm KK}}{\sqrt{g_*}} \frac{M_{\rm Pl}}{T}, \tag{16}$$

assuming that the $h_n \sim h$ are all constant for the purposes of making a numerical estimate. At T_c, $\Gamma_F/H \simeq 10^{13} h^2 N_{\rm KK}/\sqrt{g_*}$. Thus for $h^2 N_{\rm KK} \gtrsim 10^{-13}\sqrt{g_*}$, this rate will be rapid enough to produce ν_R's in the bulk. The reverse rate compared with the expansion rate is then simply $\Gamma_R/H \simeq 10^{13} h^2/\sqrt{g_*}$, so that for

$$h^2 \gtrsim 10^{-12} \sqrt{1 + \frac{N_{\rm KK}}{100}}, \tag{17}$$

this rate is also rapid and no lepton number violation occurs. This is our first main result. When this bound is violated and at the same time $h^2 N_{\rm KK} \gtrsim 10^{-12}$, then indeed the lepton and baryon number of the Universe is erased. Our second constraint is, therefore,

$$h^2 \lesssim 10^{-12}/N_{\rm KK}. \tag{18}$$

Preservation of the baryon asymmetry requires that either (17) or (18) is satisfied. These results are summarized in Fig. 2. The constraint due to insuring that the reverse process is rapid compared to H and given by Eq. (17) defines the upper boundrary of the excluded region. We note that had we included the decays of the right-handed states, this exclusion bound would be slightly weaker. At very large $N_{\rm KK}$, it is softned by a factor of approximately T_{B-L}/T_c. The lower boundary is derived by insuring that the forward process is out of equilibrium (Eq. (18)).

To see that the baryon asymmetry is effectively erased when both of the above conditions are violated, we use $H = 1/2t$, appropriate for radiation domination, and relate the temperature of the brane matter to

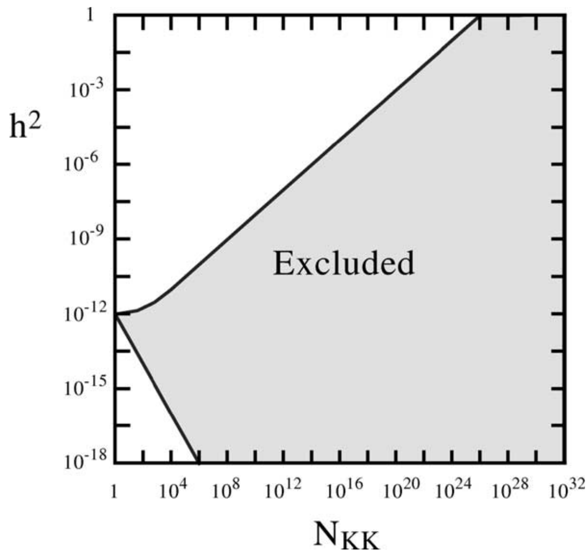

Fig. 2. The excluded region (shaded) in the h^2–$N_{\rm KK}$ plane. Above the shaded region inverse scatterings are rapid enough so that there is no effective violation of lepton number. Below the shaded region, even the forward process is too slow and right-handed states are never produced.

time as

$$t = \sqrt{\frac{45}{2\pi^2 g_*}} \frac{M_{\rm Pl}}{T^2}. \tag{19}$$

Using this expression and assuming entropy conservation, $B - L$ can be integrated from (13) to obtain

$$\frac{(B-L)[T_c]}{(B-L)[T_{B-L}]} = e^{-\gamma_0 \frac{M_{\rm Pl}}{T_c}\left(1 - \frac{T_c}{T_{B-L}}\right)}, \tag{20}$$

which gives us $B - L$ at $T = T_c$ relative to the primordial value of $B - L$ at $T = T_{B-L}$. The constant γ_0 in the exponent (20) is given by

$$\gamma_0 = \sum_{\vec{n}} \frac{27 h_t^2 h_{\vec{n}}^2}{(2\pi)^6} \sqrt{\frac{10}{g_*}} \frac{10N+3}{22N+13} \sim 10^{-3} h^2 \frac{N_{\rm KK}}{\sqrt{g_*}}, \tag{21}$$

where each emitted KK mode contributes an amount $\sim 10^{-4} h_{\vec{n}}^2$ to the sum, so that the exponent in (20) has the numerical value $\sim \sum_{\vec{n}} h_{\vec{n}}^2 10^{12}(1 - T_c/T_{B-L})$ for $T_c \sim 100$ GeV. This analysis explicitly demonstrates how a primordial $B - L$ is damped as the temperature falls from T_{B-L} to T_c. In conclusion, the underlying brane model must accommodate Yukawa couplings, which are either sufficiently large so that the reverse process (restoring the lepton asymmetry above T_c) are operative or so small so that both forward and reverse processes are frozen out in order to prevent the primordial $B-L$ from being washed out. The latter condition is clearly more severe when the number of accessible KK modes is increased. This conclusion is relevant only when the hierarchy among the neutrino masses is generated by the zero modes $f_R^{\ell\vec{0}}(\vec{y}_0)$.

2.2. Hierarchy from the 5D Yukawa couplings

A second possibility for having realistic masses and mixings for neutrinos comes via the flavor structure of the Yukawa matrix $\lambda^{\alpha\ell}$. Namely, while the zero mode wave functions $f_R^{\ell\vec{0}}(\vec{y}_0)$ are of similar size, the entries of $\lambda^{\alpha\ell}$ may possess the requisite hierarchy in the 5D Yukawa couplings to explain the data. When the neutrino masses obey a hierarchical splitting, $m_3 \approx \delta m_{\text{atm}} \gg m_2 \approx \delta m_{\text{sol}} \gg m_1$, so does the Yukawa matrix, $(h_{\vec{n}} h_{\vec{n}}^\dagger)_{33} \gg (h_{\vec{n}} h_{\vec{n}}^\dagger)_{22} \gg (h_{\vec{n}} h_{\vec{n}}^\dagger)_{11}$. In this case, we must consider separately the three forward and reverse processes. For each generation the ratio of the two is still N_{KK}. Unless all three generations, are either completely in (or out) of equilibrium, a prior asymmetry can be regenerated. Consequently, even if the heavier neutrinos can equilibrate with the Higgs boson, $\mu_{\nu_\mu} = \mu_{\nu_\tau} = -\mu_H$, the primordial asymmetry accumulated in ν_e survives the sphaleron reprocessing. Indeed, at temperatures $T \sim T_c$, there remains a non-vanishing $B-L \propto \mu_{\nu_e} + \mu_H$ which sources the observed BAU. This is precisely the scenario developed in [25] in which an individual lepton flavor asymmetry can be responsible for the BAU so long as one or two families are in (out) of equilibrium *even if* the total $B-L=0$.

In order to obtain the analogous constraint to Eq. (17) due to the absence of any effective lepton number violation one should replace h with h_1, i.e., the smallest Yukawa coupling. In this case, all three forward and reverse process will be faster than H. Similarly, the analogous constraint to Eq. (18) due to freezing out all of the interactions involving ν_R is obtained by replacing h with h_3. If we maintain that $h_1^2 \approx 10^{-5} h_3^2$, then the bounds on h_1 can be read from Fig. 2 by shifting down the upper boundary by five orders of magnitude.

3. Specific brane models

Independent of the mechanism for generating the hierarchy of the neutrino masses, the survival of the primordial $B-L$ from erasure by right-handed neutrino emission can only be determined only after the KK mass spectrum $m_{\vec{n}}$ and Yukawa spectrum $h_{\vec{n}}$ for the bulk leptons are specified. Thus in a given model in which the evolution of the universe can be tracked up to temperatures above T_c, the fate of the $B-L$ asymmetry can be determined from Eq. (12). The discussion in the last section is valid for any brane construction in which the SM spectrum is localized. It is therefore convenient to discuss the phenomenon of $(B-L)$-violation in specific brane models and show how $B-L$ evolves in the presence of right-handed neutrino emission from the brane.

3.1. Large extra dimensions

In higher-dimensional theories with flat and compact extra dimensions, a solution to the hierarchy problem requires the extra dimensions to be large [9], and therefore they drastically change the early history of the Universe. In the absence of a mechanism which stabilizes the large extra dimensions, the evolution of the universe may not be standard much above the nucleosynthesis era [26]. Therefore, temperatures above the MeV scale may not be accessible, and the generation or erasure of the BAU cannot be discussed in the context of these models. This is because unless the temperature of the Universe rises to some T_{brane} above T_c, the processes we are describing do not occur. We note that the mechanism described in [27], in which leptogenesis occurs at temperatures much below T_c overestimates the final baryon asymmetry by a very large factor. There the asymmetry was estimated as being proportional to $e^{-T_c/T_{\text{brane}}}$, and has neglected the factor of $\sim 8\pi/g_W$ in the exponent [5]. Alternatively, low scale leptogenesis is possible in orbifold GUT models [28].

3.2. Warped extra dimensions

Another higher-dimensional scenario which can solve the hierarchy problem is due to [10] where two branes, the Planck and visible (TeV) branes, are immersed in the AdS_5 bulk at respective positions $y = 0$ and $y = \pi r_c$. The extra dimension y is parametrized as $y = \pi\phi$ with $-\pi \leqslant \phi \leqslant \pi$, and the points (x, ϕ) and $(x, -\phi)$ are identified. Here r_c is the radius of the S^1/Z_2 orbifold, and it determines the size of the extra dimension. For generating the hierarchy, $M_{\rm Pl} e^{-\pi k r_c} \equiv$ TeV, one needs $r_c k \sim 10$ where k is the AdS curvature. Suppression of the higher order curvature effects in the gravity sector requires k to be smaller than the fundamental scale of gravity, $M_5 \approx (M_{\rm Pl}^2 k)^{1/3}$. Typically $k \lesssim \eta M_5$ where we have introduced the parametrization η, satisfying $0 \leqslant \eta < 1$.

Unlike large extra dimensions, the warped geometries allow for reheat temperatures $\sim \mathcal{O}$ (TeV) which is the characteristic energy scale of the visible brane, and we will see that the number of accessible KK states is $N_{\rm KK} \sim \eta^{-3/2}$. At very high temperatures, $T \gg \mathcal{O}$ (TeV), the visible brane simply does not exist as it is either shielded by the AdS horizon or pushed away from its true configuration. As the temperature drops the visible brane with a hot SM spectrum emerges at $T_{\rm brane} \sim \mathcal{O}$ (TeV) [29]. In particular, for the RS1 setup, standard evolution remains valid up to corrections of $\mathcal{O}(\rho^2)$ (see, e.g., [30]). More generally for warped extra dimensions we will distinguish two possibilities for the localization of SM fields.

3.2.1. Matter on the TeV-brane

Here, we place all SM fields on the TeV-brane, as was the case in the original RS1 model, and introduce right-handed neutrinos in the bulk [14]. A bulk fermion $\Psi^\ell(x, y)$, with Dirac mass term $\zeta_\ell k\, \text{sgn}(\phi)\overline{\Psi}^\ell \Psi^\ell$, is a Dirac spinor obeying the chiral decomposition in (1). The orbifold symmetry requires $f_L^{\ell n}(\phi)$ to be odd (even) and $f_R^{\ell n}(\phi)$ even (odd) under the Z_2 symmetry. Therefore, to generate neutrino masses on the TeV brane, the appropriate boundary conditions are

$$f_L^{\ell n}(0) = f_L^{\ell n}(\pi) = 0, \tag{22}$$

such that $f_R^{\ell 0}(\phi)$ is localized on the Planck brane with a tiny tail on the visible brane [14]. The Yukawa couplings are given by

$$f_R^{\ell 0}(\pi) = \sqrt{2\zeta_\ell - 1}\, e^{-(\zeta_\ell - 1/2)\pi k r_c},$$
$$f_R^{\ell n \neq 0}(\pi) = \sqrt{2}, \tag{23}$$

which indicates that the couplings of the zero modes are at least fifteen orders of magnitude smaller than those of the KK levels provided that $\zeta_\ell > 1/2$. Namely, the bulk fermions $\Psi^\ell(x, y)$ must have a mass parameter larger than half the AdS curvature scale in order to obtain small numbers to generate neutrino masses. On the other hand, the masses of the KK levels obey

$$m_{\ell n} = x_{\ell n} k e^{-\pi k r_c}, \tag{24}$$

where $J_{\zeta_\ell - 1/2}(x_{\ell n}) = 0$ [14]. For arbitrary η, one finds that $m_{\ell n} \approx x_{\ell n} \eta^{3/2}$ TeV which gives $m_1 \sim M_Z$ when $\eta \approx 0.1$. In general, the smaller the value of k, the smaller $m_{\ell n}$ becomes and the number of KK states excited at TeV temperatures increases; however, it is convenient to choose $\eta \sim 0.1$ as a moderate value in order to suppress the AdS curvature with respect to M_5. In order for the coupling of the zero modes (3) to approximate the existing oscillation data, the mixing with the higher modes (4) must be small, and this can be satisfied if the entries of $\lambda^{\alpha\ell} \lesssim 10^{-2}$, or in other words, $(h_n h_n^\dagger)_{\alpha\alpha} \lesssim 10^{-4}$.

Returning to the analysis of generic brane models in Section 2, we can now discuss the possible $B - L$ erasure in the RS1 model. When the neutrino textures are generated by $f_R^{\ell 0}$ in (23), we see from Fig. 2 that the entire $B - L$ asymmetry is preserved for $\eta \sim 0.1$. This is because the experimental bound [15] for suppressing $\tan\theta_\alpha$ in (4), $(h_n h_n^\dagger)_{\alpha\alpha} \sim 10^{-4}$, is roughly eight orders of magnitude higher than the bound obtained from Fig. 2, for $N_{\rm KK} \sim \eta^{-3/2} \simeq 30$.

On the other hand, as mentioned in Section 2.2, when all $f_R^{\ell n}$ are similar in size but the entries of $\lambda^{\alpha\ell}$ possess a hierarchy to generate neutrino masses, it is possible to suppress $(h_n h_n^\dagger)_{\alpha\alpha}$ for the lightest flavor for all n. Then not only is $B - L$ preserved, even if $(B - L)_{\rm total} = 0$, a baryon asymmetry will be generated if the flavor asymmetries do not all vanish [25] as would be the case for Yukawa couplings in the small window for $N_{\rm KK} \sim 30$. From Fig. 2 we see that the window becomes larger and more than one flavor symmetry could be washed out if $N_{\rm KK}$ becomes very large. Eventually for large enough $N_{\rm KK}$, all three generations are washed away, and the final baryon asymmetry is erased.

In discussing the erasure of $B - L$ we have left unspecified the origin of the primordial asymmetry. The visible brane has the natural scale $\mathcal{O}$(TeV) so that it becomes more difficult to induce leptogenesis with genuinely four-dimensional mechanisms with heavy Majorana neutrino masses. However, since the sphaleron constraint only imposes a bound on h^2/M, it is possible by tuning the Yukawa textures to achieve leptogenesis with lighter, $\mathcal{O}$(TeV), neutrino masses [31].

3.2.2. Matter on the Planck-brane

An alternative possibility is to assume that matter is localized on the Planck-brane. In this scenario supersymmetry is now required to protect the Higgs mass, and supersymmetry is broken on the TeV-brane. The warp factor is responsible for naturally explaining the TeV supersymmetry breaking scale [32]. In this set-up the appropriate boundary conditions are

$$f_R^{\ell n}(0) = f_R^{\ell n}(\pi) = 0, \tag{25}$$

where $f_L^{\ell 0}(\phi)$ is even under the orbifold symmetry and $f_L^{\ell 0}(\phi = 0)$ generates the Yukawa couplings (2) with $\psi_R^{\ell n}$ is replaced by $\psi_L^{\ell n c}$, c for charge conjugation. The KK mass spectrum has the same form as (24) with $x_{\ell n}$ being the roots of $J_{\zeta_\ell + 1/2}(x)$ instead. For $\zeta_\ell \geqslant -1/2$ the brane and bulk leptons are weakly coupled

$$\begin{aligned} f_L^{\ell 0}(0) &= \sqrt{1+2\zeta_\ell}\, e^{-(\zeta_\ell+1/2)\pi k r_c}, \\ f_L^{\ell n\neq 0}(0) &= \frac{\sqrt{2}}{\Gamma(\zeta_\ell+1/2)}\left(\frac{x_{\ell n}}{2}\right)^{\zeta_\ell - 1/2} \\ &\quad \times \frac{e^{-(\zeta_\ell+1/2)\pi k r_c}}{J_{\zeta - 1/2}(x_{\ell n})}, \end{aligned} \tag{26}$$

for both the zero modes and higher KK states. Suppose that at least the first KK level is sufficiently light: $m_1 \sim M_Z$ as was computed in the RS1 case. The smallness of the zero modes couplings in (26) can be used to obtain a good fit to the neutrino oscillation data [15] with small mixing with the higher KK levels. Then, assuming that the standard FRW evolution holds up to temperatures $T_{B-L} \sim$ TeV, one observes that the primordial $B - L$ asymmetry is not washed out. This is clear from (12) where

$$\sum_{\vec n} \left(h_{\vec n} h^\dagger_{\vec n}\right)_{\alpha\alpha} \sim \lambda^{\alpha\,\ell} e^{-(2\zeta_\ell+1)\pi k r_c} \lambda^{\dagger\,\ell\,\alpha}, \tag{27}$$

is much smaller than 10^{-12}, and so even the small window at $N_{\mathrm{KK}} \sim 30$ for Yukawa couplings $h^2 \sim 10^{-12}$ is not a problem. Therefore, by localizing matter on the Planck-brane, and keeping the TeV-brane at a finite distance, it is possible to naturally generate neutrino masses and mixings with no erasure of the primordial asymmetry.

When $r_c \to \infty$, one recovers the original RS2 limit [33], in which case the cosmological evolution depends on a new scale $M_c \equiv \sqrt{M_{\mathrm{Pl}} k}$, and it is only for temperatures $T \lesssim M_c$ that the standard FRW cosmology is recovered. For higher temperatures, the evolution does not have a FRW form and $\mathcal{O}(\rho^2)$ contributions [34] are important. For $M_c = 10^{12}$ GeV, one has $M_5 = 10^{14}$ GeV and $k = 10^6$ GeV. On the other hand, M_c can be lowered down to 1 TeV level for which $M_5 = 10^8$ GeV and $k \sim m_\nu$ [35].

The form of the Yukawa couplings (26) shows that the Dirac mass of the corresponding neutrino vanishes for infinitely separated branes. Hence the massive left-handed neutrinos as well as their mixings cannot follow from brane-bulk couplings in RS2 scheme. However, the vanishing of the Yukawa couplings as $r_c \to \infty$ does not mean that the $B - L$ damping rate (12) vanishes. Indeed, at large r_c, many KK modes become kinematically accessible and they enhance the emission rate cumulatively as is the case for gravitons [34,35]. In fact, using (26) one obtains

$$\sum_{\vec n} \left(h_{\vec n} h^\dagger_{\vec n}\right)_{\alpha\alpha} \sim \lambda^{\alpha\,\ell} \left(\frac{M_c}{2k}\right)^{2\zeta_\ell+1} \lambda^{\dagger\,\ell\,\alpha}, \tag{28}$$

at $T \sim M_c$ on the brane. When M_c is at the intermediate scale, $M_c \sim 10^{12}$ GeV, this quantity is approximately $10^{6(2\zeta_\ell+1)}$, which implies that, for $\zeta_\ell \sim \mathcal{O}(1)$, $\lambda^{\alpha\,\ell} \lesssim 10^{-15}$ for the primordial $B - L$ in $\mathbf{L}^\alpha$ to remain in tact. However, when $\zeta_\ell \to -1/2$, the cumulative effect is mild, and it suffices to choose $\lambda^{\alpha\,\ell} \lesssim 10^{-6}$ in order to preserve the primordial $B - L$. Thus, even though bulk fermions are not responsible for generating the masses and mixings of the massive neutrinos, their emission from the brane leads to a wash-out of the primordial asymmetry.

The fact that energy scales much greater than $\mathcal{O}$(TeV) are accessible on the RS2 brane enables one to invoke genuine four-dimensional mechanisms to generate neutrino masses and mixings as well as the lepton asymmetry in a natural way. Indeed, conventional Majorana leptogenesis in which the SM spectrum is augmented by three brane-localized, right-handed neutrinos $\mathbf{N}^\alpha$ can be used [6]. If the $\mathbf{N}^\alpha$ have masses at the intermediate scale $M_N \sim M_c \sim 10^{10}$ GeV then $B - L$ is not erased and a finite BAU is generated so long as the emission of right-handed neutrinos from the brane is suppressed. Furthermore we note that in compact hyperbolic extra dimensions where standard FRW evolution continues up to high temperatures, the weak scale KK spectrum is similar to that in RS1, and any primordial $B - L$ will be erased unless the brane-bulk couplings are tuned accordingly [36].

4. Conclusion

This Letter has been devoted to a discussion of $B - L$ violation induced by the emission of right-handed neutrinos from the brane. The main statement of the analysis is that unless the ν_R production and its reverse process are *both* fast or slow compared with the expansion rate at $T_c \sim 100$ GeV, the $B - L$ asymmetry of the brane can be washed out by the emission of right-handed neutrinos. A symmetry principle which can halt the neutrino emission from the brane could come through promotion of the global $B - L$ to a local invariance. However, this symmetry must be broken at an intermediate scale, and therefore, it may not be generically realizable in general brane models.

Among the brane world examples, it is not possible to discuss the preservation of a primordial asymmetry in the large extra dimensions since the temperature cannot be much above the MeV scale. In the RS geometry with matter localized on the TeV-brane, the bulk leptons are responsible for generating the neutrino masses and mixings. If the bulk field wavefunctions generate the neutrino mass hierarchy, then $B - L$ erasure only occurs for very large N_{KK}. However, when the neutrino mass splittings come from the brane-bulk couplings, then the lepton asymmetry in the lightest (or massless) flavor can remain preserved. Instead, if matter is localized on the Planck brane, we have the unique feature of generating both the neutrino masses and the primordial $B - L$ via the usual leptogenesis mechanism. The $B - L$ asymmetry generated this way is not washed out via the emission of the bulk leptons from the brane because the brane-bulk couplings can naturally be made exceedingly small.

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References

[1] A.D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32, JETP Lett. 5 (1967) 24.
[2] K. Kajantie, M. Laine, K. Rummukainen, M.E. Shaposhnikov, Phys. Rev. Lett. 77 (1996) 2887, hep-ph/9605288.
[3] J.M. Cline, M. Joyce, K. Kainulainen, JHEP 0007 (2000) 018, hep-ph/0006119;
M. Carena, M. Quiros, M. Seco, C.E. Wagner, hep-ph/0208043.
[4] V. Kuzmin, V. Rubakov, M. Shaposhnikov, Phys. Lett. B 155 (1985) 36.
[5] P. Arnold, L. McLerran, Phys. Rev. D 36 (1987) 581;
P. Arnold, L. McLerran, Phys. Rev. D 37 (1988) 1020.
[6] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[7] M. Fukugita, T. Yanagida, Phys. Rev. D 42 (1990) 1285;
B.A. Campbell, S. Davidson, J.R. Ellis, K.A. Olive, Phys. Lett. B 256 (1991) 457;
B.A. Campbell, S. Davidson, J.R. Ellis, K.A. Olive, Astropart. Phys. 1 (1992) 77.
[8] L. Maiani, in: Proceedings of the 1979 Gif-sur-Yvette Summer School On Particle Physics, 1;
G. 't Hooft, in: G. 't Hooft, et al. (Eds.), Recent Developments in Gauge Theories, Proceedings of the Nato Advanced Study Institute, Cargese, 1979, Plenum, New York, 1980;
E. Witten, Phys. Lett. B 105 (1981) 267.
[9] I. Antoniadis, Phys. Lett. B 246 (1990) 377;
N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429 (1998) 263, hep-ph/9803315;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 436 (1998) 257, hep-ph/9804398.
[10] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.

[11] J. Lykken, L. Randall, JHEP 0006 (2000) 014, hep-th/9908076;
J. Cline, C. Grojean, G. Servant, Phys. Lett. B 472 (2000) 302, hep-ph/9909496;
A.G. Cohen, D.B. Kaplan, Phys. Lett. B 470 (1999) 52, hep-th/9910132;
Z. Chacko, A.E. Nelson, Phys. Rev. D 62 (2000) 085006, hep-th/9912186;
N. Arkani-Hamed, L.J. Hall, D.R. Smith, N. Weiner, Phys. Rev. D 62 (2000) 105002, hep-ph/9912453;
P. Kanti, K.A. Olive, M. Pospelov, Phys. Rev. D 62 (2000) 126004, hep-ph/0005146.
[12] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B 557 (1999) 25, hep-ph/9811428;
N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, J. March-Russell, Phys. Rev. D 65 (2002) 024032, hep-ph/9811448.
[13] G.R. Dvali, G. Gabadadze, Phys. Lett. B 460 (1999) 47, hep-ph/9904221.
[14] Y. Grossman, M. Neubert, Phys. Lett. B 474 (2000) 361, hep-ph/9912408.
[15] H. Davoudiasl, P. Langacker, M. Perelstein, Phys. Rev. D 65 (2002) 105015, hep-ph/0201128.
[16] A.E. Faraggi, M. Pospelov, Phys. Lett. B 458 (1999) 237, hep-ph/9901299;
G.R. Dvali, A.Y. Smirnov, Nucl. Phys. B 563 (1999) 63, hep-ph/9904211;
A. Ioannisian, A. Pilaftsis, Phys. Rev. D 62 (2000) 066001, hep-ph/9907522;
R.N. Mohapatra, A. Perez-Lorenzana, Nucl. Phys. B 593 (2001) 451, hep-ph/0006278;
A. Lukas, P. Ramond, A. Romanino, G.G. Ross, JHEP 0104 (2001) 010, hep-ph/0011295;
A. De Gouvea, G.F. Giudice, A. Strumia, K. Tobe, Nucl. Phys. B 623 (2002) 395, hep-ph/0107156.
[17] Q.R. Ahmad, et al., SNO Collaboration, Phys. Rev. Lett. 87 (2001) 071301, nucl-ex/0106015;
S. Fukuda, et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 86 (2001) 5651, hep-ex/0103032.
[18] S. Fukuda, et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 85 (2000) 3999, hep-ex/0009001;
M. Ambrosio, et al., MACRO Collaboration, Phys. Lett. B 517 (2001) 59, hep-ex/0106049.
[19] K.A. Olive, D.N. Schramm, G. Steigman, Nucl. Phys. B 180 (1981) 497;
K.A. Olive, M.S. Turner, Phys. Rev. D 25 (1982) 213.
[20] J.A. Harvey, M.S. Turner, Phys. Rev. D 42 (1990) 3344.
[21] J.M. Cline, K. Kainulainen, K.A. Olive, Phys. Rev. D 49 (1994) 6394, hep-ph/9401208.
[22] B.A. Campbell, S. Davidson, K.A. Olive, Nucl. Phys. B 399 (1993) 111, hep-ph/9302223.
[23] L.J. Hall, D.R. Smith, Phys. Rev. D 60 (1999) 085008, hep-ph/9904267.
[24] E. Kiritsis, G. Kofinas, N. Tetradis, T.N. Tomaras, V. Zarikas, hep-th/0207060.
[25] B.A. Campbell, S. Davidson, J.R. Ellis, K.A. Olive, Phys. Lett. B 297 (1992) 118.
[26] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Rev. D 59 (1999) 086004, hep-ph/9807344.
[27] A. Pilaftsis, Phys. Rev. D 60 (1999) 105023, hep-ph/9906265.
[28] A. Hebecker, J. March-Russell, T. Yanagida, hep-ph/0208249.
[29] N. Arkani-Hamed, M. Porrati, L. Randall, JHEP 0108 (2001) 017, hep-th/0012148;
P. Creminelli, A. Nicolis, R. Rattazzi, JHEP 0203 (2002) 051, hep-th/0107141.
[30] J.M. Cline, J. Vinet, JHEP 0202 (2002) 042, hep-th/0201041.
[31] M.A. Luty, Phys. Rev. D 45 (1992) 455.
[32] T. Gherghetta, A. Pomarol, Nucl. Phys. B 586 (2000) 141, hep-ph/0003129;
T. Gherghetta, A. Pomarol, Nucl. Phys. B 602 (2001) 3, hep-ph/0012378.
[33] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690, hep-th/9906064.
[34] A. Hebecker, J. March-Russell, Nucl. Phys. B 608 (2001) 375, hep-ph/0103214;
S.S. Gubser, Phys. Rev. D 63 (2001) 084017, hep-th/9912001.
[35] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, N. Kaloper, Phys. Rev. Lett. 84 (2000) 586, hep-th/9907209;
S.B. Giddings, E. Katz, J. Math. Phys. 42 (2001) 3082, hep-th/0009176;
D.J. Chung, L.L. Everett, H. Davoudiasl, Phys. Rev. D 64 (2001) 065002, hep-ph/0010103.
[36] N. Kaloper, J. March-Russell, G.D. Starkman, M. Trodden, Phys. Rev. Lett. 85 (2000) 928, hep-ph/0002001;
G.D. Starkman, D. Stojkovic, M. Trodden, Phys. Rev. Lett. 87 (2001) 231303, hep-th/0106143;
D.A. Demir, M. Shifman, Phys. Rev. D 65 (2002) 104002, hep-ph/0112090.